# Axial-flow-induced Vibration of a Rod Supported by Two Translational Springs

H. S. Kang, K. N. Song, Y. H. Jung

Fuel Manufacturing Technology Development, Korea Atomic Energy Research Institute

hskang@nanum.kaweri.re.kr

## ABSTRACT

An axial-flow-induced vibration model was proposed for a rod supported by two translational springs at both ends. For the derivation of the vibration model, the normal mode method was used to solve the random vibration problem of the cylinder subjected to the randomly fluctuating pressure acting on its surface by axial flow. The first natural frequency and mode shape functions for the flow-induced vibration, so-called FIV, model were derived by using Lagrange's method based on the single mode approximation. The vibration displacement at reactor conditions were calculated by the proposed model for the spring-supported rod and by the previous model for the simple-supported(SS) rod. The resulting vibration displacement for the spring-supported rod was larger than that of the SS rod, and the discrepancy between both displacements was much larger at low flow velocity than at high flow velocity. The vibration displacement for the spring-supported rod appeared to decrease with the increase of the spring constant.

## INTRODUCTION

PWR fuel rods are exposed to reactor coolant of high flow velocity. It is known that the vibration of the fuel rod(FR) is generated by the coolant flow, and the fretting-wear by the vibration is frequently found on the surface of it. This problem of the FR is not on fluidelastic instability that causes excessive vibration and failure in short time but on turbulence-induced excitation that generates small amplitude and may cause long-term fretting-wear damage. The fretting wear by this sub-critical vibration is generally accepted as a root cause of a fuel rod failure which is not known well yet. Since the FIV obviously generates the relative motion between the FR and spacer grid, that can lead to fretting damage of the FR, it is very important to understand what is the actual vibration behavior of the fuel rod supported by spacer grids. The spacer grid has several springs in a cell in order to support the fuel rod flexibly. Therefore, the supporting method for the fuel rod is actually not a simple support but a spring support. It was reported that the spring constant of the spacer grid gave a significant effect to the modal parameters of the FR[1].

However, most works on FIV of the FR were associated with the SS cylinder. In this study, therefore, the spring support effect on the FIV of the cylinder was surveyed as compared with the SS. In order to derive the FIV model for the spring-supported cylinder, the author's previous work[2] was completely accepted, in which a procedure was described to solve the random vibration problem of the cylinder subjected to the randomly fluctuating pressure acting on the cylinder surface by axial flow. The cylinder vibration mode was obtained by using Lagrange's method based on the single mode($1^{st}$ mode) approximation. For the numerical calculations, the spring constants were selected within the range of those of the commercial spacer grid springs, and the material properties of the cylinder and flow conditions for the PWR reactor were used.

## MATHMETICAL MODEL AND NUMERICAL CALCULATION

### 1. Mathematical Model

The following Equation Of Motion(EOM) is basically used with the exception of the axial force, which was proposed by the previous study[2].

$$EI\frac{\partial^4 y}{\partial x^4}+(m_f V^2)\frac{\partial^2 y}{\partial x^2}+C\frac{\partial y}{\partial t}+M\frac{\partial^2 y}{\partial t^2}=q(x,t) \quad (1)$$

Where, $m_f$ is the added mass of fluid,

M is the total mass(sum of the $m_f$ and rod mass)

$C\frac{\partial y}{\partial t}$ is the viscous damping force which was equated to the sum of the forces $F_s$,

defined as previous study[2].

$$F_s=-\frac{1}{2}\rho DV^2C_f\frac{\partial^2 y}{\partial x^2}(\frac{L}{2}-x)+2m_f V\frac{\partial^2 y}{\partial x\partial t}+\rho DV^2C_f\frac{\partial y}{\partial x}+\frac{1}{2}\rho DVC_f\frac{\partial y}{\partial t}$$
$$+\frac{1}{2}\rho DC_D\left|\frac{\partial y}{\partial t}\right|\cdot\frac{\partial y}{\partial t} \quad (2)$$

The $C_f$ and $C_D$ in Eq. (2) are the profile and skin drag respectively for a rod. Since it is well known that the rod in axial flow is weak damping, the damping term in equation (1) can be neglected for simplicity in obtaining natural frequencies and mode shapes.

Considering the uniform mass of a cylinder and length $l$ supported on equal springs of total stiffness $k$, as shown in Fig 1.

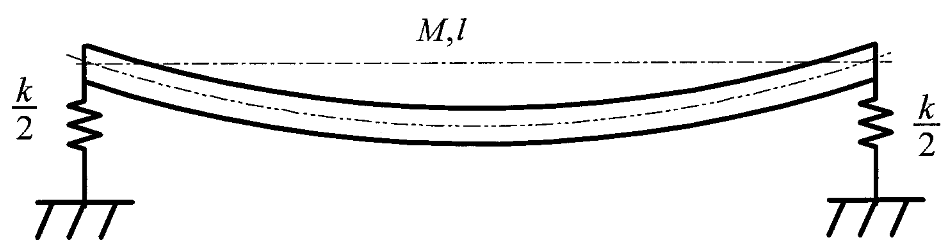


Fig. 1 A Cylinder supported by equal springs at both ends

The deflection equation of the cylinder was assumed to be

$$y(x,t) = \phi_1(x)q_1(t) + \phi_2(x)q_2(t) \tag{3}$$

The spatial equations $\phi_1 = \sin(\pi x / l)$ and $\phi_2 = 1.0$ could be chosen for the equation (3).

Using Lagrange's equation, the following were obtained.

$$\ddot{q}_1 + \frac{4}{\pi}\ddot{q}_2 + \omega_{11}^2 q_1 = 0 \tag{4}$$

$$\frac{2}{\pi}\ddot{q}_1 + \ddot{q}_2 + \omega_{22}^2 q_2 = 0 \tag{5}$$

where $\omega_{11}{}^2 = \left(\frac{\pi}{l}\right)^2 \frac{m_f V^2}{M} + \left(\frac{\pi}{l}\right)^4 \frac{EI}{M}$ ; natural frequency of the beam on rigid supports

$\omega_{22}^2 = \frac{k}{m}$ ; natural frequency of rigid beam on springs

Solving these equations, the natural frequency for the cylinder supported on the two springs can be obtained from the equations as follows:

$$\omega_n{}^2 = \omega_{22}^2 \frac{\pi^2}{2} \left\{ \frac{(T+1) \pm \sqrt{(T-1)^2 + \frac{32}{\pi^2}T}}{\pi^2 - 8} \right\} \tag{6}$$

where $T = \left(\frac{\omega_{11}}{\omega_{22}}\right)^2$ (7)

The deflection equation can be taken as followings;

$$y(x,t) = \phi(x)q(t) = (b + \sin\frac{\pi x}{l})q(t) \tag{8}$$

where,

$$b = \frac{\pi}{8}\left\{(T-1) \mp \sqrt{(T-1)^2 + \frac{32}{\pi^2}T}\right\} \tag{9}$$

## 2. Natural Frequency and Mode Shape Comparison with SS Rod and Spring-Supported Rod

For numerical calculation, material properties and geometry of the fuel rod, and spring constant range of the commercial spacer grid springs were used as summarized in Table 1. The fuel rod was assumed to be in PWR reactor. Thus, the temperature and velocity range of the coolant was assumed to be 310 °C, and 5 to 8 m/s. Since this calculation was based on the single mode approximation, only the first mode shape was calculated and compared as shown in Fig. 2. The calculation results were summarized in Table 2. Natural frequency in case of the spring support was appeared to increase gradually up to that of SS with the increase of the spring constant. Theoretically, the SS can be considered as the spring support by the infinite spring constant. Therefore, the calculation results for the spring-supported rod were judged to be reasonable.

Table 1 Material and Geometry data

| Name | Property | Value(310 °C) |
|---|---|---|
| Rod | EI | 14.03 $N/m^2$ |
| | Mass per length | 0.728 kg/m |
| | Span length | 0.522 m |
| | O.D / I. D | 9.5 / 8.22 mm |
| Spring | Constant | 50~400 kN/m |

Table 2 1st Natural Frequency of SS and Spring Support

| Case | | $1^{st}$ Natural Frequency (Hz) |
|---|---|---|
| SS-SS | | 25.35 |
| Spring Support | K= 50 kN/m | 23.52 |
| | K= 200 kN/m | 24.86 |
| | K= 400 kN/m | 25.10 |

## 3. Flow-induced-vibration Analysis

The response of the cylinder to the turbulence axial flow was derived at previous study[2]. The result is written as follows;

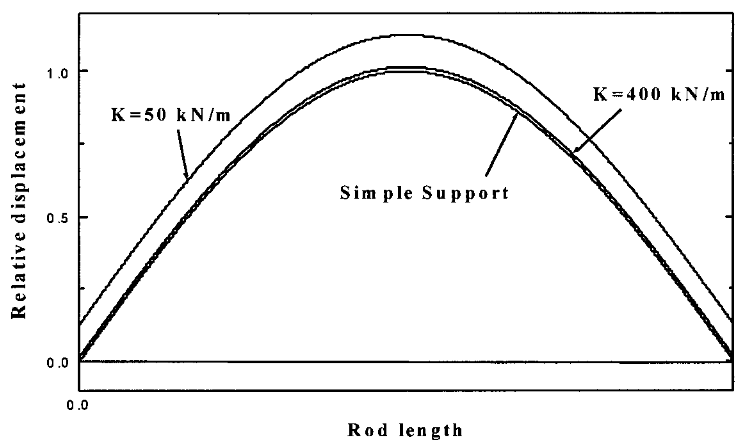


Fig. 2 First Mode Shapes of the SS Rod and the Spring-supported Rod

$$\Phi_{yy}(\xi,\xi',\omega) = 4d^2 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \phi(\xi)\phi(\xi')\, H_m(\omega)\, H_n^*(\omega)\, \chi^2\, \psi(\omega)\, Jn^2 \qquad (10)$$

where, $\Phi_{yy}$ ; Displacement PSD

$$H_n = \frac{1}{\sqrt{EI\left(\frac{n\pi}{l}\right)^4 - m_f V^2\left(\frac{n\pi}{l}\right)^2 - M\omega^2 + i2\zeta M\omega\omega_n}}$$ ; Frequency Response Function

$H_n^*$ ; Conjugate of $H_n$

$Jn$ ; Joint acceptance[2 and3]

$$\chi^2 = \frac{1}{4}\int_0^{2\pi}\int_0^{2\pi} Exp\left[-0.275\frac{\omega d}{U}\left|\theta-\theta'\right|\right]\cos\theta' \cos\theta\, d\theta' d\theta$$

$\psi(\omega)$ ; Power spectral density of wall pressure[2]

Vibration displacement of the rod can be obtained by integrating the equation (10) from zero to infinite on the frequency domain. The damping model and power spectral density of wall pressure discussed in the previous study[2] was used.

The frequency response function multiplied by its conjugate was calculated according to spring stiffness, and its results are illustrated in Fig. 3.

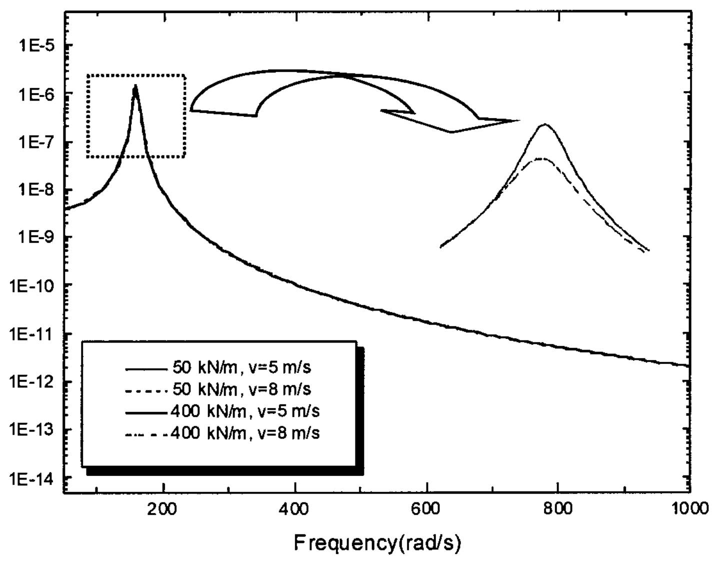


Fig. 3 Frequency Response Function for Spring Support

## 4. Numerical Calculation Results and Discussion.

Using the derived equations in section 2.1 and the data in Tables 1 and 2, the displacement PSDs for both the SS rod and the spring-supported rod were calculated for flow velocity of 5 m/s and 8 m/s, and their results were depicted in Fig. 4.

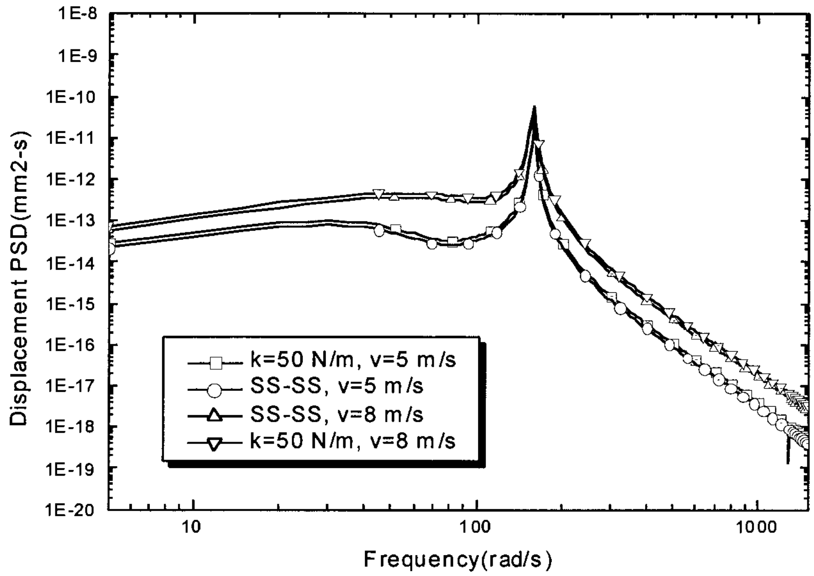


Fig. 4 Displacement PSD of Both SS Rod and Spring-support Rod

The level of the displacement PSD goes up as the flow velocity increases and the spring constant decreases while the resonant frequencies seem to have no change. The PSD level of the spring-supported rod is slightly higher than that of the SS rod at the same flow velocity.

The vibration displacement of the rod in axial flow can be obtained by integrating the PSD shown in Fig. 4 from 0(zero) to infinite theoretically. However, as reported not only in author's previous study[2] but in many, it was believed that a one-mode($1^{st}$ mode) approximation would be enough to predict the rod vibration in actual engineering point of view. Therefore, in this study, it was decided that the integration range from 0(zero) to 800 rad/s (127 Hz) was adopted to get the vibration displacement. The displacements obtained by doing that were illustrated in Fig. 5. The results came up to the expectation that the maximum displacement was obtained in the case of the cylinder exposed at the flow velocity of 8 m/s and supported on the spring constant of 50 kN/m, which was the highest flow velocity and softest spring of all cases. All results were normalized by the that of the SS rod at 5 m/s velocity. The softer the support spring is, the larger the vibration displacement is at the same flow velocity. This spring effect is stronger at the low flow velocity than at the high flow velocity. As it were, when the flow velocity is 5m/s, the displacement discrepancy between the SS rod and the rod supported by the 50 kN/m spring goes up to almost 20 %. On the other hand, the difference reduces up to half of it when the flow velocity comes up to 8 m/s. It is believed that in the range of the low flow velocity, the natural frequency and mode shape are influenced more by the spring softness than by the increase of flow velocity.

## CONCLUSION

An axial-flow-induced vibration(FIV) model was proposed for a rod supported by two translational springs at both

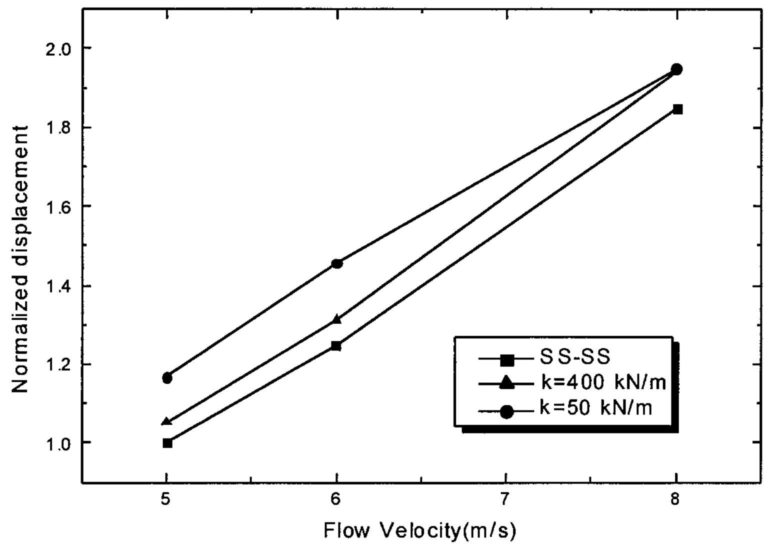


Fig. 5 Vibration Displacement of the Cylinder as a Function of Mean Flow Velocity

ends. The first natural frequency and mode shape functions for the FIV model were derived by using Lagrange's method based on the single mode approximation. The vibration displacement was calculated by the proposed model for the spring-supported rod, and the result was compared with that of the SS rod. It was concluded that the vibration displacement for the spring-supported rod was larger than that of the SS rod, and the displacement decreased with increase of the spring constant. Although the vibration displacement due to axial flow appeared to be clearly dominated by the flow velocity and considered to be small in quantity, in case of the PWR rod supported by the soft spring, 20 % more displacement than the SS rod may be possibly appreciated in the range of the flow velocity of 5 to 8 m/s, which is known to be the velocity range of the PWR reactor coolant.

## ACKNOWLEDGEMENT

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