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EQUIVALENT LINEARIZATION USING TRANSFER FUNCTION APPLIED TO PIPING SYSTEMS IN SEISMIC ENGINEERING

Thuong Anh Nguyen[1], Pierre Labbé[2], Jean-François Semblat[3], and Guillaume Hervé[2,4]

[1] Ph.D. Student, Paris-Est University, France
[2] Professor, Paris-Est University, France
[3] Professor, Paris-Est University & IFSTTAR, France
[4] IMSIA, UMR 929 EDF-CNRS-CEA-ENSTA Paris Tech, France

ABSTRACT

After the accident of Fukushima-Daïci NNP in 2011, the question of the capacity of NNP to withstand design seismic hazard has become a prior concern that has increased the need of evaluations of NNP's structures or their components within a higher level of seismic load by taking into account nonlinear behavior. This is not an easy task due to of the need of computational capacity due to many iterations required at each step in order to satisfy the equilibrium. Seismic engineers can call for equivalent linearization concepts from different criteria that allow them coping the non-linear responses without performing the non-linear transient calculation, which is a "well" established concept for reinforced concrete structures (within an important number of concepts such as the ones presented by Iwan (1980) or Chopra (1995)) but not for piping systems. In addition, despite the advantages, their assumptions are insufficient or even inconsistent. In this paper, considering a systematic and argumentative analysis based on numerical experimental plan, linear equivalent behavior has been established by considering the equivalence criterion through the transfer function from the time domain to the frequency domain. The equivalent linearization has been conceived for the elastoplastic system representing a non-linear behavior of piping systems, undergoing the filtered white noise signal Clough-Penzien. As results, the frequency degradation and increasing damping versus ductile demand have been established. Then, this result is compared with those obtained from the piping system's test campaign EPRI in 1994 (EPRI (1994)) and BARC in 2015 (Ravikiran A. (2015). This comparison highlights the similarities between the experimental results and the ones using transfer function with elastoplastic behavior in terms of the linear equivalent behavior.

INTRODUCTION

Well studied as mechanical pieces rather than as a whole structure in civil engineering, piping systems represent important elements in nuclear power plants (NNPs). Actual standard codes address failure criteria such as plastic instability, incremental collapse, and high strain-low-cycle fatigue. Nevertheless, they present a large conservatism in design. This remark is highlighted in conclusions of some experimental campaigns on piping tests. In the report of EPRI (1994), many outputs of piping components and systems tests are carried out. It is important to note that the dynamic moments excess clearly the limit moment of design during tests but does not cause the collapse, for example, the ratio of dynamic moment over limited moment for test 23 is up to 7.3. More recently, the piping systems in stainless or carbon steel have been tested with strong motions at the Bhabha Atomic Research Centre in India (Ravikiran A. (2015)). During this test, the value of primary stress of about 4.3*Sm is reached without collapse of structure (these piping systems are designed with 3*Sm with the allowable primary stress Sm equal to 140 MPa for stainless steel SS 304L) Ravikiran A. (2015). According to its nonlinear behavior, it is recommended that the design criteria bases on an analysis using the elastoplastic behavior. This remark is mentioned in RCC-M (Baylac & Grandemange (1991)).

While the structure exhibits nonlinear behavior which is characterized by hysteresis strain-stress loops during the cyclic load, the stiffness of the structure is reduced and its damping increased versus plastic deformation. These phenomena could be considered by performing the nonlinear transient calculation. However, the huge need in terms of timing and memory consummation can be prohibitive. In addition, this last calculation could raise convergence problems.

In this context, simplified methods or procedures are introduced by using the equivalent linearization analysis. These concepts are usually displacement-based criteria and well established the reinforced concrete structure based on an adequate constitutive law: Iwan (1980), Kowalsky (1994), Chopra (1995), Iwan and Guyader (2002). Unfortunately, this approach is still less studied for piping systems. The question of margin in design addressed by Labbé (2013) while examining the piping system confirms the need of effective frequency and damping for piping systems.

Based on the elastoplastic behavior, Chopra (1995) (and Chopra and Goel (1999)) has defined the equivalent linear behavior of an elastoplastic oscillator by using the secant stiffness for the Capacity Spectrum Method (CSM). This equivalent linearization consist in an equivalent linear oscillator whose frequency is linked to the slope of the straight line connecting the origin and the point of maximal deformation of the deformation-force curve during the earthquake, and the equivalent damping ratio based on the dissipated energy within the hysteretic loop corresponding to the maximal deformation. This concept has two disadvantages, at least, at the author's point of view:

1. The unloading stiffness is equal to the secant one but the dissipated energy is performed with the unloading stiffness equal to the elastic one. Therefore, there is an inconsistency in term of model of equivalent linearization,
2. The model bases completely on the loop of the maximum of the deformation time history but there are many loops of smaller deformation appearing before and after this maximum.

In the modified version proposed in ATC-40 (1996), a damping modification factor κ has been introduced in order to take into account the imperfection hysteresis loop. This value of κ is established based on different types of behavior. The maximum of κ (equal to 1) is applied to the structure representing stable, reasonably full hysteresis loops. This case seems to be closed to the piping systems.

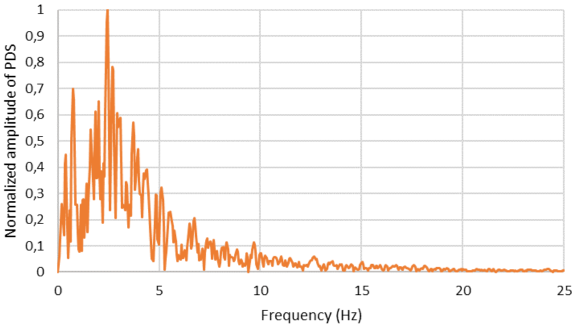
In this paper, the concept of equivalent linearization for the elastoplastic oscillator is examined. This concept is based on an argumentative and systematic analysis by using the transfer function as equivalent criterion. The filtered Clough-Penzien white noises are used as ground motions. As results, the frequency degradation and increase of damping ratio versus the ductile demand are established. This study highlighted the dependency of equivalent linearization on the relative position of initial frequency in respect to central frequency of ground motion. This important parameter has unfortunately not studied in any existing concept. These results are compared with experimental ones of EPRI (1994) and BARC (2015) for validation. In addition, the frequency degradation for elastoplastic oscillator is so much smaller than Chopra's one.

EQUIVALENT LINEARIZATION USING TRANSFER FUNCTION

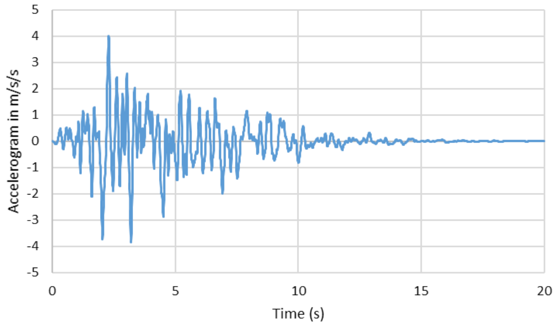
Clough & Penzien filtered White Noise

In this article, the filtered white noise of the type Clough & Penzien is used. The procedure is detailed by Zentner (2013). The signal is random stationary process multiplied by an envelope curve in time domain and characterized by its Power Spectral Density (PSD) as the figure 1.

The central frequency is about 3.3 Hz and the range of frequencies is of 0-50Hz.The signal lasts 20 seconds and the time step is 10 milliseconds.



(a): Normalized amplitude of Power Spectral Density (PSD)



(b): Signal of type Clough & Penzien filtered white noise. N=2000; Δt = 0.01s

Figure 1.

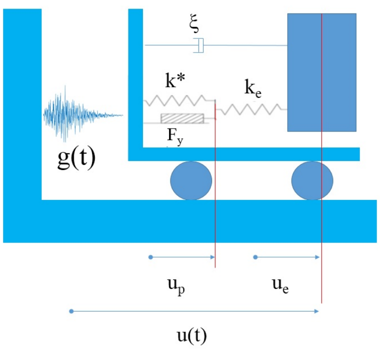
Response of elastic oscillator through transfer function

It is to consider here an elastic oscillator of frequency f_0 and damping ratio ξ_0, which is excited at its base by a strong ground motion that can be written under its complex form $x_e(t) = X_e e^{(j\omega t)}$. Its elastic response is also a harmonic signal and written under its complex form $x_s(t) = X_s e^{(j\omega t)} e^{j\phi}$. Then the complex transfer function H_t is defined as following:

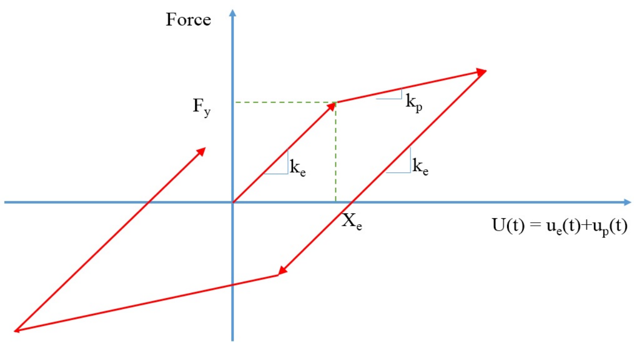
$$\frac{X_s . e^{(j\phi)}}{X_e} = \frac{\left(\frac{f}{f_0}\right)^2}{1 - \left(\frac{f}{f_0}\right)^2 + 2j\xi_0 \frac{f}{f_0}} = H_t(f) \tag{1}$$

Response of elastoplastic oscillator using β-Newmark Scheme

The elastoplastic oscillator studied in this article consists of a combination of elastic springs ($F = k_e u$), Newtonian damper ($F = C\dot{u}$) and Saint-Venant sliding element (F_y).



(a): Rheological model



(b): Displacement-Force relationship

Figure 2.

The dynamic equation of the motion of the mass of this elastoplastic oscillator is written as following:

$$\ddot{u} + 2\xi_0\omega_0\dot{u} + f(u)/m = -g(t) \tag{2}$$

Where:

u : Relative displacement of the mass m;

m : Mass of the elastoplastic oscillator, here m is taken of 1 kg;

ω_0 : Angular velocity corresponding to the elastic frequency of the oscillator $\omega_0 = 2\pi f_0$;

ξ_0 : Damping ratio corresponding to the angular velocity and the Newtonian viscosity $\xi_0 = \dfrac{C}{2m\omega_0}$;

$g(t)$: Ground motion displacement applied to the elastoplastic oscillator.

In this study, many oscillators are examined with damping ratio ξ_0 fixed at 5%. The initial frequency f_0 depends on the central frequency of ground motion fc. The ratio is taken among 0.1; 0.5; 1.0; 1.5; 2.0 in order to examine the influence of relative position of f_0 in respect to f_c equal to 3.3 Hz. In addition, the influence of the hardening on the equivalent linearization are also analyzed. Therefore, an kinematic hardening measure is defined by the ratio of hardening stiffness against the initial stiffness, so noted α_p. For numerical application, α_p takes values of 0%, 10%, 20%.

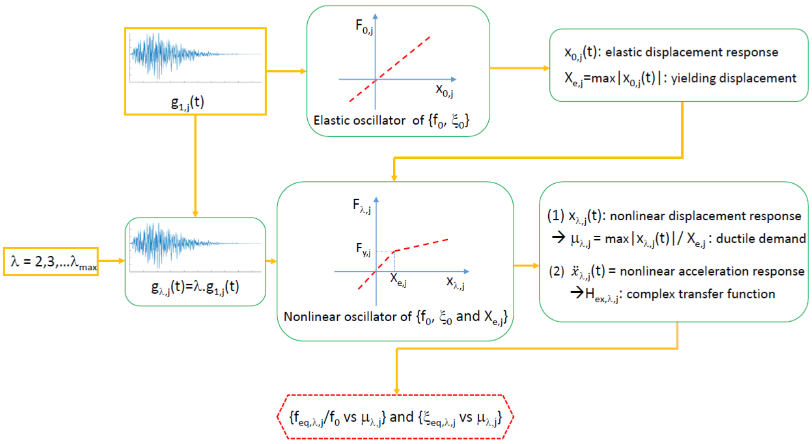


Figure 3. Plan of numerical experience

So, for each case of { f_0 and α_p }, a numerical experience as shown in the Fig. 3 are conducted.

Where:

$g_{1,j}(t)$: Ground motion of type Clough-Penzien filtered white noise with $j = 1...1000$ at level 1;

$x_{0,j}(t)$: Displacement time history of elastic oscillator of $\{f_0, \xi_0\}$ undergoing the $g_{1,j}(t)$;

$X_{e,j} = \max\left|X_{0,j}(t)\right|$: Maximal displacement of elastic oscillator of $\{f_0, \xi_0\}$ undergoing the $g_{1,j}(t)$ in m/s/s;

$\lambda = 2,3,...,9,\lambda_{\max}$: Amplification factor of the amplitude of ground motions. At this stage, it is certain to say that the $g_{1,j}(t)$ leads the elastoplastic oscillator of $\{f_0, \xi_0\}$ and $X_{e,j} = \max\left|X_{0,j}(t)\right|$ just to the yielding point, no plastic deformation is observed. Then, $g_{\lambda,j}(t) = \lambda.g_{1,j}(t)$ make these elastoplastic oscillators to exhibit plastic deformation. The value of λ_{max} is determined such as the median value of μ is equal to 20;

$u_{\lambda,j}(t)$: Displacement time history of elastoplastic oscillator of $\{f_0, \xi_0\}$ and $X_{e,j}$ undergoing the $g_{\lambda,j}(t) = \lambda.g_{1,j}(t)$, performed by using the β-Newmark scheme. Then, the ductile demand is defined as : $\mu_{\lambda,j} = \max\left|u_{\lambda,j}(t)\right| / X_{e,j}$;

$H_{ex,\lambda,j}$: Experimental complex transfer function defined as $H_{ex,\lambda,j}(f_i) = \dfrac{fft(\ddot{x}_{\lambda,j}(t))}{fft(g_{\lambda,j}(t))}$, and $fft(.)$ assigns the Fast Fourier Transform;

$\{f_{eq,\lambda,j}, \xi_{eq,\lambda,j}\}$: Equivalent linear frequency and damping ratio defined by the procedure of equivalent linearization which is defined in next section.

Equivalent linearization by the complex transfer function

For every single of $H_{ex,\lambda,j}$, the couple of $\{f_{eq,\lambda,j}, \xi_{eq,\lambda,j}\}$ is determined by minimizing "the difference" between the experimental complex transfer function $H_{ex,\lambda,j}$ and theoretical one $H_{t,f_{eq,\lambda,j},\xi_{eq,\lambda,j}}$ of the elastic equivalent oscillator. That is to say, the theoretical $H_{t,f_{eq,\lambda,j},\xi_{eq,\lambda,j}}$ is the best fit of the complex transfer function $H_{ex,\lambda,j}$. As numerically saying, the $\{f_{eq,\lambda,j}, \xi_{eq,\lambda,j}\}$ are determined by minimizing this term:

$$J\left(f_{eq,\lambda,j};\xi_{eq,\lambda,j}\right) = \sum_{f\min}^{f\max}\left(\mathrm{Re}\left(H_{ex,\lambda,j}(f_i)\right) - \mathrm{Re}\left(H_{t,\{f_{eq,\lambda,j},\xi_{eq,\lambda,j}\}}(f_i)\right)\right)^2 + \left(\mathrm{Im}\left(H_{ex,\lambda,j}(f_i)\right) - \mathrm{Im}\left(H_{t,\{f_{eq,\lambda,j},\xi_{eq,\lambda,j}\}}(f_i)\right)\right)^2 \quad (3)$$

Where Re(.) and Im(.) assign the real and imaginary part of complex number.
For the best frequency resolution, the zeros-padding method is applied. The determination of $\{f_{eq,\lambda,j}, \xi_{eq,\lambda,j}\}$ is reached by using the operator *lsqcurvefit* in matlab.

For example (Ref. figure 4), the result of $\{f_{eq,\lambda,j}, \xi_{eq,\lambda,j}\}$ determined for $\lambda = 2$ and $j = 1$ is presented. The $X_{e,1}$ is determined by the linear analysis of the elastic oscillator of $\{f_0 = 3.35\,\mathrm{Hz}, \xi_0 = 5\%\}$ and $\alpha_p = 10\%$ equal to 0.021 m. The equivalent oscillator is characterized by $\{f_{eq,2,1}, \xi_{eq,2,1}\}$ = {3.27 Hz, 13.6 %}.

According to this procedure, for each value of $\lambda = 2,3,...,9,\lambda_{\max}$, 1000 couples of $\{f_{eq,\lambda,j}, \xi_{eq,\lambda,j}\}$ and 1000 values of ductile demand $\mu_{\lambda,j}$ are obtained.

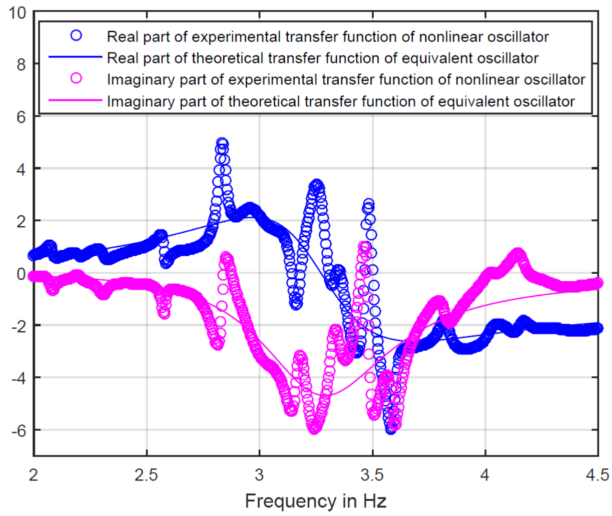


Figure 4. Example of complex transfer function regression

The regression processes is applied to the frequency degradation and increasing of damping. The formula for f_{eq}/f_0 take the value $\sqrt{\alpha_p}$ as its asymptotic and the increase of damping is inverse-proportional to f_{eq}/f_0, as observed in the Nguyen et al. (2016):

$$\frac{f_{eq}}{f_0} = \sqrt{\alpha_p} + \frac{1-\sqrt{\alpha_p}}{1+\frac{(\mu-1)^d}{b}}$$
$$\xi_{eq} = \xi_0 + a\left(1-\left(\frac{f_{eq}}{f_0}\right)^c\right) \quad (4)$$

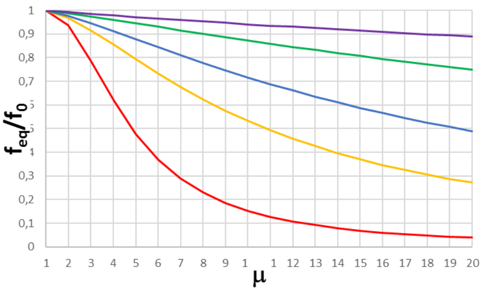
The values of a, b, c and d have determined through the nonlinear least squared regression.

Result of equivalent linearization

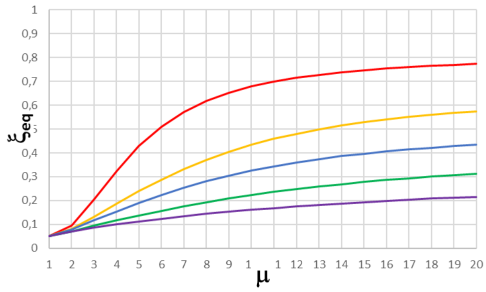
By following the procedure described above, the plan of numerical experience is applied for 3 values of kinematic hardening α_p = 0%, 10% and 20%. (Ref. figure 5)

Legend of colors	
$f_0/f_c = 0.1$	
$f_0/f_c = 0.5$	
$f_0/f_c = 1.0$	
$f_0/f_c = 1.5$	
$f_0/f_c = 2.0$	

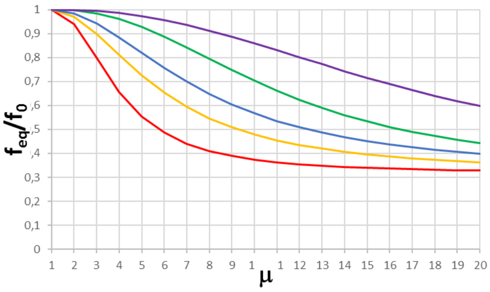
Legend of colors



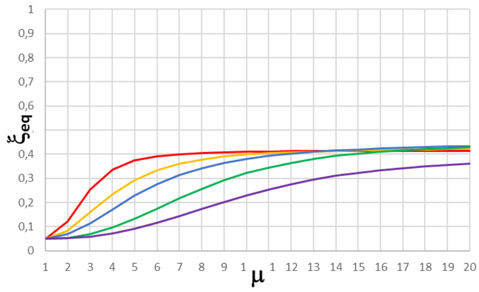
(a): $\left\{ f_{eq} \,/\, f_0 \text{ vs } \mu \right\}$ for $\alpha_p = 0\%$



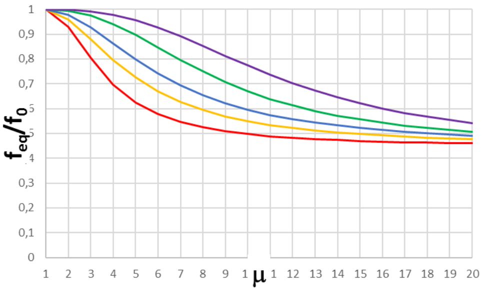
(b): $\left\{ \xi_{eq} \text{vs } \mu \right\}$ for $\alpha_p = 0\%$



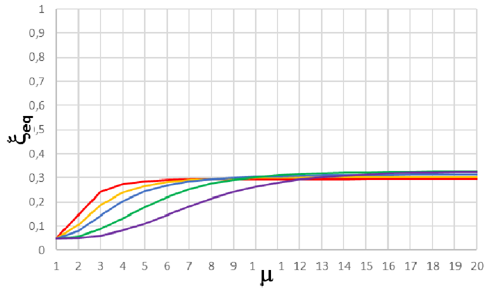
(c): $\left\{ f_{eq} \,/\, f_0 \text{ vs } \mu \right\}$ for $\alpha_p = 10\%$



(d): $\left\{ \xi_{eq} \text{vs } \mu \right\}$ for $\alpha_p = 10\%$



(e): $\left\{ f_{eq} \,/\, f_0 \text{ vs } \mu \right\}$ for $\alpha_p = 20\%$



(f): $\left\{ \xi_{eq} \text{vs } \mu \right\}$ for $\alpha_p = 20\%$

Figure 5. Results of frequency degradation and increase of damping

PIPING SYSTEM TEST CAMPAIGN OF EPRI AND BARC

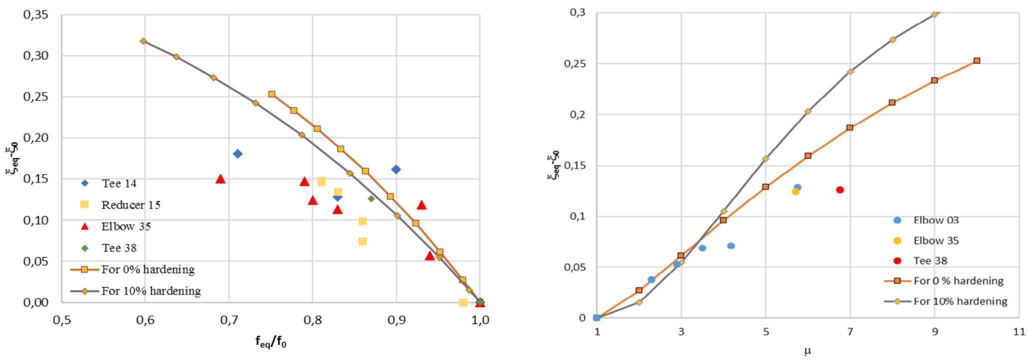
EPRI piping components tests

In the report of EPRI (1994), the outputs for pressurized piping components are found in term of effective frequencies, damping ratios and cyclic strain. Then, these outputs can be explored as:

+ Experimental effective damping versus peak cyclic strain;

\+ Experimental frequency degradation versus effective damping

Considering that the plastic yielding is classically associated to 0.2% strain, recorded strain values are divided by 0.2 to be converted into ductile demand (μ values). Then, experimental points can be plotted as following:



(a): Increasing of damping versus frequency degradation

(b): Increasing of damping versus ductile demand

Figure 6.

The experimental results can now be compared with the results obtained from our analysis of the proposed concept using transfer function. It is important to note that the parameter α_p is not available in the report, then, the kinematic hardening can be introduced with reasonable values of α_p = 0% (orange curve) and 10% (gray curve) (Ref. figure 6) (The value α_p = 5% is found for the test of BARC). According to the ratio of f_0/f_c, all components are adjusted so that the frequency at peak of signal is about 0.85% of the natural frequency of piping components, so the value of f_0/f_c equal to 1.00/0.85=1.17. Unfortunately, the result for f_0/f_c is not available from our analysis, then the interpolated value between f_0/f_c = 1.0 and 1.5 are used. According to the figure 7, the results of our proposed concept of equivalent linearization fit quietly well the experimental result. The curve for xeq-x0 versus feq/f0 for 10% is better than the curve of 0% but less in term of ξ_{eq}-ξ_0 versus μ. For μ = 3, the value of 5% of increase of damping for experimental result are observed and equal to our proposed equivalent linearization's result. This value if so smaller than Chopra's one of 32% for α_p = 10 and of 42% for α_p = 0% Chopra et al. (2001).

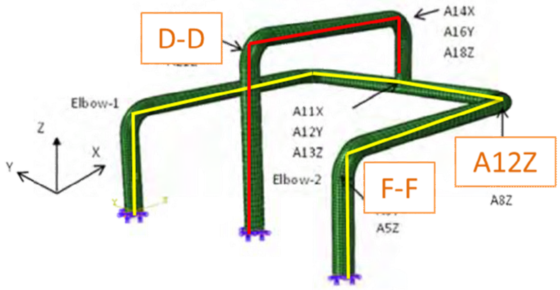
BARC piping systems tests

The carbon steel piping systems CSPS-1 and CSPS-2 were tested with strong ground motions up to 2.5 (g). According to the Labbé (2017), even the ductile demand reaches a value of 2 (equivalent to 0.4% of hoop strains), the frequency is slightly reduced, and tabulated in the following table (Ref. table 1):

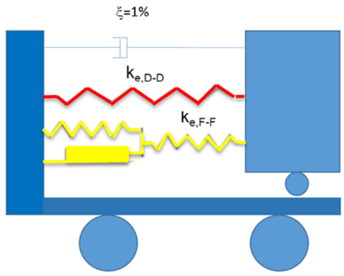
Table 1: BARC experimentation
Frequency shift under beyond design seismic input

	SSPS-1	SSPS-2
$f_{eq}(2.5g)/f_0$	0.99	0.97
Ductile demand	2	2

It is important to note that the response in vertical direction is dominant in total response. Assuming that only section F-F reaches its elastic limits and exhibit its elastoplastic behavior while all unfavorable sections such as elbows, anchors,… remain elastic. Then, the equivalent of BARC for vertical response is proposed, which consist in the parallel assemblage (Ref. figure 7) of an elastic spring (red one) which represents the mid-branch within section D-D and an elastoplastic part (yellow one) which represents the two branches within section F-F.



(a): Finite element (FE) model of piping system BARC



(b): Equivalent model for vertical response of BARC

Figure 7.

Thank to FE model, the $\frac{f_{0,\text{F-F}}}{f_{0,\text{D-D}}} = \sqrt{\frac{k_{0,\text{F-F}}}{k_{0,\text{D-D}}}}$ can be easily determined with $k_{0,\text{D-D}}$ and $k_{0,\text{F-F}}$ are successively the generalized stiffness of the first mode for structure BARC without embedded support at anchor of F-F branch and one for BARC without embedded support at anchor of D-D branch. Numerical value of this ratio is equal to 1.02. Once the section F-F exhibits its elastoplastic behavior, this elastoplastic one can be replaced by an equivalent linear one of $\{f_{eq}$ and $\xi_{eq}\}$ associated to the value of ductile demand observed in section F-F. If is sufficient to read these values from the curve of figure 5. The initial frequency of BARC is about 4.06 Hz and central frequency of seismic signal is about 8.56 Hz, then the ratio f_0/f_c = 0.47. Contrary to EPRI, experimental results are available for stabilized cycle load strain-stress relation (Goyal et al 2013) to determine the value of kinematic hardening. Here, the kinematic hardening of material SA 333 Gr.6 is about 5%. For a value μ =2, f_{eq}/f_0 is equal to 0.9664 (Ref. figure 5). Then, the total equivalent structure can be replaced by an equivalent consisting in parallel assemblage of elastic spring D-D and equivalent linear spring F-F, which lead us to the total frequency shift of $(1.02^*f_{eq,FF}/f_{0,FF}+1)/(1.02+1)$ =0.98. This value corresponds to the value obtained during the test (Ref. table 1).

CONCLUSIONS

The study carried out the result of equivalent linearization using transfer function applied to elastoplastic behavior. The result of frequency degradation and increasing of damping is established through a numerical experimental plan for an important rang of ductile demand up to 20.

The results of the proposed concept of equivalent linearization correspond to experimental of piping components and systems tests. The increase of damping on piping systems is so smaller than the value of Chopra's concept based on elastoplastic behavior, which is usually used in practice.

The frequency degradation reaches its asymptotic value of $\sqrt{\alpha_p}$ when ductile demand tends to infinity.

The results of our systematic analysis highlight the influence of an important parameter : the relative positon of initial frequency versus the central frequency (f_0/f_c). This ratio stands for the stiffness of structure. The bigger this ratio, the stiffer the structure and the smaller the frequency degradation.

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