

Large Rotation Pipe Whip with Flow Choking

N.K. Prinja

*National Nuclear Corporation Ltd., Reactor Engineering, Booths Hall, Chelford Road,
Knutsford, Cheshire WA16 8QZ, U.K.*

N.R. Chitkara

*University of Manchester/UMIST, Mechanical Engineering Dept., Sackville Street,
Manchester M60 1QD, U.K.*

Abstract

Numerical techniques employed to analyse pipe whip movement involving large rotations and large strains are briefly described. The numerical instability (flutter) encountered in using beam type finite elements proposed earlier by Hibbitt [4] are discussed. It is shown that in dynamic analysis the use of non-linear in-elastic elements give more realistic and stable solution as compared to the use of non-linear elastic elements suggested by Hibbitt [4].

1. Introduction

This article describes briefly the use of various numerical techniques employed to analyse a pipe whip movement that involves large rotations and large strains. Pipe whip has been the subject of many studies in the past. However, recent reviews have shown that most of these methods use either small displacement, small strain or thin shell assumptions to simplify the analysis. High energy piping systems in a typical advanced gas cooled reactor power plant are of relatively thick section and due to the sparse layout of the piping and the minimum use of anti-whip devices, there are many postulated rupture cases where large pipe whip movement is feasible. For such cases, small displacement or thin shell assumptions are often questioned. Our studies have shown that in such cases the non-linear properties of the material and geometric non-linearities due to large rotation of the pipe coupled with local collapse of the pipe section affect the fluid flow through the system and thus the blowdown force acting on the whipping pipe. All of these are major factors which can influence the pipe whip motion during a large rotation, and must be taken into account.

2. Numerical Techniques

Numerical solution to the pipe whip problem involving large rotations and large strain is obtained in three stages:

2.1. Pipe section collapse

The first stage of the analysis consists of considering the local plastic collapse behaviour of a pipe section which occurs at a plastic hinge. Once the hinge is formed, since the local inertia effects are not important, the motion is dominated by the gross inertia of the whipping pipe. The behaviour of the plastic hinge may thus be regarded as quasi-static as observed in the experimental work reported by Dupuy et al [1] and Miyazaki et al [2]. A simplified method to analyse quasi-static post collapse behaviour of a pipe section has been reported by the authors [3]. Application of the generalised plane strain finite elements to study collapse of a straight pipe in bending has been discussed in detail by Hibbitt [4]. Prinja et al [5] have also shown good agreement with their experimental work in which deformations at the collapsed section were measured and compared after a large rotation static bend test on initially straight pipes.

2.2. Flow choking due to pipe section collapse

The discharge of the fluid from the whipping pipe eventually becomes controlled by

the cross-sectional flow area at the collapsing pipe section as was observed during the experiments by Baum [6]. A simple numerical procedure was suggested by Prinja et al [5] to determine the reduction in blow down force as a consequence of flow choking and additional pressure losses due to plastic collapse of the pipe section.

The reduction in blow down force was presented in terms of the known pipe deformation and flow area reduction at the throat developed at the plastic hinge.

2.3. Dynamic structure analysis

For dynamic structural analysis of the piping system undergoing large rotation and large strains, normally, the local collapse cannot be computed within the context of structural theory. A recent finite element technique suggested by Hibbitt [4] is where, the local collapse behaviour is accounted for by specifying up to four non-linear elastic section behaviours (two bending modes, axial and torque) for beam type finite elements and suggests that these elements may be employed to obtain rapid and inexpensive evaluation of the pipe whip event involving very large motion of parts of the pipe provided that the loading is essentially monotonic. In practice, our analyses have shown that these elements (given their non-linear elastic behaviour) encounter numerical instability problems when moment reduces due to formation of hinges. Our experience with such numerical instability encountered in using the numerical procedure suggested by Hibbitt [4] for large rotation pipe whip application is discussed in the next section.

3. Some Numerical Problems in Dynamic Analysis

3.1. Non-symmetric stiffness matrix

In finite element solution of non-linear problems usually tangent stiffness matrix for the structure is computed. As discussed by Hibbitt [11], a follower force given by $p(u, \lambda)$ where load p depends on displacement u and magnitude λ contributes to the tangential stiffness as follows:

$$dp = \frac{\partial p}{\partial u} du + \frac{\partial p}{\partial \lambda} d\lambda \quad (1)$$

For a finite element let $k_{\sigma} = \frac{\partial p}{\partial u}$ which is a preload stiffness usually called the 'load stiffness matrix'. Generally, the global load stiffness matrix $[K_{\sigma}]$ (obtained by usual summation of k_{σ}) is symmetric and stresses (σ) contributing to matrix $[K_{\sigma}]$ maintain constant relative magnitude, so that changing one by a factor β changes all others by the same factor and the load stiffness matrix remains symmetric and becomes $\beta [K_{\sigma}]$. Considering virtual displacement (δD) in a static case, the condition to maintain equilibrium leads to the following eigenvalue problem:

$$\begin{aligned} [K] + \beta [K_{\sigma}] (\delta D) &= 0 \\ [K_T] (\delta D) &= 0 \end{aligned} \quad (2)$$

It is well known that if tangential matrix $[K_T]$ is real symmetric, the eigenvalues β will also be all real, but if K_T becomes unsymmetric some eigenvalues may be imaginary and the solution may show oscillations. As shown by Argyris [9] the load stiffness matrix for non-conservative systems do become unsymmetric. In case of a large rotation pipe whip problem as shown in Fig 1, if blowdown force is F and ϕ is the total rotation of end nodal point, the load vector is given as:

$$(p) = (0 \ 0 \ 0 \ \dots \ -F \cos \phi \ F \sin \phi \ 0)$$

Assuming that (p) depends only on the rotation ϕ , the load stiffness matrix is given as:

$$[K_{\sigma}] = F \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 & \sin \phi \\ 0 & 0 & 0 & \dots & 0 & 0 & \cos \phi \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix}$$

which is unsymmetric.

3.2. Numerical instability (flutter)

Break down of the numerical procedure when analysing beam type structures portraying 'falling-moment' hinges was discussed by Wood [7] and Lay [8]. It was shown by them that in an encastered beam with falling moment characteristics, no portion of the beam could make use of the falling moment part of the curve, even when the strains were monotonically increased. Furthermore the structure in our problem is subjected to the follower force i.e. the applied load (blow down force) rotates with rotation of the beam. If the piping system is represented by elastic beams and is subjected to non-conservative (follower) forces, the system may exhibit instability when no non-trivial equilibrium configuration is found in the next increment. This loss of stability is shown in the form of oscillations with increasing amplitudes. One such example is shown in Fig 3(a). Such instability phenomenon is usually termed as flutter. This phenomenon has been discussed in detail by Argyris et al [9 and 10] who have shown that for elastic systems subjected to non-conservative loading, the total tangent stiffness matrix normally used in non-linear finite element analysis becomes non-symmetric and often leads to flutter. Effect of non-symmetry of the stiffness matrix on instability is discussed in the next section.

4. Numerical Analysis

4.1. Non-linear elastic elements

Finite element analyses have been performed to analyse large rotation - large strain pipe whip motion of a piping system shown in Fig 1. The piping system was modelled by beam elements with non-linear elastic beam section behaviour shown in Fig 2. The dynamic analyses were conducted using computer code Abaqus [12] as suggested by Hibbitt [4]. Figures 3 and 7 show bending moment time history of the collapsed element and of the adjacent element. It can be seen that at large rotation the solution becomes unstable and the flutter phenomenon is clearly shown in Figs 3 and 4.

The analysis was repeated by considering the full non-symmetric stiffness matrix. This made only a little difference to the solution.

Solution instability in the numerical solution is much more evident when the bending moment time history of an element adjacent to the collapsed element (hinge) is considered. Figures 3 and 4 show such plots for mesh sizes of 23 and 53 nodes. Although, bending moment at the collapsed element (hinge) does not exhibit any instability (flutter) for any mesh sizes as shown in Fig 5, when the attention is given to their adjacent, elements flutter is clearly shown.

The instability seems to occur earlier and growth in amplitudes is faster for the finer mesh size. This indicates that flutter worsens as the element size is reduced. This rather unexpected phenomenon is akin to 'explosive failure' mentioned by Wood [7].

Physical interpretation of the above phenomenon is provided by inspection of the strain energy densities for both the collapsed and the adjacent element shown in Fig 6 and 7 for a fine (53 noded) and a coarse (17 noded) mesh sizes respectively. It may be seen from Fig 6 and 7 that as the element at the hinge collapses, the surrounding elements unload thus providing a reservoir of strain energy mainly to be dissipated by the collapsed element only. Whilst the strain at the collapsed element increases rapidly the adjacent elements simply unload. The discontinuity in strains increase sharply and for smaller elements the increase in strain discontinuity is even faster.

4.2. Non-linear in-elastic element

To obtain a realistic solution to the pipe whip problem use is made of in-elastic beam elements [12]. In such a case, if an element collapses the surrounding elements will simply unload elastically thereby removing the strain energy from the system by plastic dissipation thus the solution remains stable as shown in Fig 8. This is also borne by the results given in Fig 5 which shows a comparison of the bending moment at the collapsed element for both the elastic and in-elastic type elements for three different mesh sizes used in the analyses.

5. Conclusions

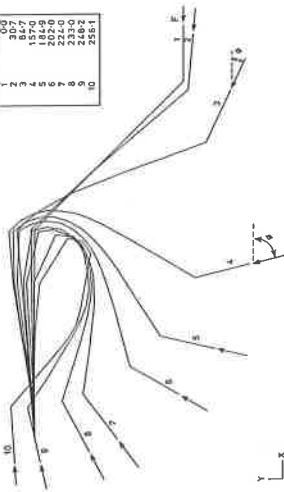
- (i) Large rotation pipe whip analysis using non-linear elastic elements will exhibit numerical instability (flutter).
- (ii) Flutter will occur earlier and the amplitudes will grow faster when smaller element sizes are used.

- (iii) Use of in-elastic elements will most certainly delay any occurrence of flutter in the solution.
- (iv) Numerical solution is less sensitive to mesh or element size when in-elastic elements are used.

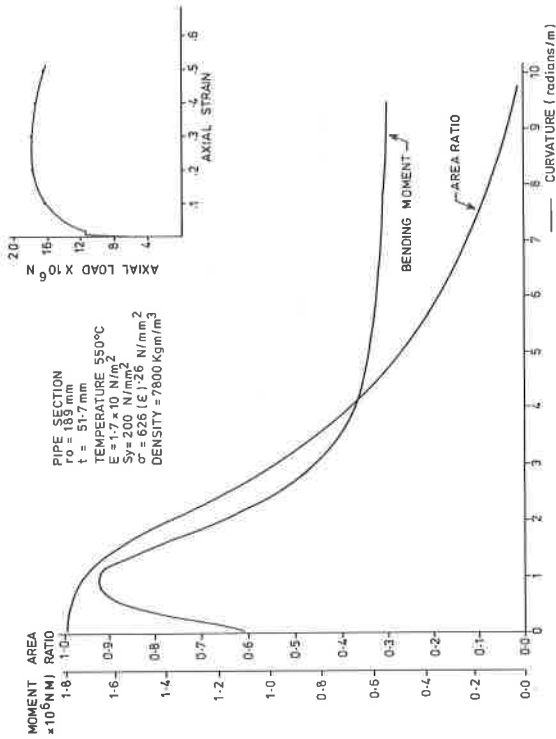
6. References

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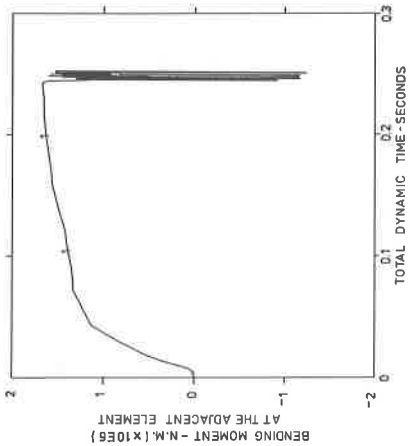
PLOT	TIME (sec)
1	0.0
2	30.7
3	61.4
4	157.0
5	314.0
6	471.0
7	628.0
8	785.0
9	1570.0
10	2851.0



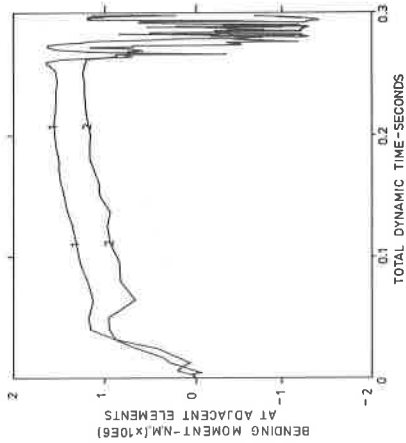
1. An Example of Large Rotation Pipe Whip Of A Typical High Energy Piping System



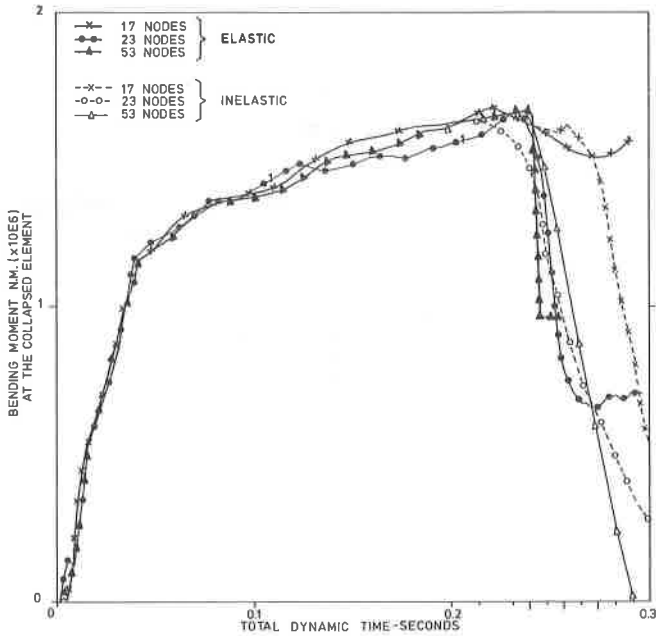
2. Non-linear Beam Section Behaviour Used In Pipe Whip Example



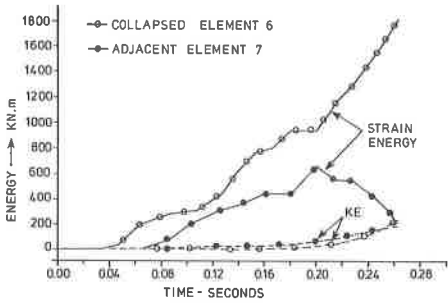
3. Elastic Elements Showing Instability (Flutter) In Bending Moment At Adjacent Element In 53 Noded Mesh



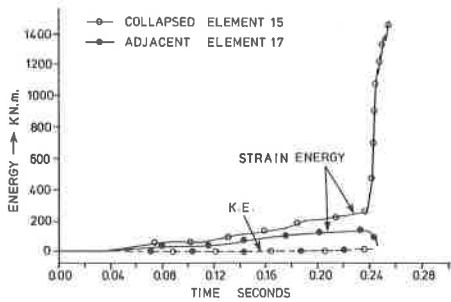
4. Elastic Elements Showing Instability (Flutter) In Bending Moment At Adjacent Elements In 23 Noded Mesh



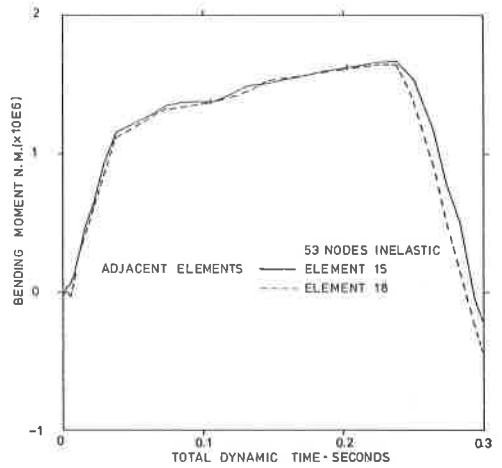
5. Comparison Of Bending Moment At The Collapsed Element 'Hinge' For Different Mesh Sizes Using Elastic And Inelastic Behaviour



6. Time History Of Element Energies For 17 Noded Mesh



7. Time History Of Element Energies for 53 Noded Mesh



8. Inelastic Elements Showing Stable Solution