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ASSESSMENT OF PROGRESSIVE DEFORMATION ON THE BASIS OF ELASTIC ANALYSIS

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INTRODUCTION

The behaviour of structures subjected to cyclic loading is complex. The structure may be in inelastic or plastic shakedown state or exhibit the ratchetting phenomenon. For reasons related to operation (functional play), geometric instability (buckling) and damage, it is important to estimate the maximum deformation reached on the structure when it stabilizes. A proposed solution to this problem is offered by the rule of the efficiency diagram [1] based on a set of experimental results but, in certain cases, this method is impossible or difficult to apply.

In this paper, we propose a general theoretical approach to the efficiency diagram and this will allow us to extend its field of application to cases of structures subjected to null primary loading. For this purpose, we demonstrate that, in certain cases, there is a coupling between primary and secondary loading.

A new definition of primary stress, identified with the former definition in simple cases, is proposed.

Finally, we will apply this method to structures bitubes and shells at free level, under thermomechanical loading and, therefore, generating secondary stresses liable to work in progressive deformation mode.

1 HYPOTHESIS AND NOTATIONS

We consider a solid, S, occupying a certain reference configuration, a compact domain Ω, with non-empty inside and "sufficiently regular" boundary $\partial\Omega$. We assume that this boundary is partitioned into :

$\partial\Omega_u$: section on which variable kinematic conditions in relation to time are imposed,

$\partial\Omega_F$: the complement of $\partial\Omega_u$ (which may be empty). The forces exerted on this section may be considered to be constant with respect to time.

The structures studied are, thus, subjected to two types of loading : a strain or displacement controlled loading which is variable with respect to time, referred to as secondary loading, and force controlled loading which is constant with respect to time, referred to as primary loading.

We use the term primary stress to refer to the stress field balancing the primary loading. This will be noted σp. We use the term secondary stress to refer to the stress field defined by : $\sigma q(t) = \sigma(t) - \sigma p$

$\sigma(t)$ being the actual stress field in the structure at time t. The equivalent stress is defined in the Von-Mises sense : $\sigma^* = ((3/2)\, \sigma : \sigma)^{1/2}$ (: is the contracted product)

Similarly, the equivalent strain is defined by : $\epsilon^* = (3/2)\, \epsilon : \epsilon)^{1/2}$

We will consider the following hypothesis for loading.

Hypothesis 1

We will suppose that the secondary loading is periodic as period T.

The imposed displacement is written : $U_d = U_{d0} + U_{d1} \sin \omega t$

The imposed strain is written : $\epsilon_d = \epsilon_{l0} + \epsilon_{l1} \sin \omega t$

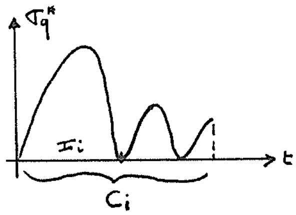
Hypothesis 2

The primary loading must remain fairly low in order to avoid plastifying the structure.

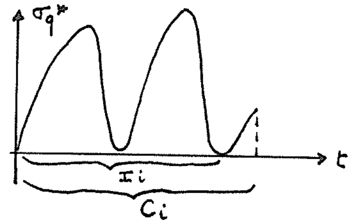
In general and, at least, for the first cycles, the response in σq is not periodic.

Let C_i be the i^{th} cycle of loading and let $I_i \subset C_i$ be the interval where the structure is in the plastic domain.

I_i is of the following form :



or



On I_i we model the variation of secondary stress by means of : $\sigma q(t) = \sigma q_{0i} + \sigma q_{1i} \cos(\omega t + \psi_i)$

2 ENERGY PRINCIPLE

After applying the primary loading, at each cycle I_i the volume receives a quantity of energy :

$$W_i = \int_V \int_{I_i} \sigma : \dot{\epsilon} \, dvdt \qquad (1)$$

$$<==> \qquad W_i = \int_V \int_{I_i} \sigma q : \dot{\epsilon} \, dvdt + \int_V \int_{I_i} \sigma p : \dot{\epsilon} \, dvdt$$

$$<==> \qquad W_i = \int_V \int_{I_i} \sigma q_{1i} : \dot{\epsilon} \cos(\omega t + \psi_i) \, dvdt + \int_V \int_{I_i} (\sigma p + \sigma q_{0i}) : \dot{\epsilon} \, dvdt$$

$$<==> \qquad W_i = \int_V \int_{I_i} \sigma q_{1i} : \dot{\epsilon} \cos(\omega t + \psi_i) \, dvdt + \int_V (\sigma p + \sigma q_{0i}) . \delta\epsilon_{Pi} \, dv$$

with $$\delta\epsilon_{Pi} = \int_{I_i} \dot{\epsilon} \, dt$$

2.1 Interpretation and analysis

We can therefore state that : $W_i = Q_i + W_{Pri}$ (2). Let N be the cycle number :

$$(3) \quad W_T = \sum_{i=1}^{N} W_i = \sum_{i=1}^{N} Q_i + \sum_{i=1}^{N} W_{Pri} = Q_T + W_{Pr}$$

Definition 1

As long as term W_{Pri} is not null, we say that the structure shows progressive deformation. A plastic strain on increment appears at each loading cycle.

Definition 2

The term Q_i is identifiable with the plastic energy obtained during a closed elastoplastic cycle.
The stress amplitude of this cycle can be either increased (cyclic hardening) or decreased (cyclic softening).

2.2 Study of the various types of possible variation

Let Ns be the cycle number at the point of stabilization.
a) After a finite number of cycles Ns we have : $\forall\, i \geq Ns \; W_{Pri} = 0 <=> W_i = Q_i$
- If $Q_i = W_i = 0$ the structure is in an elastic shakedown state
- If $Q_i = W_i = W_S$ the structure is in plastic shakedown state, W_S being the energy dissipated at each cycle. In this case, a fatigue phenomenon can occur.

b) If Ns is infinite
- if $\lim_{i \to \infty} W_{Pri} = 0$ the structure is then either in elastic or plastic shakedown state in an infinite number of cycles
- if $\lim_{i \to \infty} W_{Pri} \neq 0$ we then observe the ratchetting phenomenon. A strain increment appears at each cycle until rupture occurs

2.3 Development

Hypothesis 3

If a stabilized state is reached, there then exists an intrinsic relation between $\Delta\sigma^*$ and $\Delta\epsilon^*$. This relation, which is formed via the cyclic curve, is independent of the primary loading and of the mean plastic strain. This hypothesis means that, if the structure is stabilized, then the term σq_{1i}^* tends towards $1/2\ \Delta\sigma^*$.
In what follows, we will identify the energy, W_i, with the energy dissipated by the structure subjected to the same secondary loading as the structure studied but not showing progressive deformation.
Example : for a tube under tensile/twisting load (see Figure 2), energy W_i is identified with the energy dissipated by the structure in the absence of tensile force (under twisting with imposed angle only).

$$W_i = \int_V \int_{I_i} \sigma' q_i : \dot{\epsilon}\ dt\ dv$$

Hypothesis 3 and the definition of W_i bring us to restrict the field of application of this theory to materials presenting cyclic hardening. Indeed, in this case, the convergence of $\Delta\sigma' q^*$ leads to the convergence of $\Delta\sigma q^*$ as we have : $\Delta\sigma q^* \leq \Delta\sigma' q^* \leq \Delta\sigma^*$

Hypothesis 4

Let σq^{el} be the secondary stress calculated elastically and let ϵq^{el} be the corresponding elastic strain.
We will assume that the functions ξ and ξ' exist so that : $\sigma' q_{1i} = \xi'_i\, \sigma q_1^{el}$ and $\sigma q_{1i} = \xi_i\, \sigma q_1^{el}$
Using these various hypothesis, the different terms of equation (3) are written as follows :

$$Q_i = \int_V \int_{I_i} \sigma q_{1i} : \dot{\epsilon} \cos(\omega t + \psi_i)\ dtdv = \int_V \int_{I_i} \sigma q_{1i} : \dot{\epsilon}^{el} \cos(\omega t + \psi_i)\ dtdv$$

(σq_{1i} being an auto-equilibrated stress field) whence :

$$\sum_{i=1}^{Ns} Q_i = \sum_{i=1}^{Ns} \int_V \int_{C_i} \omega . \sigma q_{1i} : \epsilon q_1^{el} \cos(\omega t + \psi_i) \cos \omega t\ dtdv$$

and thus $Q_T = \sum_{i=1}^{Ns} \int_V \pi \cos\psi_i\, \xi_i\, (\sigma q_1^{el} : \epsilon q_1^{el}) dv$

For W_T we have $W_T = \sum_{i=1}^{Ns} \int_V \pi \cos\psi_i\, \xi'_i\, (\sigma q_1^{el} : \epsilon q_1^{el})\ dv$

For W_{Pr} we have $W_{Pr} = \sum_{i=1}^{Ns} \int_V P_i\, \delta\epsilon_{pi}^*\ dv$ With $P_i = (\sigma p + \sigma q_{oi})^*$

Finally equation (3) is written as follows :

$$(5)\quad \sum_{i=1}^{Ns} \int_V \pi\, (\xi'_i - \xi_i) \cos\psi_i\, (\sigma q_1^{el} : \epsilon_1^{el})\ dv = \sum_{i=1}^{Ns} \int_V P_i\, \delta\epsilon_{pi}^* dv$$

Remark :In order to cancel the first member of this equation, it is sufficient that $\xi_i = \xi'_i$
Two cases may then occur :

- $\cos\psi_i \neq 0$ which corresponds to plastic shakedown
- $\cos\psi_i = 0$ which corresponds to elastic shakedown with the stresses and strains being in phase.

Ignoring the elastic energy due to the variation of the volume of the structure, we obtain:

$$(5) \Longrightarrow (6)\quad \sum_{i=1}^{Ns} \int_V (1/6).(1+\nu)\, \pi(\xi' i - \xi i) \cos\psi i\, \Delta Q^2/E\ dv = \sum_{i=1}^{Ns} \int_V P_i\ \delta\epsilon_{pi}^*\ dv$$

With $\Delta Q = 2.\sigma q_1^{el*}$.

3 SIMPLIFIED METHOD

Hypothesis 5

We will assume that the energy density is uniform in the areas presenting progressive deformation.
This means that we can write equation (6) in a local manner as follows :

$$(7)\quad K\ \Delta Q^2 = \sum_{i=1}^{Ns} P_i\, \delta\epsilon p_i^*$$

Hypothesis 6

The value of P_i is assumed to be constant. This hypothesis is conservative as the mean secondary stress working in the strain increment can, in general, only decrease.

We finally obtain the following equation :

$$K\,\Delta Q^2 = P \sum_{i=1}^{Ns} \delta\epsilon_{pi}^{\ *} = P\,\delta\epsilon \quad (8)$$: P is referred to as a modified primary stress

Definition 3

Effective primary stress

The stress-strain law can be expressed in the following form : $\epsilon = k\sigma^{n(\epsilon)}$ in a given strain interval ($n(\epsilon)$ is constant in that interval).Let ϵ_T be the mean total strain around which the stabilized cycle varies :

The effective primary stress will, by definition, be a stress such that :$\epsilon_T = k\,Peff^{neff}$ (9)

with $\epsilon_T = (P/E) + \delta\epsilon$ (10) , neff thus depends on P and $\delta\epsilon$.

With equation (8), (9) and (10), we obtain:

(11) $1 + K\,E\,SR^2 = (E/P)\,k\,Peff^{neff}$, with $SR = \Delta Q/P$. SR is referred to as the secondary ratio.

And then: $1 + K\,E\,SR^2 = 1 / v^{neff}$.

Where $v = P/Peff$ and $k.P^{neff} \simeq P/E$, v is referred to as the coefficient of efficiency.

Identification of E.K and neff

We will seek a pair (EK_0, $neff_0$) which is independent of the loading and the characteristics of the material so as to obtain : $V = 1 / (1 + E.K_0.SR^2)^{1/neffo}$

Any experimental point characterized by (v,SR) will be such that $v > V \Longrightarrow Peff = P/V > Peff_{exp}$

$Peff_{exp}$ is the effective primary value which can be determined experimentally on the basis of a behaviour law and the total deformation reached in the stabilized state). This ineaquality will be effectively verified experimentally (see figure 5).

If $E\,K_0\,SR^2 >> 1$ then $V \simeq 1 / (E.K_0\,SR^2)^{1/neffo}$

The current efficiency diagram and Bree diagram give, in the case where $SR \geq 4$:$V = 1 / (SR)^{1/2}$

By identifying the two relations, we find :$E\,K_0 = 1$ and $neff_0 = 4$

V is written in the following form : $\mathbf{V = 1 / (1 + SR^2)^{1/4}}$

The other methods give :

a) Bree method

$0 < SR < 4 : V = 1 / (1 + SR / 4)$; $SR > 4 : V = 1 / (SR)^{1/2}$

b) Efficiency diagram method

$0 < SR < 0.46 :\quad V = 1$; $\quad 0.46 < SR < 4 :\quad V = 1.093 - 0.926.(SR / (1 + SR))^2$;

$SR > 4 :\quad V = 1 / (SR)^{1/2}$

The graphic comparison of these three methods is shown in Figure 1.

4 APPLICATION

4.1 Thin tube under tensile - twisting load [2]

A thin tube is subjected to a tensile force which is constant with respect to time, on the one hand, and a twisting with an imposed angle, on the other hand (Figure 2).

In this context, all the previous hypotheses are immediately verified.

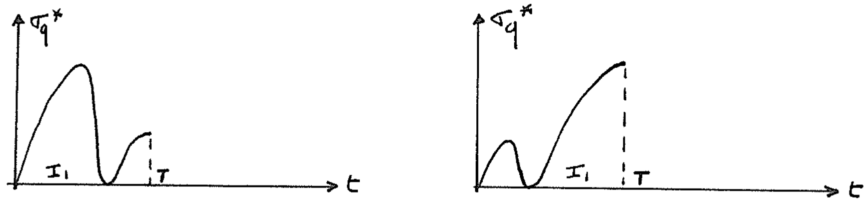
We have : $P = F / S$ and $\Delta Q = 2.\Delta\tau^{el}$

A few experimental results are shown in figure 2 an included in the efficienty diagram (figure 5)

4.2 Bitubes structure [3]

This structure is made up of two coaxial tubes rigidly linked at the top. It is subjected to, on the one hand, an axial force which is constant with respect to time and, on the other hand, cyclic heating of the outer tube with the inner tube remaining cold (see Figure 3). Several types of tube geometry and several types of loading were tested. The secondary stress was obtained by an elastic finite element computation. The modified primary stress P was deduced from an elastoplastic calculation on one cycle.

The variation of σq^* at the centre of the inner and outer tube is as follows :



- Analysis of the inner tube (cold tube): $P = (\sigma p + \sigma_{max}) / 2$

σp being the primary axial stress and σ_{max} being the maximum stress reached under the loading. (Unloading was performed elastically for all the tests).

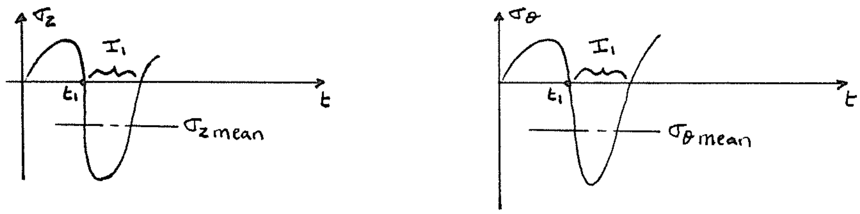
- Analysis of the outer tube : $P = (\sigma p + \sigma_{max}) / 2$

σ_{max} being the maximum stress reached on unloading. (Loading was performed elastically for all the tests).

Remark : The behaviour curve taken to analyze the results will be the cold curve as the strain increment occurs in cold conditions in both tubes. A few experimental results are shown in Figure 3 and included in the efficiency diagram (figure 5).

4.3 Vinil test [4]

This was a test on a shell at free level, submitted to an axial temperature gradient moving along the wall. The secondary stress is obtained by an elastic finite element computation. The modified primary stress was deduced from an elastoplastic calculation on one cycle. The point studied here is located on the outer skin and the variation of stresses for the first cycle is as follows :



Between instant 0 and t1, the point studied remains elastic. We take : $P = \sigma^*_{mean}$ on I_1.

The calculation gives :

$\sigma\theta_{mean} = -37.7$ MPa ; $\sigma z_{mean} = 58$ MPa => P = 83 MPa Furthermore : $\Delta Q = 343$ MPa

Remark : The behaviour curve considered for the analysis will be the "hot curve" (620°C in this case) as the strain increment occurs in the hot condition. The result is shown in Figure 4 and included in the efficiency diagram (figure 5).

CONCLUSION

A method generalizing the field of application of the efficiency diagram was proposed. This new method gives promising results but, even so, requires an elastoplastic calculation on one cycle in order to determine the primary stress in the general case.

The efficiency diagram is shown here as a minimum curve. The identification of parameters EK and neff in relation to the loading and the behaviour curve may be envisaged and would be a means of improving some of the method's predictions which are still too conservative.

REFERENCES

[1] RCC-MR - Règle de conception et de construction des matériels mécaniques des îlots nucléaires, RNR.

[2] Lebey J. and Roche R. "Tests on mechanical behaviour of 304 L stainless steel under constant stress associated with cyclic strain" - Fatigue of Eng. Nat. and struct." Vol. 1 n° 3 - May 1979 - Pages 307-318.

[3] Taleb L. "Structure métallique sous chargement thermomécanique. Effet des surcharges mécaniques de courte durée". Thèse : Génie Civl - Structure INSA - 273 pages.

[4] CABRILLAT M.T, GATT J.M., SKIARA A., SCHOULGUINE P. "Evaluation of thermal ratchetting on axisymmetric thin shells at the free level of sodium. Experimental results and elastic analysis" SMIRT 12 - Stuttgart - August 93 - Paper E05/3.

FIG 1

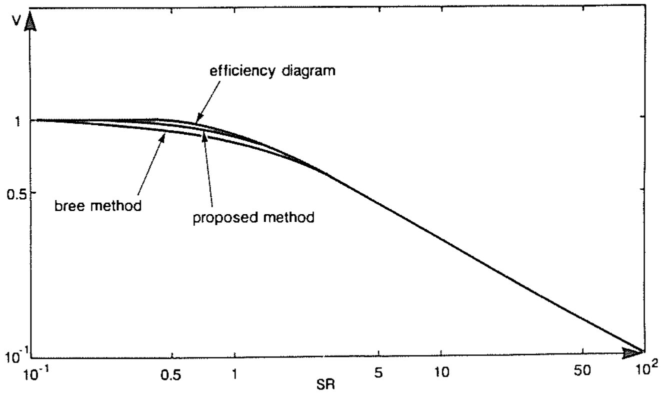


Fig 2 Tensile-Twisting

Test reference	P (MPa)	ΔQ (MPa)	$Peff_{exp}$ (MPa)	SR	V
4	220	1390	353	6.32	0.62
5	220	1390	369	6.32	0.60
6	220	1762	460	8.00	0.48
8	220	483	307	2.19	0.72
9	129	1762	296	13.66	0.44
10	85	483	183	5.68	0.46
11	85	1762	279	20.73	0.30
12	129	483	216	3.74	0.27
13	129	1390	305	10.78	0.42
14	85	1390	253	16.35	0.34

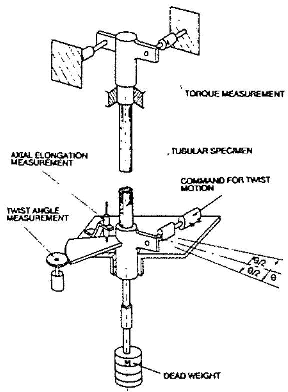


Fig 3 Bitubes

Test reference	P (MPa)	ΔQ (MPa)	$Peff_{exp}$(MPa)	SR	V
11a	212	357	285	1.68	0.74
21a	233	500	345	2.15	0.67
21b	292	500	395	1.71	0.74
31a	244	506	338	2.07	0.72
b11	172	515	260	2.99	0.66
b12	158	453	240	2.87	0.66
b13	172	444	254	2.58	0.68
b141	169	375	245	2.22	0.69
b142	159	459	230	2.89	0.69
b15	176	500	270	2.84	0.65

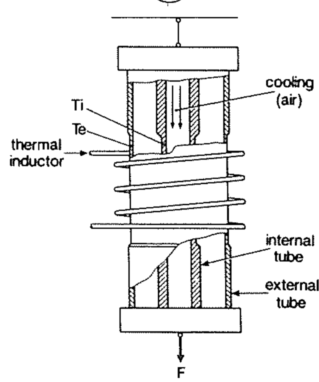


Fig 4 Vinil test

Test reference	P (MPa)	ΔQ (MPa)	$Peff_{exp}$ (MPa)	SR	V
1	83	343	158	4.13	0.52

FIG 5