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SPECTRUM COMPATIBLE NONSTATIONARY EARTHQUAKE MODEL

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INTRODUCTION

It is standard practice currently to specify seismic inputs in terms of their response spectrum. This spectrum is inverted into a suitable compatible accelerogram for further time history response studies. There are well documented approaches [1,2,3] for generating spectrum compatible accelerograms. The weakness of this approach is the non-uniqueness of the generated accelerogram. Since more than one earthquake-like accelerogram can be generated from a given spectrum, logically it is necessary to use several of them in time history studies. Obviously this becomes an expensive procedure. A viable alternative to this is the generation of a compatible stochastic model which can be used in a random vibration study of the structure under consideration. Compatible stationary models, represented in terms of their power spectral density (PSD) functions are available in the literature [4, 5, 6]. Stationary models are simple to use but are not realistic, since earthquakes are transient in nature. Another drawback of the current compatible PSD generation is the use of an arbitrary peakfactor to relate the standard deviation and the expected maximum response. The SSE spectrum is generally defined as a percentile level, (usually at 84.1%) spectrum. Thus, this information rather than the peakfactor should be reflected in the spectrum compatible models. Spanos and Vargasloli [7] have considered the question of generating a nonstationary spectrum compatible model. Their approach is based on an approximate closed form expression for the response statistics of a SDOF system obtained using the Markov process approach. A multiplicative nonstationary model is selected for matching. The PSD of the stationary part is derived by matching the mean realized spectra with the smooth design response spectrum (SDRS) through an empirical peakfactor. In the present paper, a more rational approach for relating the SDRS on a probabilistic basis with the compatible model is developed. The proposed approach takes into account the percentile level of the specified SDRS also.

NONSTATIONARY MODEL

The ground acceleration $\ddot{u}_g(t)$ is modelled as a modulated stationary process of the form

$$\ddot{u}_g = V(t)\ u(t) \tag{1}$$

Here V(t) is a known deterministic function controlling the build up and decay of the amplitudes. u(t) is a zero mean stationary gaussian process with unknown PSD function. The response of a SDOF system to the above input is considered. The relative displacement and velocity are given as

$$x(t) = \int_0^t \ddot{u}_g(\tau)\ h(t,\tau)d\tau \tag{2}$$

$$\dot{x}(t) = \int_0^t \ddot{u}_g(\tau)\ \dot{h}(t,\tau)d\tau \tag{3}$$

where, $h = \omega_d^{-1}\ e^{-\eta\omega t}\ \sin\omega_d t$

$$\dot{h} = \omega_d^{-1}\ e^{-\eta\omega t}\ (\omega_d\cos\omega_d t - \eta\omega\sin\omega_d t)$$

$$\omega_d = \omega(1-\eta^2)^{1/2} \tag{4}$$

If the PSD of u(t) is $S(\lambda)$, the autocorrelation of the nonstationary input can be written as

$$R(t_1, t_2) = V(t_1)\ V(t_2) \int_0^\infty S(\lambda)\ \cos\lambda\ (t_2-t_1)d\lambda \tag{5}$$

Now, from the theory of random vibration, the variance and correlation of (x, x) are

$$\sigma_x^2 = \int_0^\infty S(\lambda)\ H(\lambda,t)\ d\lambda$$

$$\sigma_{\dot{x}}^2 = \int_0^\infty S(\lambda)\ H_1(\lambda,t)\ d\lambda$$

$$r_{x\dot{x}}(t) = (\sigma_x\ \sigma_{\dot{x}})^{-1} \int_0^\infty S(\lambda)\ H_2(\lambda,\ t)d\lambda \tag{6}$$

where,

$$H(\lambda,\ t) = \int_0^t \int_0^t V(\tau_1)\ V(\tau_2)h(t-\tau_1)\ h(t-\tau_2)\cos\lambda(\tau_2-\tau_1)d\tau_1 d\tau_2$$

$$H_1(\lambda, t) = \int_o^t \int_o^t V(\tau_1)\, V(\tau_2)\dot{h}(t-\tau_1)\dot{h}(t-\tau_2)\cos\lambda(\tau_2-\tau_1)\, d\tau_1 d\tau_2$$

$$H_2(\lambda, t) = \int_o^t \int_o^t V(\tau_1)\, V(\tau_2)h(t-\tau_1)\dot{h}(t-\tau_2)\cos\lambda(\tau_2-\tau_1)\, d\tau_1 d\tau_2 \tag{7}$$

Since the response is a gaussian process, the probability distribution of the highest peak of x(t) in (0, T) is governed by

$$F_{x_p}(\alpha) = \text{Prob}\ (x_p \le \alpha) = \exp[-\int_o^T N(\alpha,t)dt] = p_\alpha \tag{8}$$

Here $N(\alpha, t)$ is the average rate of upcrossings of the level α, given by

$$N(\alpha,t) = (2\Pi)^{-1}\ (\sigma_{\dot{x}}/\sigma_x)\ (1-r^2)^{1/2}\ [\exp(-\mu^2)+\mu r\sqrt{\Pi}\ \exp(-.5\alpha^2/\sigma_x^2)$$
$$\{1+\text{erf}(\mu r)\}]$$
$$\mu^2 = 0.5\alpha^2/[\sigma_x^2(1-r^2)] \tag{9}$$

Here σ_x, $\sigma_{\dot{x}}$ and r are the time varying response moments given by eq. (6). If α_p is the peak relative displacement at a known percentile level p_α, the acceleration spectrum is given by

$$S_a(\omega) = \alpha_p\omega^2 \tag{10}$$

Thus, the problem reduces to one of finding the unknown $S(\lambda)$ from the given spectrum $S_a(\omega)$, such that equation (10) is satisfied.

COMPATIBLE PSD

The unknown PSD is expressed as a linear function between discrete sample points. Thus

$$S(\lambda) = S_i + [(S_{i+1} - S_i)/(\lambda_{i+1}-\lambda_i)](\lambda-\lambda_i);\ \lambda_i \le \lambda \le \lambda_{i+1} \tag{11}$$

The time dependent frequency response functions, H, H_1 and H_2 are to be computed. For an efficient and simple way of evaluating these functions, they are expressed as

$$H(\lambda,t) = [\int_o^t V(\tau)h(t-\tau)\cos\lambda\tau d\tau]^2 + [\int_o^t V(\tau)h(t-\tau)\sin\lambda\ \tau d\tau]^2$$
$$= (Q^2 + R^2) \tag{12}$$

Similarly, $H_1(\lambda,t) = (\dot{Q}^2 + \dot{R}^2)$ and $H_2(\lambda,t) = Q\dot{Q} + R\dot{R}$

Instead of integrating Q, $\dot{Q}$, R, $\dot{R}$ in closed form it is expedient to find them by numerically solving the differential equations

$$\ddot{Q} + 2\eta\omega\,\dot{Q} + \omega^2 Q = V(t)\cos\lambda t$$

$$\ddot{R} + 2\eta\omega\,\dot{R} + \omega^2 R = V(t)\sin\lambda t \tag{13}$$

These are solved using the Runge-Kutta-Gill scheme, with initial conditions $Q(0) = \dot{Q}(0) = R(0) = \dot{R}(0) = 0$.

The response quantities, σ_x, $\sigma_{\dot{x}}$ and r are found at a frequency `ω' in terms of the unknown PSD ordinates S_i ($i = 1,2,...N$). Once these response moments are known, the peak relative displacement α_p with a specified probability of non-excedance p_α is obtained by solving the transcendental eq. (8). This is used to find the corresponding spectral acceleration from eq. (10). An iterative procedure is set up starting with a whitenoise approximation to arrive at the final compatible PSD function $S(\lambda)$.

NUMERICAL EXAMPLE

In fig. 1, the SSE spectrum for which the compatible model is required is shown. The nonstationary modulation is taken as

$$V(t) = A\,(e^{-bt} - e^{-ct})$$

Here (b, c) control the rise and decay and hence the duration of the ground motion. A is a scale factor chosen such that the maximum value of V(t) is unity. To study the effect of changing the duration, the following three sets of parameters are used: $b = 0.5$, $c = 0.1$, corresponding to $T = 10$ sec. ; $b = 0.25$, $c = 0.5$, corresponding to $T = 20$ sec. ; $b = 0.15$, $c = 0.3$, corresponding to $T = 30$ sec. The spectrum is taken as specified at level $p_\alpha = 0.841$. The realized spectrum for $T = 10$ sec. is compared with the specified spectrum in fig. 1. The compatible PSD function $S(\lambda)$ is shown in fig. 2 for three different durations.

CONCLUSION

A method for specifying spectrum compatible nonstationary stochastic earthquakes has been developed. The percentile level of the specified spectrum can be precisely included in the analysis. This leads to seismic inputs consistent with the risk level adopted in the design procedure.

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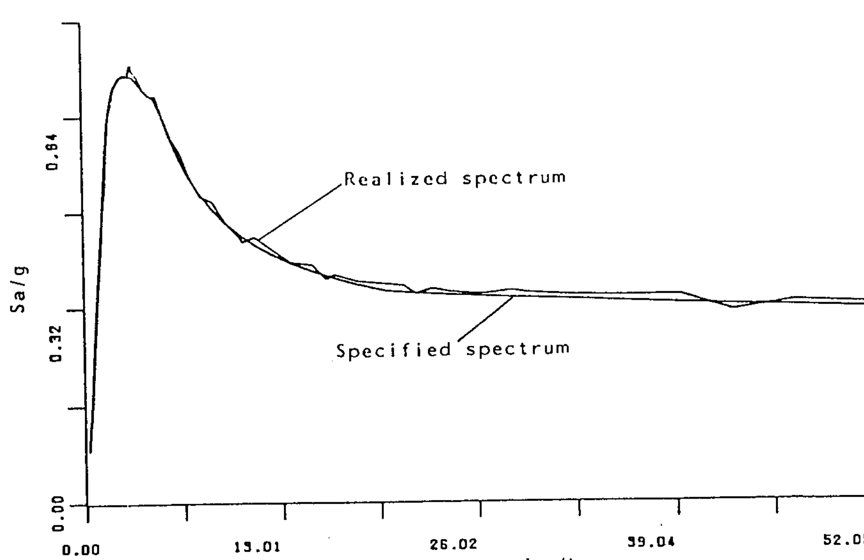

Fig.1a Target and realised spectra.
15.9% exceedance level; T=10Sec.

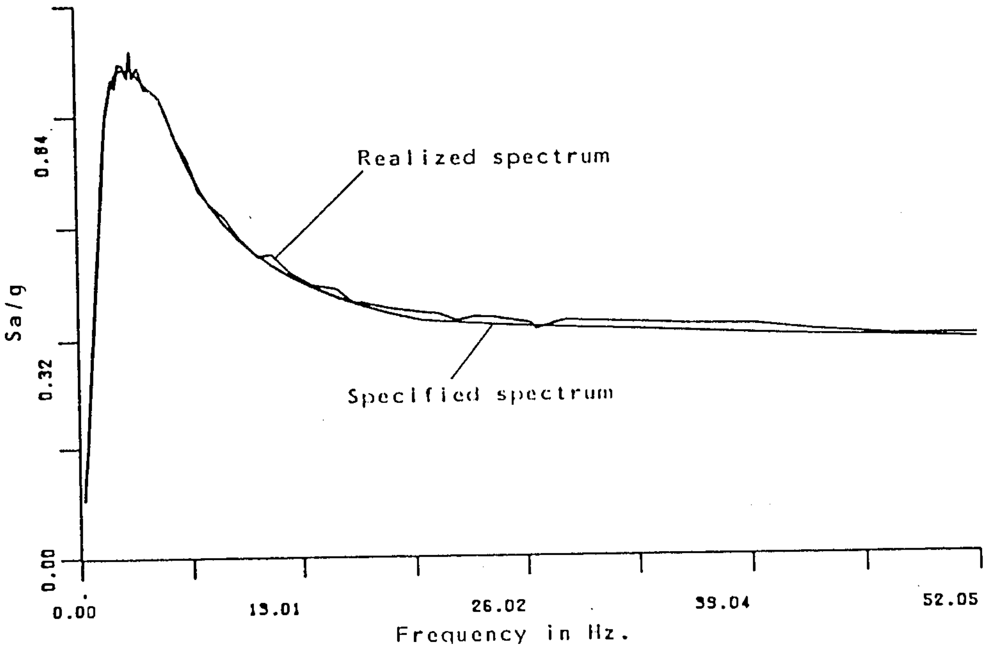

Fig.1b Target and realised spectra.
15.9% exceedance level; T=20Sec.

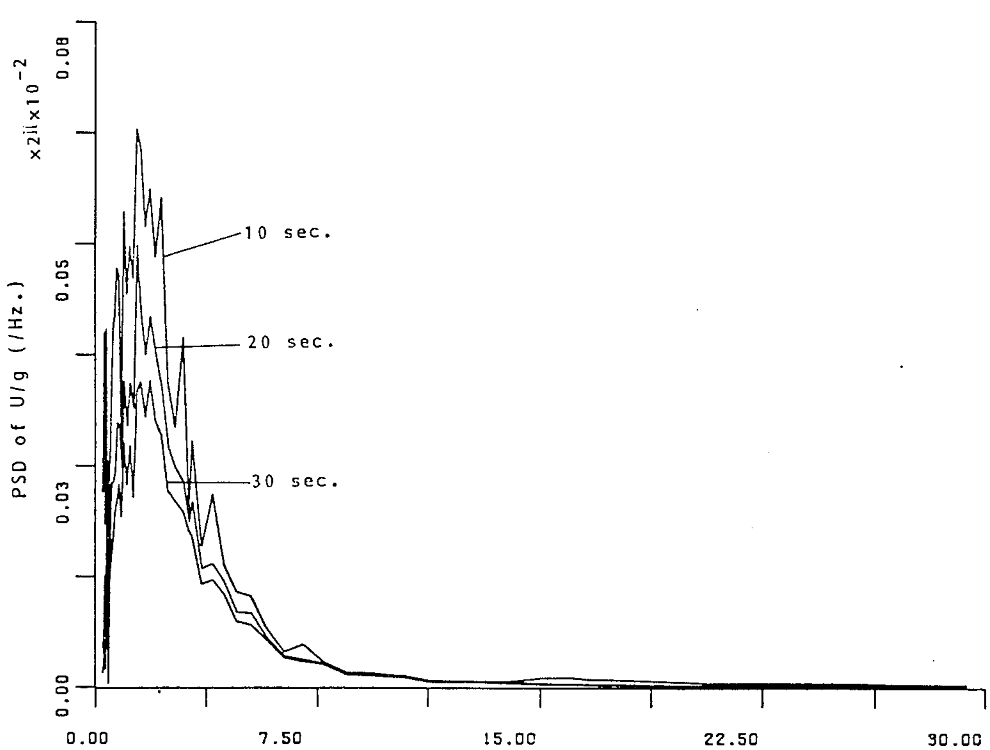

Fig. 2 - Effect of earthquake duration on generated PSD function (15.9% exceedance)