DYNAMIC ANALYSIS OF FUEL ELEMENTS IN BOILING WATER REACTOR

Y. SASASI, Y. SASAKI, H. NIWA

Tokyo Shibaura Electric Company, Ltd., Research and Development Center, Komukai Toshiba-cho 1, Kawasaki City, Japan

SUMMARY

Antiseismic design of Boiling Water Reactor core is one of the most important parts of BWR nuclear power plant design. The range of the natural frequency of the fuel assemblies is similar to that of the natural frequency of the reactor building and the predominant frequency of the earthquake. We are afraid that if BWR nuclear power plant undergoes a great earthquake, it will receive considerable damage. So we must investigate the antiseismic analysis of the reactor core seriously.

It is very difficult to investigate the dynamic behavior of the reactor core, because the core consists of many fuel assemblies and is in water. The difficulty mainly depends upon the fact that the reactor core is not only considered as the multidegrees of freedom system, but also the interaction between the water and the fuel assemblies must also be taken into consideration.

Fuel assemblies are allocated in the circular shape shroud in actual design. We replace the circular form of the shroud and the core arrangement as the square form. Assuming the flow of the water is two-dimensional, we have obtained the fluid force on the fuel assemblies by applying potential theory. The dynamic equations of the fuel assemblies are obtained by using these external hydraulic forces. If the core consists of $n \times n$ fuel assemblies, we must make $2n \times n$ equations and these can be represented by a matrix equation. We have solved these equations numerically and have obtained some results.

On the other hand, we have planned a core model for testing in which the number of fuel assembly models can be changed and which we can vibrate using our testing machine. A tank in which 21 fuel assembly models on the scale of about a quarter are set in a rectangular lattice shape is placed on the shaking table and vibrated. These fuel assemblies have multi-peak response curve and three main peaks are observed. They move in two-dimensional way because of the coupling effect of water. Moreover, they move in the same phace and their response is maximum at the most predominant peak. As for the natural frequencies, the mode shapes and the dynamic response of these assemblies, the experimental results are well coincident with the theoretical one. We are convinced that the dynamic characteristics of fuel assemblies in the core can be calculated by the approximate theory and that the assumption that the fuel assemblies in the core move in the same phase is reasonable.

1. Introduction

The anti-seismic design of core fuel assemblies is one of the most important parts of the BWR nuclear power plant design. The range of natural frequencies of the fuel assemblies is similar to that of the natural frequency of the reactor building and the predominant frequency of the earthquake. The damage of the fuel assembly may lead the radio-active accident, so we must investigate the anti-seismic analysis of the reactor core seriously.

Because the core consists of many fuel assemblies and is in water, it is very difficult to investigate the dynamic behavior of the reactor core. This difficulty mainly depends upon the fact that the reactor core is not only considered as the multidegrees of freedom system, but the interaction between the water and the fuel assemblies must also be taken into consideration. In the case of a small number of fuel assemblies, however, we can calculate the dynamic behavior of the core theoretically on the assumption that the fluid has two dimensional potential flow, and that the fuel assemblies are rigid beams supported by springs at the both ends and move two dimensionally in water.

We have prepared 21 fuel assembly models on the scale of about a quarter, and set them in a rectangular lattice shape in the tank, and carried out the vibration tests using the high power vibration testing machine. The theoretical results coincide with the experimental results. We are convinced that the dynamic characteristics of fuel assemblies in the core can be calculated by the approximate theory and that the assumption that the fuel assemblies in the core move in the same phase is reasonable.

2. Experiment

2-1 Experimental apparatus

We have planned a core model for testing in which the number of fuel assembly models can be changed and which we can vibrate using our testing machine. We have also planned the core model on the basis of the following analysis. We have analyzed the core model on the assumption that the fuel assemblies can be regarded as one elastic beam supported at the both ends, and that the fluid around the fuel assemblies has two dimensional potential flow. The fundamental dynamic equation of the fuel assemblies can be obtained on these assumptions,

$$\left[\rho A + \pi \rho_F r_1^2 \times \frac{\left(\frac{r_2}{r_1}\right)^2 + 1}{\left(\frac{r_2}{r_1}\right)^2 - 1}\right] \ddot{Y} + EI \frac{\partial^4 Y}{\partial x^4} = -(\rho A - \rho_F A) \frac{\partial^2 \eta}{\partial t^2} \qquad (1)$$

where Y: the relative displacement between the tank and fuel assemblies

ρ: the density of fuel

ρ_F: the density of water

A: the cross sectional area of fuel assemblies

r_1 : the radius of the core
r_2 : the radius of the tank

The natural frequency of fuel is influenced by the virtual mass,

$$\pi \rho_F r_1^2 \times \frac{\left(\frac{r_2}{r_1}\right)^2+1}{\left(\frac{r_2}{r_1}\right)^2-1} \tag{2}$$

and also the external force in water is influenced by the fluid participation factor,

$$\frac{\rho A - \rho_F A}{\rho A + \pi \rho_F r_1^2 \times \frac{(r_2/r_1)^2+1}{(r_2/r_1)^2-1}} \tag{3}$$

From the above equations we can see that the natural frequency of the fuel models decreases according to the increment of the virtual mass, and that the external force in water is influenced by the virtual mass and the displaced mass of water. The fundamental test plan is as follows,

(i) To increase the number of the fuel assembly models as far as possible.

(ii) To increase the virtual mass in order to clear the difference between the natural frequency in air and that in water. For this purpose, the gaps between the tank wall and the core are decreased as possible.

(iii) To increase the difference between the density of fuel and that of water in order to increase the external force.

(iv) To keep the natural frequency of the tank considerably higher than that of the fuel assembly models.

Based on this fundamental plan, we have decided that the scale of the fuel assembly model is about 1/4, and set them in a rectangular lattice shape in the tank. The cross section of the tank and the details of this model fuel assembly are shown in Figs. 1 and 2.

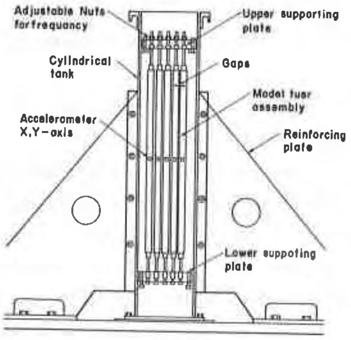

Fig. 1 Experimental model

The fuel assembly models supported flexibly on the plenum plates at the both ends. And their natural frequencies are adjustable in some range.

To treat the reactor core as two dimensional theoretical model, we planned to prepare the wide spaces above and below the fuel models, and filled the spaces with water. These spaces are provided so that the flow of the water may be two dimensional in this experimental apparatus.

2-2 Experiment

The tank in which 21 fuel assembly

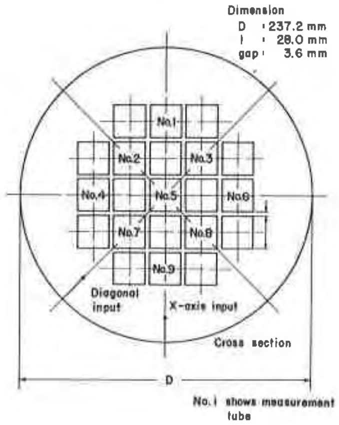

Fig. 2 Cross section of experimental apparatus

models are set is placed on the shaking table and excited. The input signals were supplied by a sweep oscillator and the acceleration level was kept constant during the sweep test. The frequency of the oscillation were swept up and down automatically from 10 to 50 Hz. The sweep rate was 0.02 Hz/sec to keep the peak value of the acceleration more than 90% of the real response peak. The acceleration responses in the middle point fuel assemblies were recorded by the X-Y recorder. Their abscissa shows the exciting frequency and the coordinate shows the acceleration in the middle point of the fuel assemblies. the measurements were mainly carried out using the force balance-type acceleration pick-ups. Two pick-ups were installed to the inside of fuel models in the middle points of them so that X and Y direction can be measured simultaneously. The number of instrumented fuel assemblies are 9 out of 21 and they are located symmetrically as shown in Fig. 2.

2.3 Experimental results

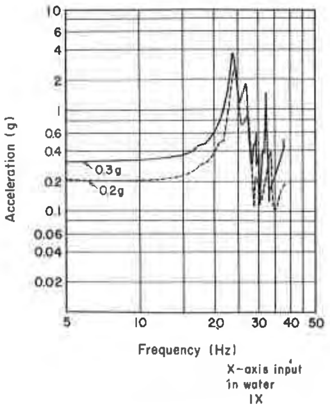

Fig. 3 Response curves of fuel model

One of the typical example of response curve from the vibration tests is shown in Fig. 3. Fig. 3 shows the response curves of the center element in the X-axis direction excitation. The input acceleration levels are 20%G and 30%G. The natural frequency of the single element is about 38 Hz in air and even for the assemblied models there are no other peaks in the range from 10 Hz to 50 Hz, therefore the authors judged that they do not interact each other.

However in water, as you can see in Fig. 3, they have multi-peak response curve. The three main peaks corresponding to 24, 27 and 31 Hz are observed. These response curves of 9 measuring model fuel assemblies are all coincident with each other. As we will show you later, the elements move in two dimensional way, because of the coupling effect of water.

The vibration mode shapes corresponding to the main peak of 24 Hz is shown in Fig. 4. The acceleration signals of X and Y directions on the

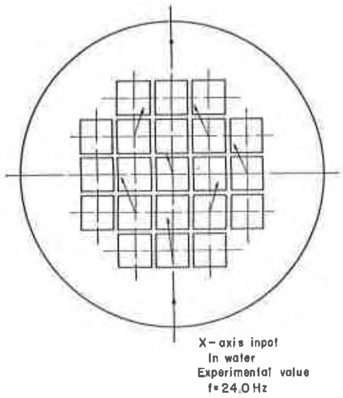

Fig. 4 Mode shape of 21 fuel elements

oscilloscope make a Lissajou's diagram. And they are mainly straight at the main peaks. Although at 24 Hz all fuel assembly models move in the same direction, at 27 Hz only one fuel at the center moves in the opposite direction of all other fuel assembly models, and this is true in theoretical result. At 30.3 Hz the direction of vibration of each element is not symmetric but makes some pattern. The study on four elements makes it easier to understand the above. These mode shapes appears to be due to a pumping action as the water is forced through the gaps between the vibrating fuel elements. Distribution of these natural frequencies and mode shapes coincide with the theoretical results.

3. Theoretical analysis

The objective of the core model vibration testing is to obrain the dynamic behavior of the core having so many fuel assemblies as the real reactor core. For this purpose, we have fabricated the experimental apparatus which has 21 fuel assembly models and carried out the vibration tests. We have tried to consider the mathematical models of the system and solved them.

3-1 General view of the theory

As the fuel assemblies are alocated in the circular shape shroud, in actual design, the gaps between shroud and the core are irregular form. It is very difficult to analyze the motion of the fluid or water in such model. So we replace the circular form of the shroud and the core arrangement as the square form. And we tried to keep both areas of the shroud and the core to be equal by this transformation form circle to square. The error caused by this replacement might be negligible small when the gaps are narrow. These are shown in Figs. 5-8. It is assumed that the flow velocity distribution between the gaps is uniform.

This assumption holds approximately well because of the narrow gaps and the small vibration amplitude. To derive the dynamic equation of each element, the hydrodynamic force should be calculated. In Figs. 5-8 X and Y are absolute coordinates, and the motion of the relative coordinate is x and y. Passages of flow form grid, and we calculate the flow balance at each node. The mesh point is nominated by (i,j). Total number of elements is n x n. Equation (4) is the fundamental equation of the flow.

$$\left.\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial P}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{\partial P}{\partial y} \end{aligned}\right\} \qquad (4)$$

Here all variables are assumed to be small and only the terms of variation.

The fluid forces per unit length F_x, F_y on each element are obtained from integrating this pressure variation over the surface of the element as equations (5).

$$\left.\begin{aligned} F_x &= -\int_c P\cos\theta\, ds = -\int_c P\, dy \\ F_y &= -\int_c P\sin\theta\, ds = -\int_c P\, dx \end{aligned}\right\} \qquad (5)$$

where ds: infinitely short length

θ: angle between the direction of the external force and the normal direction to the surface

A: the cross sectional area of the fuel models

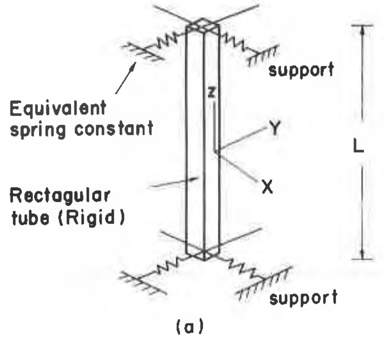

Fig. 5 Fuel element

Along the small passage between mesh points (i,j) and (i+1,j) we introduce coordinate ξ, as shown in Fig. 6, and then the $P(\xi)$ between the gaps can be described as equation (6) where $\ddot{h}$ is the second delivative of the amount of gap.

$$P(\xi) = \frac{l-\xi}{l}P(0) + \frac{\xi}{l}P(l) - \frac{\rho\xi(l-\xi)}{2\bar{h}}\ddot{h} \qquad (6)$$

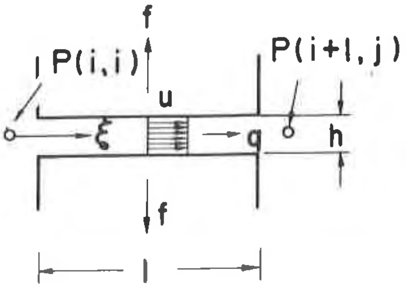

Fig. 6 Flow pattern in gap

Therefore the force acting on the element on the element particular in y direction is shown by equation (7)

$$F = \int_0^l P(\xi)d\xi = \frac{l}{2}[P(0)+P(l)] - \frac{\rho l^3}{12\bar{h}}\ddot{h} \qquad (7)$$

where h: the gaps between the fuel assembly models.

$\bar{h}$: the average of the gap

At the arbitrary mesh point the flow balance is shown by equation (8), where q is flow rate in the gap between (i,j) and (i',J').

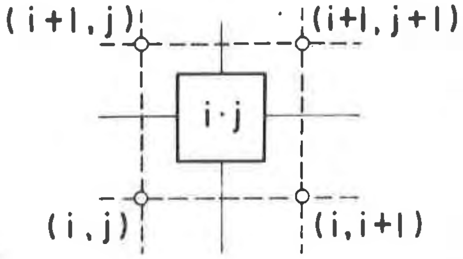

Fig. 7 Arrangement of mesh points

$$\sum_{(i',j')} q_{i,j;i',j'} = 0 \qquad (8)$$

Then the relation of the mesh point (i,j) is as follows. Equation (9) is for $P(\xi)$ and force along the gap between (i,j) and (i',j') is shown by equation (10).

$$\sum_{(i',j')}[P_{i,j} - P_{i',j'}] = -\frac{\rho l^2}{2\bar{h}}\sum_{(i',j')}\ddot{h}_{i,j;i',j'} \qquad (9)$$

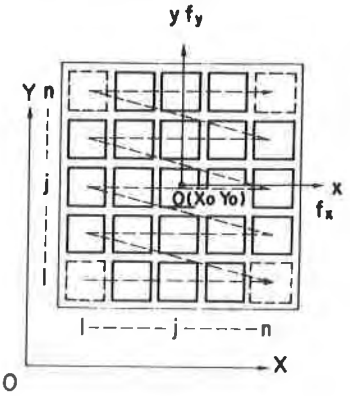

Fig. 8 Disposition of fuel elements

$$F_{i,j;i',j'} = \frac{1}{2}\left[P_{i,j} + P_{i',j'}\right] - \frac{\rho l^3}{12 h}\ddot{h}_{i,j;i',j'} \qquad (10)$$

Referring to Fig. 7, the force acting on the arbitrary element to x-direction $F_{x,i,j}$ is obtained by subtracting the force $F_{i,j+1;i+1,j+1}$ along the gap between (i, j+1) and (i+1, j+1) from the force $F_{i,j;i+1,j}$ along the gap between (i,j) and (i+1,j). The force acting on the element to y-direction $F_{y,i,j}$ is obtained by subtracting the force $F_{i+1,j;i+1,j+1}$ along the gap between (i+1,j) and (i+1,j+1) from the force $F_{i,j;i,j+1}$ along the gap between (i,j) and (i, j+1).

The forces F_x and F_y only represent hydraulic force, therefore absolute forces are shown by equation (11).

$$\left.\begin{aligned} F'_x &= F_{x,i,j} + \rho A \ddot{X}_0 \\ F'_y &= F_{y,i,j} + \rho A \ddot{Y}_0 \end{aligned}\right\} \qquad (11)$$

The dynamic equations of the element are obtained by using the these external hydraulic forces F'_x, F'_y.

$$\left.\begin{aligned} m_{i,j} L[\ddot{x}_{i,j} + \ddot{X}_0] + k_{x,i,j} \cdot x_{i,j} &= F'_{x,i,j} \cdot L \\ m_{i,j} L[\ddot{y}_{i,j} + \ddot{Y}_0] + k_{y,i,j} \cdot y_{i,j} &= F'_{y,i,j} \cdot L \end{aligned}\right\} \qquad (12)$$

where $m_{i,j}$ is unit mass of each element and $k_{x,i,j}$ and $k_{y,i,j}$ is overall stiffness of each direction. Then the gap is assumed to be shown by eq. (13), which is two dimensional model.

$$\left.\begin{aligned} h_{i,j;i,j+1} &= y_{i,j} - y_{i-1,j} \\ h_{i,j;i+1,j} &= x_{i,j} - x_{i,j-1} \end{aligned}\right\} \qquad (13)$$

Introducing the matrix notation, equations (9),(10) and (12) are represented as follows,

$$\left.\begin{aligned} &[M](\{\ddot{x}\}+\{\ddot{X}_0\})+[K]\{x\}=\{F\} \\ &\{F\}=[A][P]-[B]\{\ddot{x}\} \\ &[C][P]=-[D]\{\ddot{x}\} \end{aligned}\right\} \qquad (14)$$

where $\{F\}$ is an external hydraulic force vector.

From the equation (14) we eliminate the force $\{F\}$ and pressure $[P]$, taking care of the coefficient matrix on $[P]$ not to be singular. $([M]-[M_D])$ is effective mass of element for external excitation and $[M_A]$ is virtual mass of fluid.

$$\left.\begin{aligned} &([M]+[M_A])\{\ddot{x}\}+[K]\{x\} = -([M]-[M_D])\{\ddot{X}_0\} \\ &[M_A]=[A][C]^{-1}[D]+[B] \quad , \quad [M_D]=\rho A \end{aligned}\right\} \qquad (15)$$

This equation (15) is the fundamental equation to describe the motion of fuel assemblies in the core, and the form of eq. (15) does not change if the number of fuel assemblies become several hundreds and the arrangement of them be square. The coefficient matrices ($[M]+[M_A]$) and $[K]$ are symmetric matrices when the number of the fuel assemblies is N^2. Let the right hand term of equation (15) be set to zero, then the natural frequencies and the modes are calculated. Transforming the coordinate X to the normal coordinate by the canonical transformation $\{X\}=[U]\{\xi\}$, we obtain the following normal equations like equation (16) for each vibration mode,

$$\ddot{\xi}_i + \omega_i^2 \xi_i = -\frac{m - m_D}{m + m_A}\left\{ \ddot{X}_0 \sum_{j=1}^{N\times N} u_{i,j} + \ddot{Y}_0 \sum_{j=N\times N+1}^{2\times N\times N} u_{i,j} \right\} \tag{16}$$

where ω_i is the circular frequency of the mode, $U_{i,j}$ is coefficient of ith mode transformation of each element. Participations factor $\sum_{j=1}^{n} \phi_{i,j}$ is defined as follows

$$\left.\begin{aligned} \sum_{j=1}^{n} \phi_{i,j} &= \frac{m - m_D}{m + m_A} \sum_{j=1}^{N\times N} u_{i,j} \\ \sum_{j=1}^{n} \phi_{i,j} &= \frac{m - m_D}{m + m_A} \sum_{j=N\times N+1}^{2\times N\times N} u_{i,j} \end{aligned}\right\} \tag{17}$$

where the upper equation is participation factor in X-direction and the lower one is that in Y-direction.

3-2 Theoretical results on the core model

In order to understand the result of experiment of 21 fuel assembly models, we tried to calculate 5 x 5 lattice case.

As shown in Fig. 8, there is no fuel assembly at each corner. It is very difficult to analyze such complicated configuration of core theoretically. So we filled these four blanks with imaginary fuel assembly model and treated them as 5 x 5 square.

The distribution of natural frequencies and the participation factors are shown in Table 1. The mode shapes corresponding to the 5th, 8th and 9th coincide with the experiments and the participation factors of these mode shapes are larger than the others.

Order	Theoretical value (Hz)	$\sum_{i=1}^{n}\phi_{ij}$	Experimental value (Hz)	Notes
1	15.90	0.0136		
2	17.03	0.0212		
3	18.64	0.0229		
4	20.95	0.0043		
5	24.33	4.602	24.0	Max. Resp. Acc
6	25.06	0.263		
7	26.26	0.433		
8	27.03	1.095	27.0	
9	30.44	1.174	30.3	
10	31.02	0.953		
11	34.43	0.236		
12	37.00	0.153		

Table 1 Natural frequencies & participation factors

The mode shape of the largest participation factors are shown in Fig. 9. The analytical results of the mode shape show that at 24 Hz all fuel assembly models move in the same phase, at 27 Hz only one fuel placed at the center of the core moves to the opposite direction of all the other fuel assembly models, and at 31 Hz the direction of vibrated fuel assembly models does not seem to move regularly. This is quite similar to that obtained by the experiment.

By assumed value of damping co-effi-

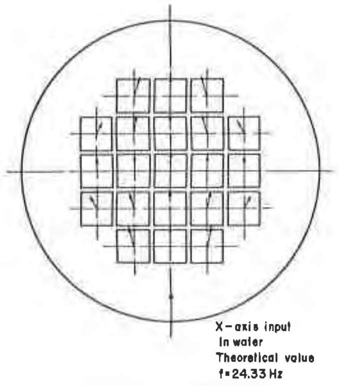

Fig. 9 Theoretical mode shape

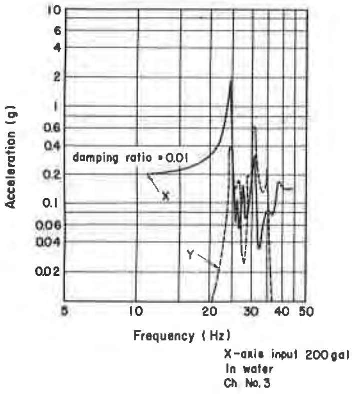

Fig. 10 Theoretical response curve

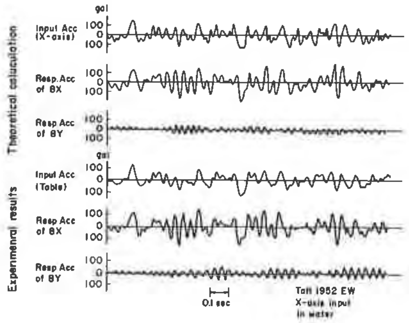

Fig. 11 Reponses of fuel elements to earthquake wave

cient, we tried to fit the theoretical response curve to that by experiment. When we chose the value of 2% damping as the modal damping, it fit well. Some of the results are shown in Fig. 10. They considerably coincide with the experimental results. Of course, we can observe the response of Y-direction also as shown by the dotted line. The experimental responses to the earthquake wave are shown in Fig. 11. The theoretical responses to the earthquakes which are obtained by means of the modal analysis using 2% damping ratio are also shown in Fig. 11. The comparison of the theoretical results with the experimental results shows that the response wave forms and the amplification factors are well coincident with each other.

4. Conclusion

In order to find the way of analysis for design of the reactor core which has many fuel assemblies, we made the testing experimental model to simulate the dynamic behavior of the BWR core and carried out the vibration tests and investigated the response to the external force.

To understand the experimental results, we have developed a theory in order to analyze the fuel motion on the assumption that the core is multi-degree of freedom system which consists of the elastic beams surrounded by water, and that the motion of the water is two dimensional. We have applied this theory to the core model with 21 fuel assembly models, and obtained some results.

(i) The theoretical results show that the group of the fuel assembly models has many normal vibration modes which correspond to its natural frequencies. When the perticipation factor has the largest value, the mode shape shows that all of the fuel assembly models vibrate to the same direction.

(ii) The experiments in water show that the response curve of the fuel assembly models has many peaks. The reason of this phenomenon is understood that the fuel models are coupled by the fluid forces. The response curve in this case has three main peaks, and the vibration mode of the most predominant peak of them is the same phase motion, i.e., all of the fuel assembly models vibrate to the same direction. This result agrees with the theoretical result of mathematical simulation. Therefore single-degree freedom system is applicable to the real reactor core which has several hundreds of fuel assemblies.

We wish to thank Dr. H. Shibata, Prof. of Tokyo Univ., for his kind advice and we also thank gratefully for the kind operatiom of Tokyo Electric Power Co. on this regard.

References

[1] Lamb, H. "Hydrodynamics", Cambridge Press, Sixth edition, 1932.

[2] Shimogo, T. and Inui, T. "Coupled Vibration of Many Elastic Circular Bars in Water" Proceedings of the 21st Japan National Congress for Applied Mechanics, 1971, pp 495-505

[3] S. S. Chen "Dynamic Response of Two Parallel Circular Cylinder in a Liquid" Trans. ASME. Journal of Pressure Vessel Technology, 1975, pp 78-83