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BENDING AND PUNCHING STUDIES ON IMPACT LOADED PLATE

Arja Saarenheimo[1], Kim Calonius[2] and Markku Tuomala[3]

[1] Research Team Leader, VTT Technical Research Centre of Finland
[2] Research Scientist, VTT Technical Research Centre of Finland
[3] prof. emeritus, TUT Finland

ABSTRACT

A shear punching test series is carried out at VTT. The target structure is a two way simply supported concrete plate with a span of 2 m and a thickness of 25 cm. The mass of the deformable stainless steel projectile is 50 kg. The capabilities of different numerical methods for assessing global and local deformation with possible shear punching are studied. Numerical solutions are compared with experimental results of test X4.

INTRODUCTION

A combined bending and shear punching test series, called X, is carried out at VTT. The target structure is a two-way simply supported concrete plate with a span of 2 m and a thickness of 25 cm. The deformable projectile is made of stainless steel tube with a shallow spherical dome nose and its mass is 50 kg. In the first two tests, X1 and X2, the outer diameter of the missile was 253 mm and the wall thickness was 3 mm. The impact velocity was 166 m/s. No clear shear punching occurred in these tests. In order to achieve shear punching, the missile was modified for test X3. The diameter of the missile was 219 mm and the wall thickness was 6.35 mm. For test X4 the impact velocity was increased from 144.7 m/s to 168.6 m/s. Capabilities of different calculation methods in assessing both global bending deformation and local shear deformation and possible shear punching are studied. The models and methods comprise a two-degree-of-freedom (TDOF) model CEB (1989) and two finite element (FE) programs, an in-house code and a commercial general purpose code Abaqus (2014). Additionally, some semi-empirical formulae are used for comparisons. Tests X1 and X2 have been analysed in Borgerhoff et al. (2013).

TARGET STRUCTURE

The dimensions of the simply supported slab used in the stainless steel missile impact tests at VTT are: the span l = 2 m, the thickness h = 0.25 m and the effective thickness d_e = 0.225 m (assuming concrete cover of 20 mm and rebar diameter of 10 mm). The geometry of the slab and the cross-sections are shown in Figure 1. The bending reinforcement consists of steel bars (A500HW) with a diameter of 10 mm and a spacing of 90 mm, which gives A_{sx}=A_{sy}= 8.73 cm^2/m and a ratio of 0.35% each way and each face. The shear reinforcement is made with 6 mm closed stirrups. The amount of shear reinforcement in tests X1, X3, and X4 was A_{ss}=17.45 cm^2/m^2. In test X2 it was A_{ss}=11.67 cm^2/m^2. The material properties of the concrete slab used in the analyses of test X4 are as follows: the modulus of elasticity (nominal strength K50/C40) E_c=24.32 GPa, the Poisson ratio ν=0.2, the compressive cubic strength f_{cu}=46 MPa, the cylindrical strength f_c=41.7 MPa and the splitting tensile strength f_t=2.26 MPa. The properties of bending reinforcement (A500HW) are: the modulus of elasticity E_s=210 GPa, the yield stress σ_y=537 MPa, the ultimate stress σ_u=629 MPa, reached at a strain value of ε_u=0.1 The properties of shear reinforcement are: the modulus of elasticity E_s=210 GPa, the yield stress σ_y=629 MPa, the ultimate stress σ_u=702 MPa, reached at a strain value of ε_u=0.15.

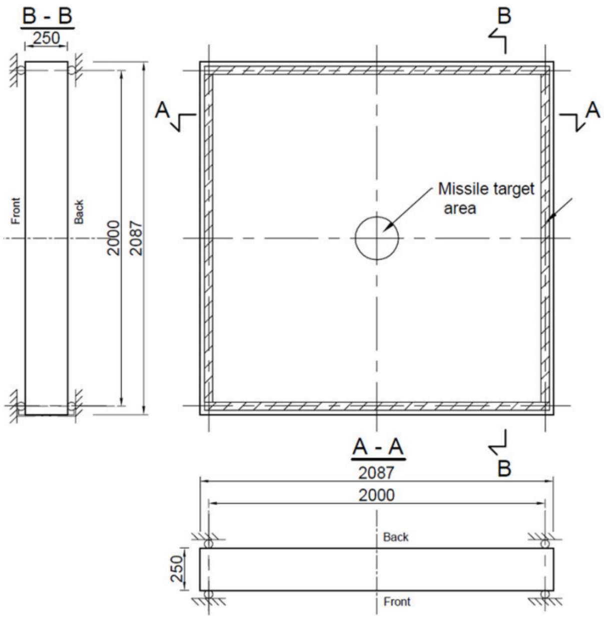

Figure 1. Test slab with dimensions.

DEFORMABLE STAINLESS STEEL MISSILE

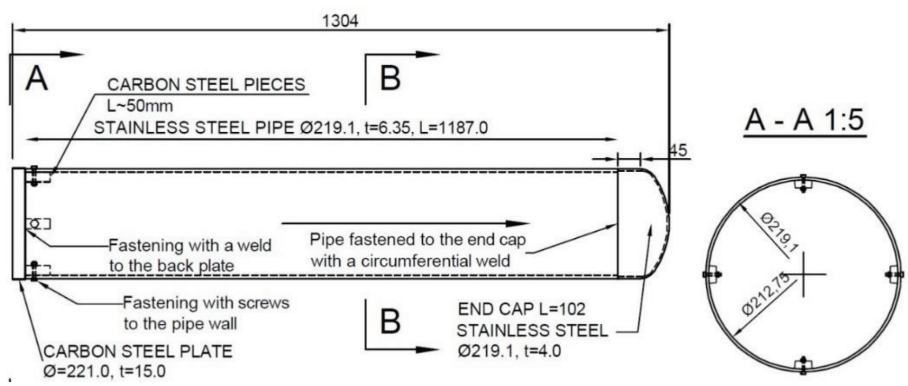

Figure 2. Stainless steel missile.

The missile used in test X4 is shown in Figure 2. The measured mass of the missile was 49.96 kg and its total length before the impact was 1.304 m. The missile fuselage is made of stainless steel (EN1.4404 or 316L in ASME) pipe with a wall thickness of 6.35 mm and an outer diameter of d_0= 0.2191 m. The wall thickness of the front cap with a height of 57 mm and a short tube with a length of 45 mm is 4 mm. The impact velocity of the missile in the test X4 was 168.6 m/s. The properties of missile steel are: the modulus of elasticity E_s=210 GPa, the yield stress σ_y=304 MPa, the ultimate stress σ_u=595 MPa, reached at a strain value of about ε_u=0.5, based on static material tests. For stainless steel of missile in the Cowper–Symonds equation D = 1522 1/s and q = 5.13 are assumed, Schleyer and Langdon, (2003).

CALCULATION METHODS AND MODELS FOR REINFORCED CONCRETE PLATE

Dynamic punching capacity of plate

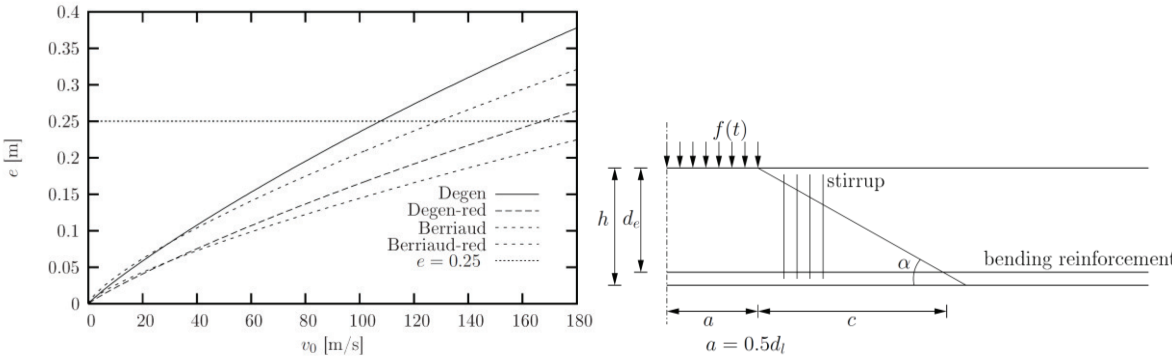

Figure 3. a)Perforation thickness of plate by assuming a softening factor of α=0.7, DOE (1996) and b) Shear cone.

For hard projectiles various formulae for predicting perforation thickness have been developed, e.g. in Berriaud et al. (1978), Degen (1980). According to DOE (1996) these formulae can be applied for deformable missiles using a softening factor. For the present missile Figure 3a shows the perforation thickness as a function of impact velocity. Curves labelled 'red' are calculated using a softening factor of $\alpha = 0.7$.

Jowett and Kinsella (1989) proposed for the punching load of reinforced concrete slab subjected to a soft missile impact the formula

$$F_p = 8170\left(\rho_p f_{cu}\right)^{\frac{1}{3}} \pi d_e (d_l + 2.5 d_e) \tag{1}$$

in which ρ_p is the average fraction of the tensile reinforcement, [%], f_{cu} is the compressive strength of concrete (cube), [Pa], d_e is the depth of slab from load face to the centre of rear face reinforcement, [m], d_l is the diameter of loaded area, [m]. Formula (1) predicts the punching load most accurately, when the load duration exceeds the time needed to reach the maximum plate deflection. The scabbing limit, F_s, is obtained using a coefficient 7040 instead of 8170, Jowett and Kinsella (1989). The capacity F_p is compared to the dynamic load caused by the missile impact and the plate must fulfil the condition $F_a \leq F_p$, where an average force is calculated by the Riera formula, Riera (1968). The average force used in the following is obtained by dividing the impulse of the force-time function by the duration of the impact load.

The punching capacity is straightforwardly assumed to be the sum of the concrete and stirrup contributions. The shear capacity due to stirrups alone can be determined assuming a circular load area with a diameter of d_l and a shear cone angle of inclination of α measured from the horizontal plane, Figure 3b. The required amount of shear reinforcement $A_{ss}$for load F_{sh} is obtained from

$$A_{ss} f_y = \frac{F_{sh}}{\pi[(a+c)^2 - a^2]} \tag{2}$$

in which $c = d_e/\tan\alpha$ and $a = d_l/2$ is the radius of loaded area. Thus the shear capacity due to stirrups in the case X4 is $F_{sh} = 0.37$ MN when assuming $\alpha = 45^o$. In the bending reinforcement case $A_{sx} =$

$A_{sy} = 0.000873$ m²/m and $f_{cu} = 46$ MPa, the punching and scabbing capacities by Equation (1) become $F_p = 1.17$ MN and $F_s = 1.01$ MN, respectively, and the shear capacity of the plate is 1.58 MN. The averaged force F_a in case X4 is 1.63 MN. In previous tests X1 and X2 the cubic compression strength values of concrete were 41.6 MPa and 46 MPa, respectively. The reinforcement in slab X1 was similar to the reinforcement used in slabs X3 and X4. The shear reinforcement was reduced for test X2 and the shear capacity due to stirrups is 0.24 MN. Punching capacities predicted by formula (1) for tests X1, X2, X3 and X4 are compiled in Table 1. These results are in agreement with the experimental observations. Except case X4 the shear punching strength of the slab has not yet been exceeded. According to these calculations, scabbing capacity was exceeded in tests X3 and X4. The observed scabbing area was about 0.7 m² after the test X3. No scabbing was observed in test slabs X1 and X2 and this is also in agreement with the calculation results shown in Table 1. Only in the case X4 the average value of time dependent force resultant exceeds the total punching capacity of the wall.

Table 1. Punching and scabbing capacities [MN].

Test	F_p	F_s	F_{sh}	$F_p + F_{sh}$	F_a
X1	1.13	0.97	0.37	1.5	0.67
X2	1.17	1.01	0.24	1.41	0.67
X3	1.23	1.0	0.41	1.64	1.46
X4	1.17	1.01	0.41	1.58	1.63

TDOF model

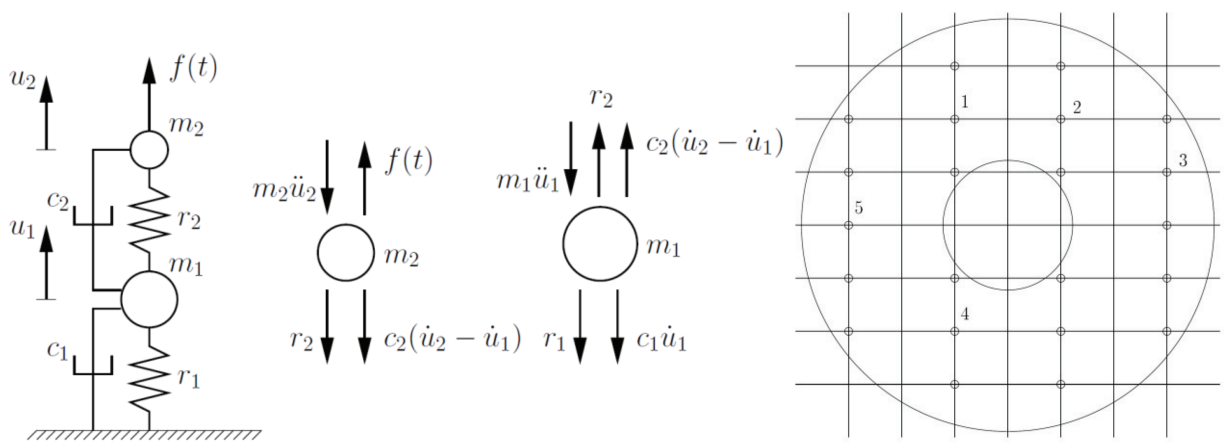

Figure 4. a) TDOF model used in bending and punching studies of impacted concrete slabs and b) assumed punching cone and strain gauge locations 1 to 5 in test X4.

A two-degree of freedom (TDOF) elasto-plastic oscillator, CEB (1988), can predict bending mode deformation but also possible local failure of impact loaded plate. TDOF model is shown in Figure 4a. Spring 1 and mass 1 are connected to the global bending deformation. Spring 2 and mass 2 are used in describing the local shear behaviour in the impact area. A visco-plastic constitutive law is used for stirrups.

Rayleigh damping, Chopra (1995), is assumed with damping factors $\varsigma_1 = 0.07$ (based on earlier studies) and $\varsigma_2 = 0.02$ for modes 1 and 2, respectively. Shear strength of spring 2 consists of concrete, stirrup and bending reinforcement contributions. The assumed punching cone with the contributing shear reinforcement in test X4 is shown in Figure 4b. Also the locations of strain gauges in stirrups are presented by numbers.

Abaqus 3D solid model

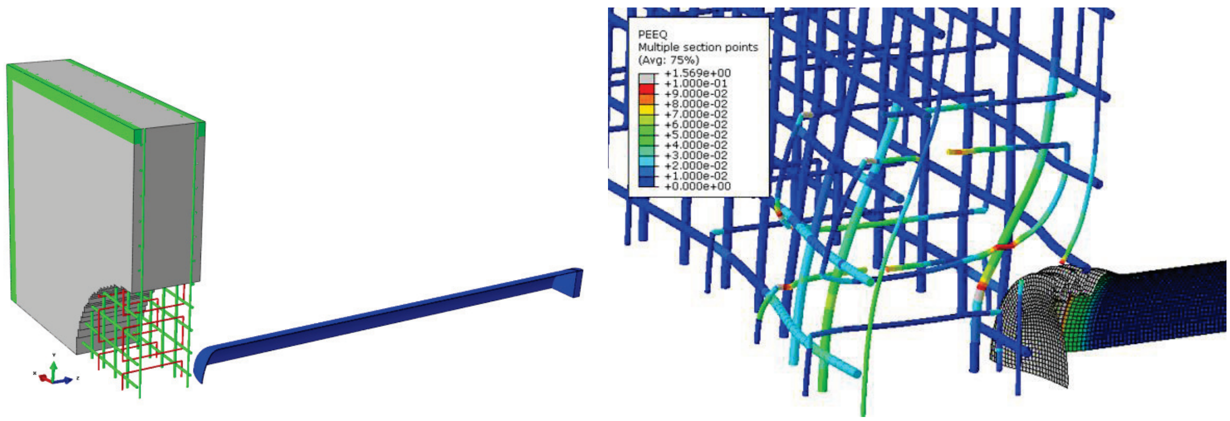

Figure 5. a) 3D solid element model and b) equivalent plastic strains in reinforcement and missile.

One quarter of the test slab and missile is modelled in more detail with FE code Abaqus, Abaqus, (2014) using eight-noded 3D solid elements for the slab and four-noded shell elements for the stainless steel missile. Reinforcement is modelled with beam elements. This model, shown in Figure 5a, contains over half a million degrees of freedom. Bending reinforcement and steel plates on slab edge are shown in green. Shear reinforcement is shown in red. Some solid elements (grey) representing concrete are removed from the plot.

The *concrete damaged plasticity* model of Abaqus is used for solid elements. Material damage is taken into account both in compression and in tension. Maximum principal strain criterion, 20 %, was used as an erosion criterion for concrete elements. A special code for eroding elements was used, PRINCIPIA (2012). The stress-strain curves based on the tensile material test results are used for reinforcement steel and stainless steel material of the missile. Strain rate sensitivity of missile and reinforcing steel is taken into account by the Cowper-Symonds formula for uniaxial tension or compression. For mild steel of reinforcement parameter values D = 40 1/s and q = 5, Jones, (1989), and for stainless steel of missile D = 1522 1/s and q = 5.13, Schleyer and Langdon (2003), are assumed. The nonlinear dynamic analyses are conducted with explicit time-integration using Abaqus/Explicit solver.

Plate element models

Complementary numerical results are computed also with an in-house finite element program. The used Bogner-Fox-Schmit (BFS) plate element has 16 degrees of freedom (DOF), Bogner et al. (1965). The nodal degrees of freedom are:.$w, w_{,x}, w_{,y}, w_{,xy}$ (twist). The BFS element is based on the Kirchhoff plate theory, which does not consider the transverse shear deformation. The Reissner-Mindlin plate theory, RM-theory, includes the effect of transverse shear deformation. A four-noded 12 DOF element with nodal degrees of freedom w, θ_x, θ_y (deflection and rotations) is used here. For transverse shear deformation special interpolation is needed for stable numerical behaviour, Bathe and Dvorkin (1985).

A simplified resultant formulation is adopted in which the material nonlinearities of concrete and reinforcement are formulated in terms of the stress resultants and curvatures. In reverse motion cracked state stiffness is assumed. Stress resultant formulations for concrete shells and plates have been considered in Ibrahimbegovic and Frey (1993) and Koechlin and Potapov (2007). Based on nonlinear uniaxial stress strain curve bending moment curvature relationship is determined and this relationship is used in determining the principal moments from the principal curvatures.

For transverse shear bilinear elastic plastic relationship is assumed between the principal transverse shear force and shear strain. Before reaching the yield value linear elastic relation is used. The transverse shear stress capacity at plate section is calculated from

$$\tau_p = f_t + A_{ss} f_{ys} \tag{3}$$

where f_t is the tensile strength of concrete A_{ss} is the amount of shear reinforcement [m^2/m^2] and f_{ys} is the yield stress of reinforcement. The shear force capacity of plate section is obtained from

$$q_p = \tau_p d_e \tag{4}$$

where d_e is the effective thickness of the plate. The effective thickness of plate X4 is 0.225 m and $q_p = 0.9$ MN/m. Plastic yield in transverse shear is assumed independent of bending stress state

RESULTS

With Abaqus both missile and plate are modelled. Calculated equivalent plastic strain distribution in reinforcement at t = 4.25 ms is shown Figure 5b. One bending reinforcement bar and some stirrups are broken. In the calculation the missile perforates the plate with a residual velocity of about 30 m/s, which is slightly larger than the corresponding velocity observed in the test.

For TDOF and other FE analyses the load was determined in advance by the Riera method. An average folding model and a model taking into account the actual formation of folds were used, Saarenheimo (2010). Rigid target was assumed in these load calculations, while in test the target turned out to be deformable. The load deflection curves are shown in Figure 6. The deformed shape of the missile after the test with corresponding deformed shapes obtained with FE calculation and analytically are shown in Figure 7.

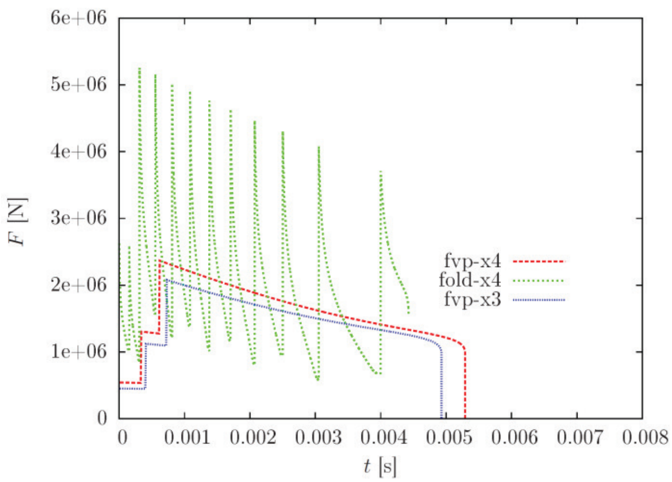

Figure 6. Load function of steel missile in test X4 with impact velocity of v_0=168.6 m/s obtained with a folding visco-plastic model assuming σ_y = 416 MPa (ideally plastic), D = 1522 1/s and q = 5.13. Curve fvp refers to average folding model and curve fold to folding model with front part thickness t = 4 mm.

Figure 8 shows deflection histories of plate X4 at point 1 to point 4. Point 1 is located 360 mm above the central point and points 2 to 4 are at 200 mm intervals to the right of point 1. Curve RM is obtained with FE method using RM elements and a 21 by 21 mesh. Curve TDOF is the displacement of mass 1 which corresponds to the deflection at D1.

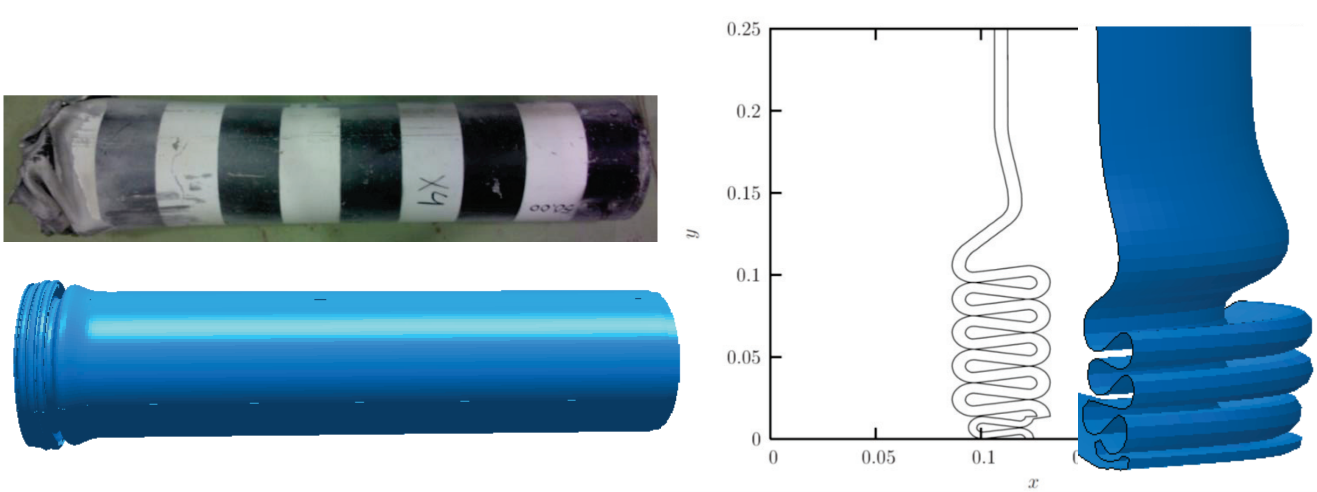

Figure 7. Deformed missile after test X4, final shape of the FE model and analytically obtained shape of X4 missile.

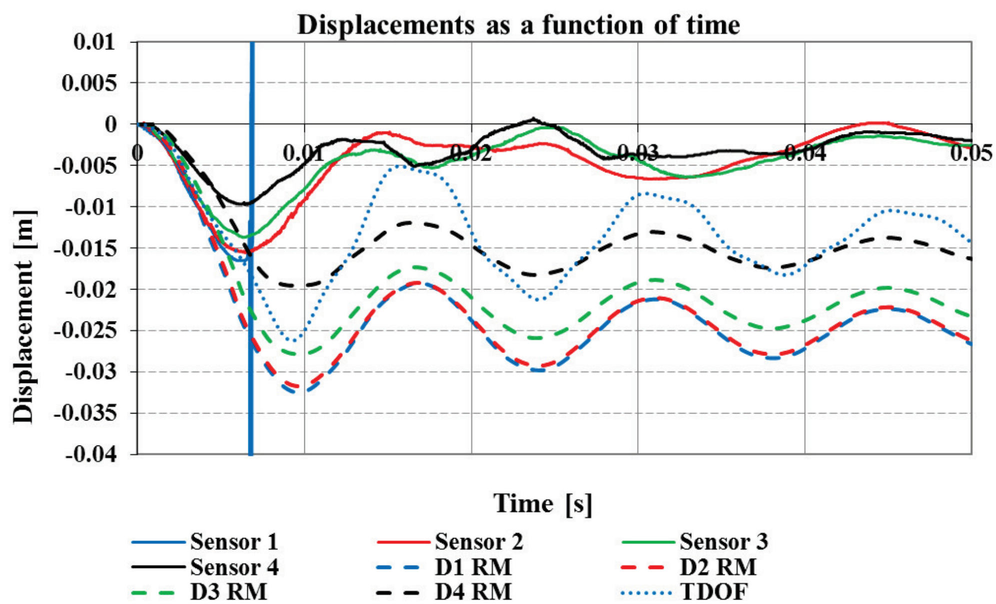

Figure 8. Deflection histories of plate X4 with curves RM obtained with FE method using RM elements and 21 by 21 mesh. Curve TDOF is the displacement of mass 1 which corresponds to deflection at D1.

Deflected shape and deflected profile of plate X4 at time 0.0084 s calculated by a FEM model with a 21 by 21 RM element mesh for one quadrant are depicted in Figure 9. The deflected profile, labelled X4-q, on the right hand side figure is calculated by assuming a large value of transverse shear strength. Large local shear is seen to take place predicting local failure of plate X4. Because missile is not explicitly modelled and no fracture criterion is applied complete severance does not happen. This affects also the deflections at points 1 to 4 which are larger than in test.

Calculated and measured strains in bending reinforcement at location 6 and 7, 135 mm and 315 mm to the right of impact point, are presented as a function of time in Figure 10. Calculated stresses and strains along in a broken stirrup bar are presented in Figure 11a and b. At the location where the strain value exceeds the ultimate tensile strain value (pink curve in Figure 11b) also the stress drops suddenly to zero (pink curve in Figure 11a).

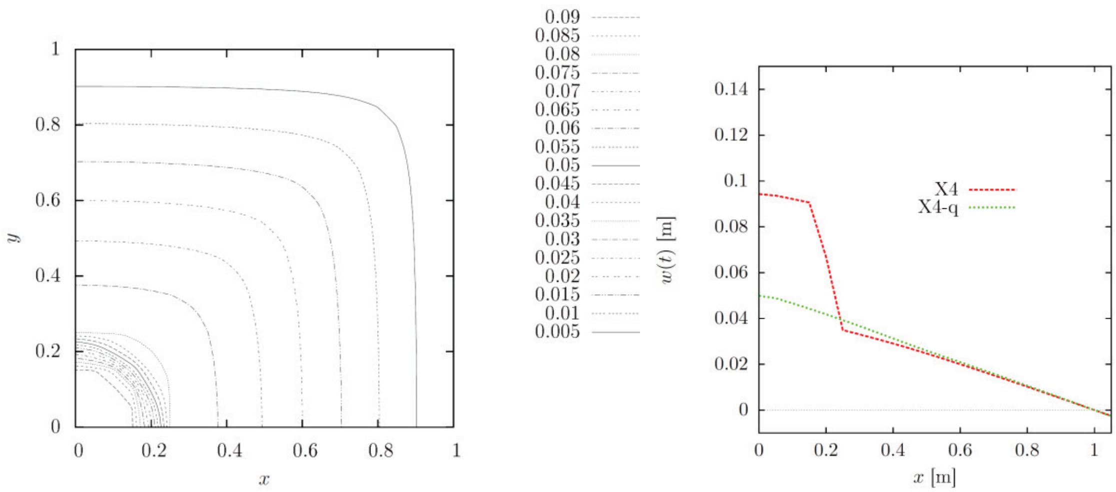

Figure 9. Deflected shape and deflected profile of plate X4 at time 0.0084 s calculated by a FEM model of 21 by 21 RM elements for one quadrant.

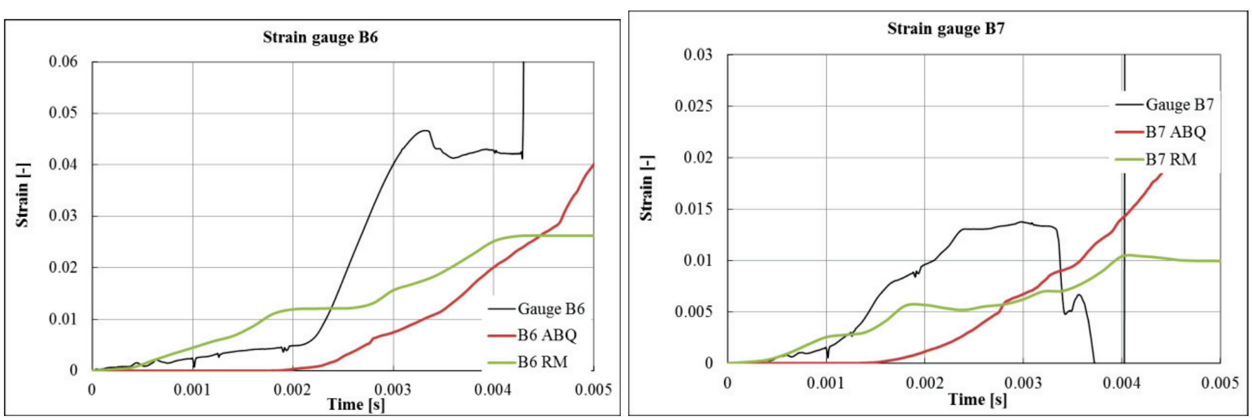

Figure 10. Strains in bending reinforcement at locations 6 and 7 of plate X4 by FEM model of 21 by 21 RM elements for one quadrant and by Abaqus solid element model.

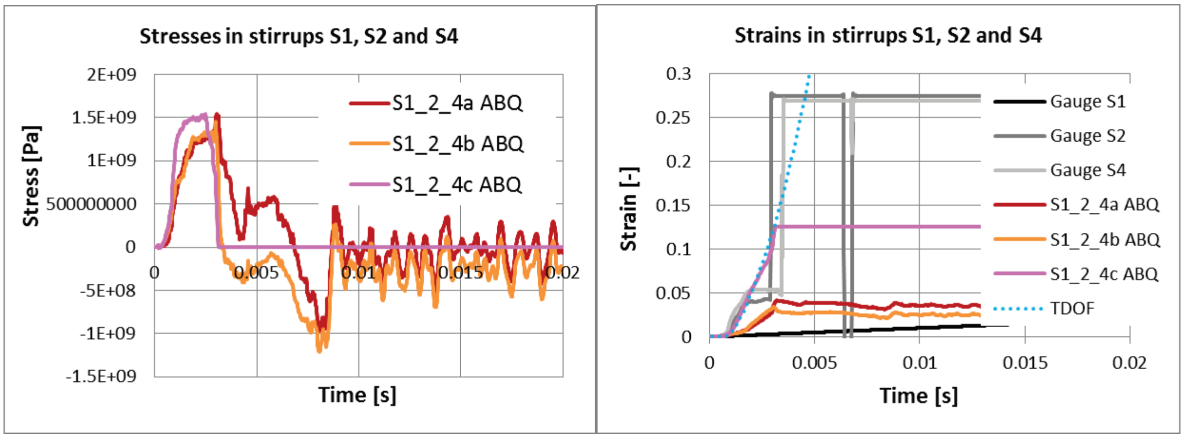

Figure 11. a) Stresses and b) strains in shear reinforcement at strain gauge locations S1, 2 and 4.

Velocity of the missile as a function of the time is presented in Figure 12. The calculated residual velocity of the missile is about 30 m/s. The recorded residual velocity in the test was 25 m/s.

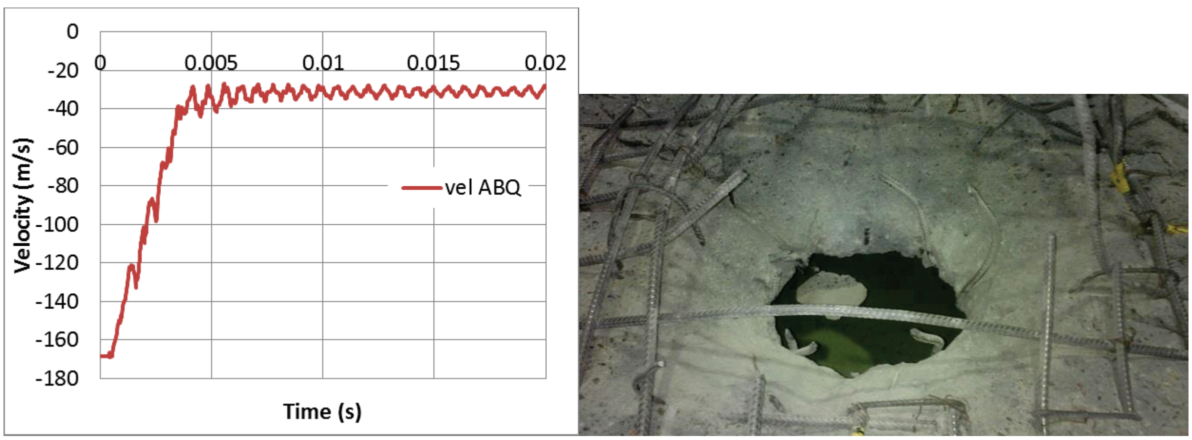

Figure 12. Missile velocity as a function of time. Local damage is shown on the right.

CONCLUSION

Punching and combined bending and punching of plates subjected to deformable missile impact can be analysed with different models of varied sophistication.

Experimentally based formulae and simplified models are valuable for preliminary studies. The Degen hard missile formula with reduction factor for missile deformation predicted just perforation in the studied case, where perforation took place with a residual velocity of 25 m/s. The experimentally derived formulae of Jowett and Kinsella (1989) predicted scabbing and punching consistently in various test cases X_i. A simplified TDOF-model time history solution can give an indication of punching with the bending behaviour predicted as well. This model needs a pre-determined shear cone angle.

FE models with nonlinear transverse shear deformation included and the impact load history determined beforehand e.g. by the Riera method can predict an indication of punching. However, for detailed local deformation studies 3D FE models are required. Also these models need parameter assumptions and model calibration. The Abaqus-model was able to predict various damage mechanisms such as cratering, penetration, scabbing, crushing of missile and rupture of reinforcement.

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