Direct Analysis Method of Probabilistic Fracture Mechanics

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1. INTRODUCTION

For the purpose of evaluating the reliability of nuclear plants with pipes or vessels which assume to have growing cracks, the Monte-Carlo method is commonly used. In the present paper, a direct analysis method which does not depend on the Monte-Carlo method is proposed. The load stress on the walls during the plant lifetime is supposed to be deterministic. This is an essential assumption in the formulation. It is possible to extend to the situation the load stress being generated stochastically, but the present formulation might be a first step and there would be a certain availability in the practical design studies of nuclear plants.

2. FORMULATION

The crack geometry is assumed to be half-elliptic, with crack depth a, and half length b. Let

$$\alpha = \frac{a}{h}, \tag{1}$$

where h is the thickness of the wall, accordingly $0<\alpha\leq 1$. The aspect ratio

$$\gamma = \frac{a}{b} \tag{2}$$

is supposed to be $0<\gamma\leq 1$. The upper bound 1 is not essential from the viewpoint of the fracture mechanics, but as a matter of convenience, $\gamma\leq 1$ is assumed throughout the discussion.

The crack growth is supposed to be caused only by load stress cycles, namely the fatigue crack growth. The creep effect is omitted, but a slight modification makes it be possible to be considered.

Let us consider the process of the vector (α^n, γ^n) with load stress cycles, or discrete time steps $n=0,1,\ldots$. This could be said to be a *stochastic process*, but does not have a real sense of *stochasticity*, as stated above. The random variables α^0 and γ^0 are supposed to be independent, and their distributions are supposed to be prescribed:

$$\begin{cases} F^0(x) = \mathbb{P}(\alpha^0<x) \\ G^0(y) = \mathbb{P}(\gamma^0<y). \end{cases} \tag{3}$$

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The crack grows from (a^n, b^n) to (a^{n+1}, b^{n+1}) according to the fracture mechanics,

$$\begin{cases} a^{n+1} = a^n + C\Big[\Delta K_a(a^n\,,\,b^n)\Big]^\nu \\ \\ b^{n+1} = b^n + C\Big[\Delta K_b(a^n\,,\,b^n)\Big]^\nu \end{cases} \tag{4}$$

in every step n, where ΔK_a and ΔK_b are the stress intensity factors which are functions of the crack geometry. Hence (α^n, γ^n) also grows to $(\alpha^{n+1}, \gamma^{n+1})$. There is dependence between α^{n+1} (or γ^{n+1}) and (α^n, γ^n), but α^{n+1} and γ^{n+1} are independent each other, because of the independence of α^n and γ^n. So we can determine the distribution of the process (α^n, γ^n), in the following manner.

From (4), the following relation is obtained:

$$\begin{cases} \alpha^{n+1} = \phi(\alpha^n\,,\,\gamma^n) \\ \\ \gamma^{n+1} = \psi(\alpha^n\,,\,\gamma^n). \end{cases} \tag{5}$$

Our goal is to determine the distribution functions F^n and G^n of α^n and γ^n respectively. We have

$$F^{n+1}(x) = \mathbb{P}(\alpha^{n+1} < x\,) = \int_0^1 \mathbb{P}(\phi(\alpha^n, y) < x\,)\mathbb{P}(y < \gamma^n < y + dy\,). \tag{6}$$

This is the Chapman-Kolmogorof's equation. As for G^n, the similar equation holds. For fixed y, denote $\phi^{-1}(\,\circ\,, y)$ for the inverse function of $\phi(\,\circ\,, y)$, and for fixed x, $\psi^{-1}(x\,, \circ\,)$ for that of $\psi(x\,, \circ\,)$. Then the following iterative *forward equation* is obtained.

$$\begin{cases} F^{n+1}(x) = \displaystyle\int_0^1 F^n(\phi^{-1}(x\,,\,y))G^n(dy) \\ \\ G^{n+1}(y) = \displaystyle\int_0^1 G^n(\psi^{-1}(x\,,\,y))F^n(dx). \end{cases} \tag{7}$$

3. NUMERICAL SOLUTION

In order to obtain the numerical solution of the equation (7), the both intervals $0 \leq \alpha \leq 1$ and $0 \leq \gamma \leq 1$ are divided into subintervals:

$$\begin{cases} 0 = x_0 \leq x_1 \leq \cdots \leq x_{N_\alpha} = 1 \\ \\ 0 = y_0 \leq y_1 \leq \cdots \leq y_{N_\gamma} = 1. \end{cases} \tag{8}$$

For each point x_i and y_j, the function values F_i^n and G_j^n are obtained for n=0,1,.... The Newton method and the numerical integration are employed in this procedure. The former is used to evaluate values of the two functions $\phi^{-1}(\circ, y_j)$ and $\psi^{-1}(x_i, \circ)$.

1. FAILURE PROBABILITY AND PRE-SERVICE INSPECTION

We have acquired the complete information of the process (α^n, γ^n). Accordingly, related various quantities can be calculated.

The leak probability p^n is

$$p^n = \mathbb{P}(\alpha^n \geq \alpha_c) = 1 - F^n(\alpha_c), \tag{9}$$

where α_c is a threshold value (ratio) of crack depth. The fracture probability q^n is, if using the so-called net section stress criteria,

$$q^n = \mathbb{P}(b^n \geq b_c(a^n)) = \int_0^1 G^n(\gamma_c(x))F^n(dx), \tag{10}$$

where b_c is the threshold value of half crack length, which is a function of crack depth a, and γ_c is the corresponding aspect ratio, which is a function of α. As for the fracture criteria, the J-criteria also could be applicable.

The pre-service inspection is taken into account as follows. Let $P_{ND}(\alpha, \gamma)$ be the probability of not detecting cracks in the inspection procedure. Instead of (3), the distribution functions $F^0(x)$ and $G^0(x)$ are then to be conditional distribution functions $F_+^0(x)$ and $G_+^0(y)$, under the condition that the crack is not detected in the inspection:

$$\begin{cases} F_+^0(x) = \dfrac{\int_0^x F^0(d\xi)\int_0^1 P_{ND}(\xi, \eta)G^0(d\eta)}{\int_0^1 F^0(d\xi)\int_0^1 P_{ND}(\xi, \eta)G^0(d\eta)} \\ \\ G_+^0(y) = \dfrac{\int_0^y G^0(d\eta)\int_0^1 P_{ND}(\xi, \eta)F^0(d\xi)}{\int_0^1 G^0(d\eta)\int_0^1 P_{ND}(\xi, \eta)F^0(d\xi)} . \end{cases} \tag{11}$$

Starting from $F_+^0(x)$ and $G_+^0(y)$, the iterative equation (7) also gives distribution functions $F_+^n(x)$ and $G_+^n(y)$, under the condition of undertaking the pre-service inspection.

In the equations (10) and (11), the numerical integration is employed, as in section 3.

2. NUMERICAL EXAMPLES

Numerical examples on piping reliability problems are given. The following, rather hypothetical conditions are assumed.

- The material: type 304 stainless steel.
- The pipe inside radius and the wall thickness: 292.1mm and h=12.7mm respectively.
- The stress intensity factors K_a and K_b: the Newman-Raju equation is used.

∘ The fatigue crack growth parameters in the equation (4): C=6.3×10^{-10}, ν=3 (the unit of ΔK is kgf/mm$^{3/2}$).

∘ The load stress: the bending and membrane stress both vary from 0 to 18kg/mm^2.

∘ The number of loading cycles: totally 300 steps. Assuming 30 year plant lifetime, 10 steps per year.

∘ The initial distribution of the crack depth ratio α: exponentially distributed with the distribution function

$$F^0(x) = \frac{1 - e^{-\lambda hx}}{1 - e^{-\lambda h}}, \tag{12}$$

where λ=0.5878.

∘ The initial distribution of the aspect ratio γ: log-normally distributed with the distribution function

$$G^0(y) = c \int_0^y \frac{1}{\sqrt{2\pi}St}\exp\left[-\frac{1}{2S^2}\left[\log\frac{t}{\mu}\right]^2\right]dt, \tag{13}$$

where μ=1.336, S=0.538; c is a normalizing factor which corresponds to $G^0(1)$=1.

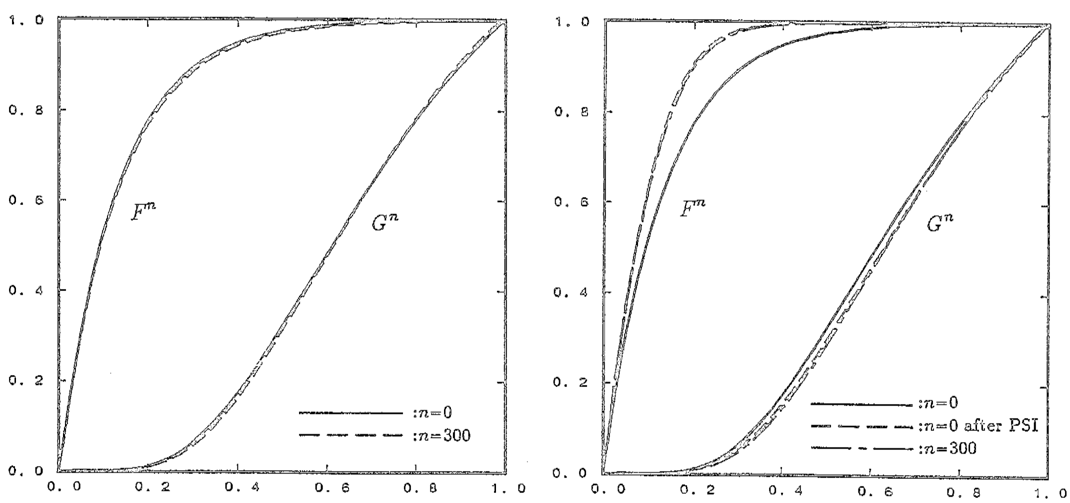
∘ The non detection probability in the pre-service inspection procedure[1]:

$$P_{ND}(A) = 0.4975\int_{1.33\log(A/A^*)}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}dt + 0.005, \tag{14}$$

where A is the crack area, A^*=$(\pi/4)a^*D_B$, a^*=1.27mm, D_B=25.4mm.

∘ The partition of α and γ axes: the interval [0,1] is divided ununiformly into 63 points, for both axes.

Two examples are given. One is the case which is not applied the pre-service inspection and



(a) PSI not applied (b) PSI applied

Fig.1 Variation of distribution functions the other is applied. Fig.1 shows the distribution functions and their variations. Fig.1 (a) shows that the curve of the distribution function of α goes downward according to the repeated load stress, which is intuitively clear. The distribution functions of the aspect ratio γ turns out to cross the original curve, which might depend on the characteristics of the fracture mechanics in the present context. Fig.1 (b) shows that the application of the pre-service inspection drastically improves the distributions as compared to the original ones. The leak and fracture probabilities of the both cases are shown in Fig.2. The inspection lowers the probabilities by about 0.007.

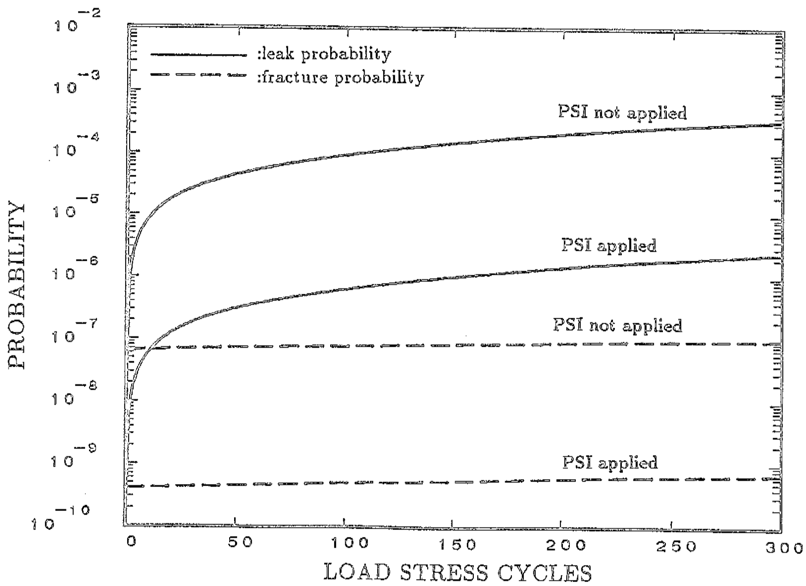


Fig.2 Failure probability as a function of load stress cycles

To perform the calculation, an engineering workstation Sun-4 was used. It took about 80 minutes to execute one case. The execution time depends greatly on the number of partitions of the intervals of both axes (and of course on the number of the load stress cycles). Roughly speaking, it is necessary to execute about

$$2\times(63\times63)\times1.7\times2 \approx 27000$$

times of the lifelong fracture mechanics (300 steps in the present situation), where the first '2' means the necessity of calculating F and G, '63' is the number of the partitions, '1.7' is the average number of repetition of the Newton method and the latter '2' means that one step of the Newton method needs two times calculations of the fracture mechanics. This number 27000 corresponds to the number of samples of the Monte-Carlo method.

The execution time is *not* extremely short, but the obtained probability curve is remarkably smooth, as shown in Fig.2, which could not be attainable by employing the Monte-Carlo method with 27000 samples.

6. CONCLUSION

A direct analysis method of probabilistic fracture mechanics is proposed. By means of this method, the distribution functions of the crack depth and aspect ratio are obtained explicitly at each load stress cycle or time step. In addition, various conceptions of the fracture mechanics could be taken into account in common with the Monte-Carlo method. This method brings forth smooth curves of the failure probability as compared with the Monte-Carlo method, which means calculational accuracy.

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