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IMPACT BEHAVIOUR OF ROCKING RIGID BODIES SUBJECTED TO SEISMIC LOADS

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ABSTRACT

Rocking of unanchored components (e.g. storage casks for spent fuel) is a highly nonlinear phenomenon, which cannot be completely described analytically. Therefore, the rocking motion of a rigid body under various conditions will be analysed with different models. The first model is an analytical model which is solved numerically by using the Runge-Kutta method. The other models are mostly plane finite element (FE) models. The FE models consider a rigid 2D (or in some cases 3D) block on a flexible ground without sliding. With the FE method it is possible to investigate the influence of the properties of the ground, the contact conditions, slight deviations from the ideal rectangular corner geometry and the use of anti-slip pads especially on the impact. The impact is described by the coefficient of restitution. The FE models, which take into account the above mentioned influences, lead to higher coefficients of restitution than those derived from the simplified model published by Housner (1963). For hard ground materials like steel or concrete the agreement between results from finite element analyses and the theoretical model with two rotation centres at the corners of the body is very good if the coefficients of restitution are based on best estimate values from FE analyses. For the model with the anti-slip pad the theoretical model is not usable. A classical resonance does not occur but a dynamic increase of the amplitude during the first cycles up to a factor of about 2.5 is possible.

INTRODUCTION

In nuclear power plants unanchored components can cause unacceptable damage particularly in the case of an earthquake. This also applies if the components, such as storage casks, are designed for the loads from overturning or hitting. To avoid damage it must be shown that no overturning or impermissible movement of unanchored components occurs. In principle it is the dynamic of a rigid, elastic or plastic body on a rigid, elastic or plastic foundation. The mechanical problem of the motion for a rigid body looks simple at the first sight, but becomes difficult due to the impacts. In literature the question of rocking oscillations of a rigid body has been found to be of great interest, so that there are a large number of publications on this subject. The fundamental works were done by Housner (1963). Based on these works, the motion of unanchored bodies will be analysed analytically and numerically by numerical integration of the equations of motion and using the FE method. Especially the influence of the supporting flexibilities, the material properties of the ground, the contact conditions und slight deviations from the ideal geometry on the impact properties described by the coefficient of restitution will be investigated. In contrast to most other studies a more compact body is considered.

THEORY OF ROCKING

A rigid body (block) in a plane subjected e.g. to an earthquake can perform the following motions: Rest, rotation (rocking), sliding, sliding rotation, translation and rotation jump (translation and rotation). A plane model to describe the rotation or rocking motion of a rigid body on a rigid ground is shown in Figure 1 (CG - centre of gravity). The symbols are explained in Table 1.

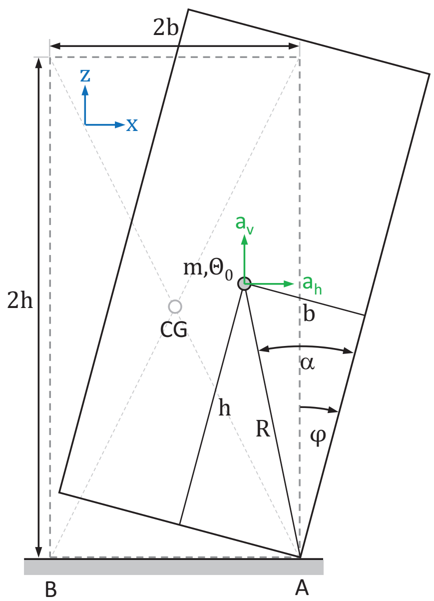

Figure 1. Plane model of a rigid body

Symbol	Parameter
$2h$	height
$2b$	width
$R = \sqrt{b^2 + h^2}$	length of the half diagonal
φ	angle of rotation
m	mass
I_0	moment of inertia about CG
$I = I_0 + mR^2$	moment of inertia about A/B
α	critical angle of rotation
$b/h = \tan\alpha$	aspect ratio
g	acceleration of earth gravity
$a_h(t)$	horizontal seismic acceleration
$a_v(t) + g$	vertical seismic acceleration
$p = \sqrt{mgR/I}$	frequency parameter
r	coefficient of restitution
E	modulus of elasticity

Table 1: Important characteristics and parameters

The dynamic equilibrium conditions for the moments around the points A and B give the following equations for the rocking motion (no sliding and/or jumping):

$$\begin{aligned} &\ddot{\varphi} - p^2\left\{\frac{a_v(t)}{g}\sin(\alpha-\varphi) + \frac{a_h(t)}{g}\cos(\alpha-\varphi)\right\} = 0 \;, \quad \varphi > 0 \qquad \text{(Point A)} \\ &\ddot{\varphi} + p^2\left\{\frac{a_v(t)}{g}\sin(\alpha+\varphi) - \frac{a_h(t)}{g}\cos(\alpha+\varphi)\right\} = 0 \;, \quad \varphi < 0 \qquad \text{(Point B)} \end{aligned} \tag{1}$$

In the case of pure rocking the Equations (1) give the motion before and after the impact. These equations are coupled by the angular velocities immediately before and after the impact. Housner (1963) has derived a simple model to calculate these angular velocities. The model uses the following assumptions:

- The body and the ground are rigid.
- There is no bouncing or complete lifting off of the body.
- The impact is a point impact (point contact).
- The time interval of the impact is very short.
- The body remains at the same position during the impact time.

Under these assumptions the principle of conservation of angular momentum immediately before and after the impact gives the following equation for the rigid body:

$$\frac{\dot{\varphi}_{(+)}}{\dot{\varphi}_{(-)}} = r = 1 - \frac{2mR^2}{I}\sin^2\alpha \qquad \text{or for a rectangular block} \qquad \frac{\dot{\varphi}_{(+)}}{\dot{\varphi}_{(-)}} = 1 - \frac{3}{2}\sin^2\alpha \tag{2}$$

($\dot{\varphi}_{(-)}$ - angular velocity before the impact, $\dot{\varphi}_{(+)}$ - angular velocity after the impact)

The parameter r denotes the coefficient of restitution. With Equations (1) and (2) it is possible to solve the problem numerically. For the ratio of the kinetic energy before W_k^- and after the impact W_k^+ follows:

$$\frac{W_k^+}{W_k^-} = \frac{\frac{1}{2} I \dot{\varphi}_{(+)}^2}{\frac{1}{2} I \dot{\varphi}_{(-)}^2} = r^2 = \left[1 - \frac{2mR^2}{I} \sin^2 \alpha\right]^2 \tag{3}$$

Further attempts to understand the impact and the motion are given by Ishiyama (1982), Lipscombe and Pellegrino (1993), Pena et al. (2007), Yilmaz et al. (2009) and Kounadis (2010).

NUMERICAL ANALYSES

Analysis methods

The numerical solution of the Equations (1) is done by using the (explicit) classical fourth-order Runge-Kutta method with a maximum time step of 0.001 s and an additional method for the accurate determination of the contact time. For every new time step the contact condition $\varphi_n \cdot \varphi_{n+1} \leq 0$ (φ_n - rotation of the current step, φ_{n+1} - rotation of the new step) is checked. In the case of contact, the time interval will decrease until the specified accuracy of the angle φ is reached. Because the Runge-Kutta method also provides the velocity for every time step, the velocity immediately before the impact $\dot{\varphi}_{(-)}$ is given. The procedure is restarted with the initial conditions for the angular displacement $\varphi(t_c) = 0$ and the angular velocity $\dot{\varphi}_{(+)}$ immediately after the impact according to Equation (2), where the time t_c denotes the contact time. The numerical solutions of Equations (1) and (2) will be used as a reference (quoted as theoretical curve) for the further finite element (FE) analyses.

Because the impacts have the main influence on the further movement the properties of the coefficient of restitution are investigated in most analyses. FE analyses allow the consideration of elastic or plastic properties of the material, especially of the ground. Additionally, the influence of geometrical parameters and their variations from the ideal geometry can be studied. By using infinite elements for the ground it is also possible to take the radiation of energy in the foundation into account. The coefficient of restitution can be calculated from the ratio of angular velocities before and after the impact and/or from the kinetic energies of two successive maxima. The FE analyses use the implicit code of the FE program Abaqus. Because the coefficients of restitution only depend on the velocities before the impact, the geometric conditions and the (material) properties of the ground (the body is assumed as rigid) the excitation can be done by an initial angular displacement $\varphi_0 < \alpha$. After preliminary calculations a maximum time increment of 0.001 s (typical 0.0002 s) and automatic time stepping will be used.

Determination of the coefficient of restitution with FE analyses

The basic models used in the analysis are shown in Figures 2 to 4. The first model in Figure 2 (model 1) is a discrete and abstract model using rigid R2D2 and MASS/ROTARY elements for the rigid body and SPRING1 and DASHPOT1 elements for the ground. The value c denotes the spring stiffness and k the damping constant. The second model in Figure 3 (model 2) uses CPS4 elements for the block and CPS4 and infinite CINPS4 for the ground. The CINPS4 elements do not reflect elastic waves at the boundaries of the model and allow the modelling of an infinite ground. These elements lead to a dissipation of energy into the ground. The last model in Figure 4 (model 3) also uses CPS4 and CINPS4 elements. The ground is an infinite plate. In contrast to model 2, the ground is finite thick. The dimensions of the rectangular block are 2 m x 2 m x 4 m (width/depth/height). The mass is 124.8 t (similarly to a storage cask). The block is rigid in model 2 and 3 by using RIGID BODY definitions. It is noted, that the models are only used to study the influence of typical parameters and boundary conditions. The models describe not the properties of real grounds.

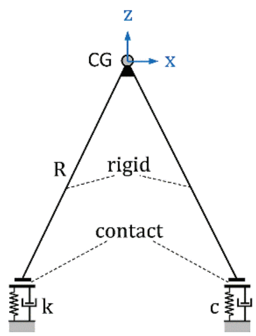

Figure 2. Simple model with spring and damper (model 1)

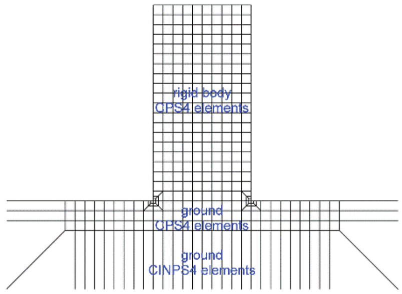

Figure 3. Plane model with an infinite ground (model 2)

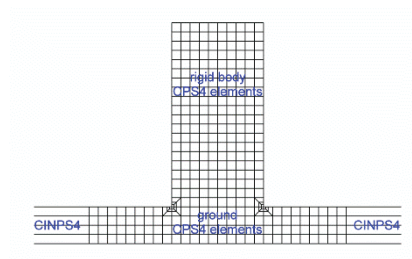

Figure 4. Plane model with an infinite plate as ground (model 3)

The results for the simple rigid model with $c = 2 \cdot 10^{10}$ N/m and $k = 1{,}4 \cdot 10^{8}$ kg/s are shown in Figures 5 to 7. There is no bouncing or jumping with these values for the parameters c and k. Other combinations of c (especially large c) and k often produce bouncing and/or jumping. The comparison between the curves of the FE analysis and the theoretical curves with r = 0.7 from Equation (2) show a very good agreement. A relevant dependency of the coefficient of restitution from the amplitude of the excitation is not visible. A problem is the determination of the “real” stiffness and damping of the ground.

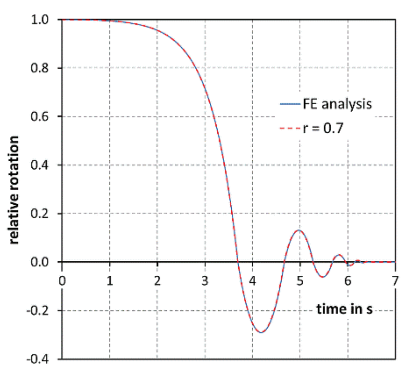

Figure 5. Time histories of the rotation for model 1

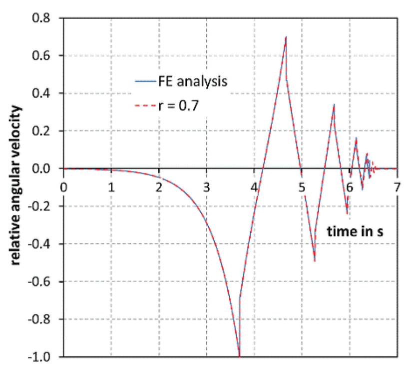

Figure 6. Time histories of the angular velocity for model 1

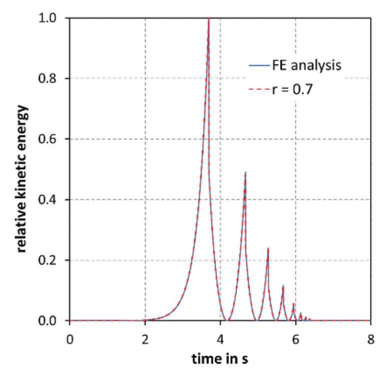

Figure 7. Time histories of the kinetic energy for model 1

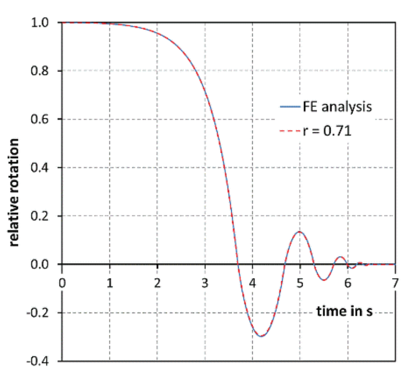

Figure 8. Time histories of the rotation for model 2 with 2p contact and a material like steel

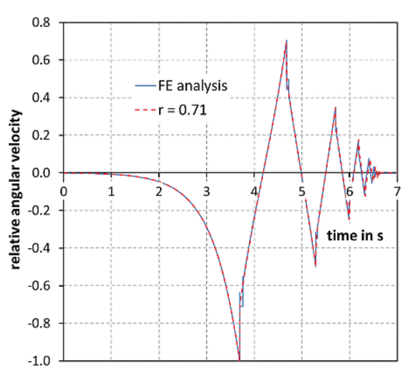

Figure 9. Time histories of the angular velocity for model 2 with 2p contact and a material like steel

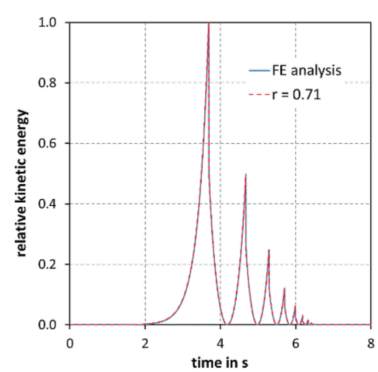

Figure 10. Time histories of the kinetic energy for model 2 with 2p contact and a material like steel

The results for the plane model with an infinite elastic ground of a material like steel (modulus of elasticity

E = 200 GPa, density ϱ = 7800 kg/m³) and a two-point (2p) contact (points A and B in Figure 1) are shown in Figures 8 to 10. The comparison between the curves of the FE analysis and the theoretical curves with the best estimate r = 0.71 show a very good agreement. Equation (2) gives with r = 0.7 a slightly lower value.

The results for the plane model with the same ground but a three-point (3p) contact (points A and B, middle point of the bottom edge) are shown in Figures 11 to 13. The curves of the FE analysis and the theoretical curves with the best estimate value r = 0.75 show a very good agreement. The best estimate value 0.75 is larger than the value 0.7 from Equation (2) and the value 0.71 for the 2p contact. A plane model with a complete line contact and the same materials gives also r = 0.75 as best estimate value and shows also a very good agreement with the theoretical curves.

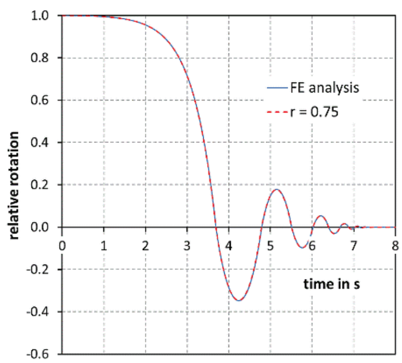

Figure 11. Time histories of the rotation for model 2 with 3p contact and a material like steel

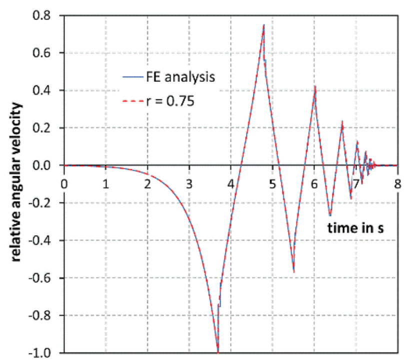

Figure 12. Time histories of the angular velocity for model 2 with 3p contact and a material like steel

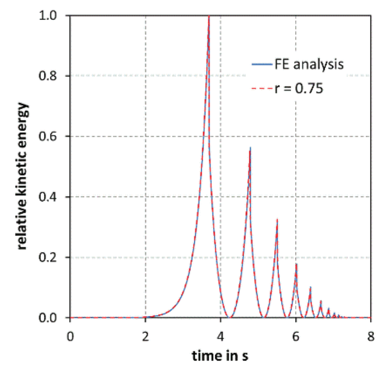

Figure 13. Time histories of the kinetic energy for model 2 with 3p contact and a material like steel

To study the influence of slight deviations from the ideal rectangular corner geometry model 2 is modified with bevelled corners (1 mm over 200 mm length). The material properties are the same as in the previous calculations. The results are shown in Figures 14 to 16. There is a remarkably good agreement between the FE curves and the theoretical curves with the best estimate value r = 0.82. It is noted, that this small deviations from the ideal geometry has a significant influence on the results. The value 0.7 from Equation (2) is not useable.

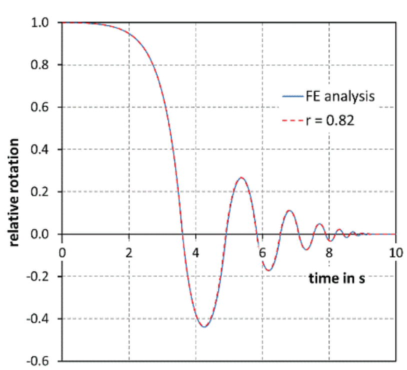

Figure 14. Time histories of the rotation for model 2 with bevelled corners and a material like steel

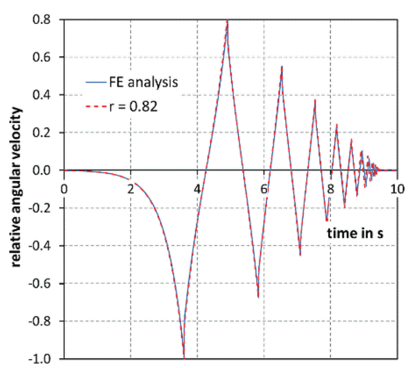

Figure 15. Time histories of the angular velocity for model 2 with bevelled corners (steel)

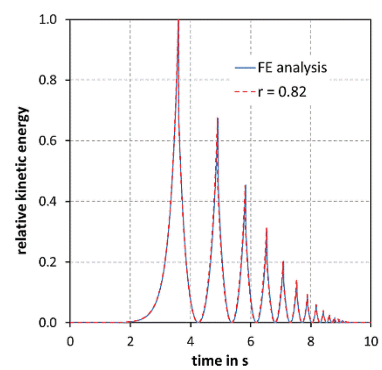

Figure 16. Time histories of the kinetic energy for model 2 with bevelled corners (steel)

The results for model 3 with line contact and a material like steel are shown in Figures 17 to 19. In this model the ground is a finite thick plate which is infinitely extended in the horizontal direction. The obtained coefficient of restitution is identical with the coefficient for model 2 with a half-infinite ground. The agreement between the FE curves and the theoretical curves are very good for higher amplitudes.

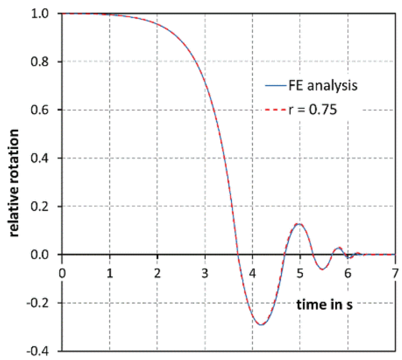

Figure 17. Time histories of the rotation for model 3 with line contact and a material like steel

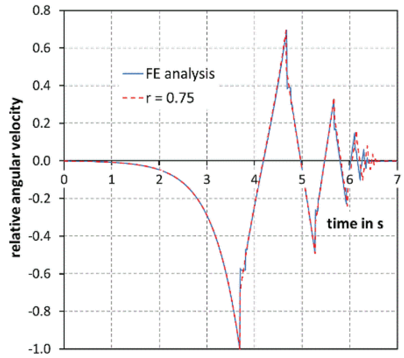

Figure 18. Time histories of the angular velocity for model 3 (line contact) and a material like steel

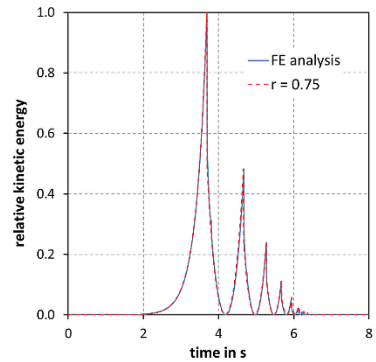

Figure 19. Time histories of the kinetic energy for model 3 (line contact) and a material like steel

In many cases anti-slip pads are used to prevent sliding. To investigate the effect of such a pad on the motion, model 2 is modified by a 10 mm thick pad between the block and the ground. The material of the pad is assumed to be elastic with a modulus of 50 MPa (density 1200 kg/m³, Poisson ratio 0.49). The results with line contact and a material like steel for the ground are shown in Figures 20 to 22.

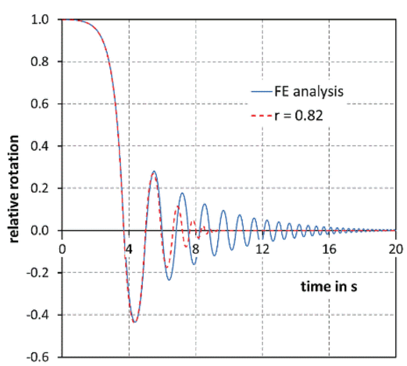

Figure 20. Time histories of the rotation for model 2 with line contact and an anti-slip pad

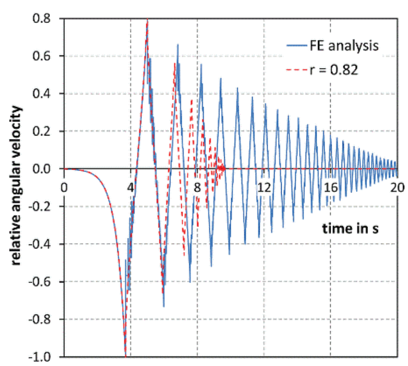

Figure 21. Time histories of the angular velocity for model 2 with line contact and an anti-slip pad

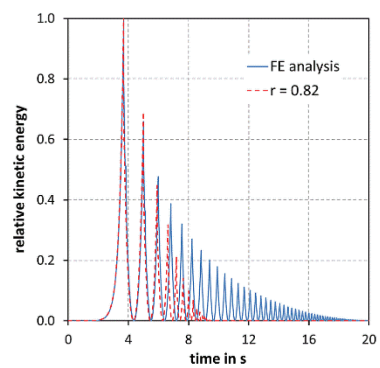

Figure 22. Time histories of the kinetic energy for model 2 with line contact and an anti-slip pad

The summary of the obtained results for the coefficients of restitution is given in Table 2. The ideal elastic-plastic material models use a yield stress of 200 MPa for steel and 50 MPa for concrete (modulus of elasticity E = 35 GPa, density ϱ = 2500 kg/m³). The coefficients of restitution for the models with 2p contact are nearly identical with the coefficient 0.7 from Equation (2). In all other cases the obtained coefficients of restitution are always higher than the theoretical value of 0.7. With the exception of the model with the anti-slip pad the agreement between the FE and theoretical curves are very good if the coefficients of restitution are based on the FE analyses. The results in Table 2 show that geometric deviations cause the largest differences. For the model with the anti-slip pad Equations (1) are not usable for the description of the motion of the block. In this case the coefficients of restitution strongly depend on the amplitude of the rocking motion. Additional studies using 3D models (rectangular block) show that the obtained results are also valid for these models.

Table 2: Summary of the obtained coefficients of restitution

model	block	ground	contact	r
1	rigid	spring, damper	2-point contact	0.700
2	rigid	elastic, steel	2-point contact	0.705
2	rigid	elastic, steel	3-point contact	0.752
2	rigid	elastic, steel	line	0.752
2	rigid	ideal elastic-plastic, steel	line	0.760
2	rigid	elastic, concrete	line	0.733
2	rigid	ideal elastic-plastic, concrete	line	0.746
2	rigid	elastic, steel	line, bevelled	0.823
3	rigid	elastic, steel	line	0.752
2	rigid	elastic, steel, pad	line	0.82 … 0.91

Remarks to seismic excitation

The dynamic response of the body does not only depend on the geometric parameters and the coefficient of restitution but also on the type and strength of the excitation. Therefore, the excitation is further investigated. Since the previous studies have shown a very good agreement between the numerical solutions of Equations (1) with the best estimate coefficient of restitution and FE analyses, the calculations are performed by using the numerical integration of the equations of motion.

For a closer study the block is excited by sine vibrations with an amplitude of 1 g and frequencies between 0.81 and 1.0 Hz. The results for a coefficient of restitution of 0.82 (largest value from Table 1) are shown in Figure 23. A classical resonance like for a harmonic oscillator does not occur. Nevertheless, the rotation amplitude increases during the first cycles by a factor of about 2.5 for a pure sine excitation. The maximum amplitude occurs at a frequency of 0.81 Hz in half cycle 7. For a verification of the results, some FE analyses using the model 2 have been performed. The obtained results confirm the previous statements on the very good agreement between theoretical and FE results.

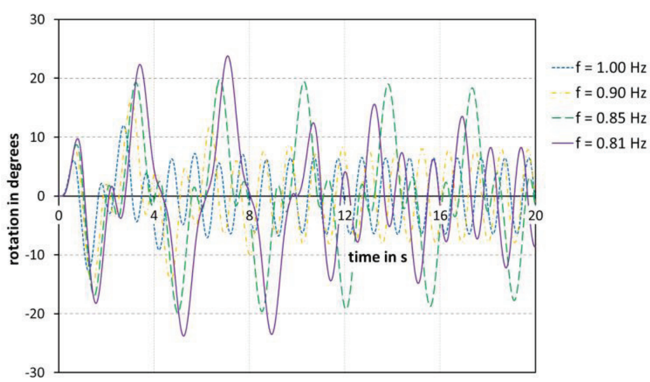

Figure 23. Time history of the rotations for r = 0.82 and sine excitation

For higher frequencies the maximum rotation amplitudes are significantly reduced in accordance with the Housner model. Additional calculations with normalized time histories (1.0g PGA) using earthquake ground accelerations from the European strong-motion database give smaller maximum rotation angles. The reason for this behaviour is that the low frequencies are almost not included in these time histories.

Additional FE analyses with 3D models (rectangular block) give similar results and dependencies.

CONCLUSIONS

The rocking motion and especially the impact of unanchored bodies under seismic excitation were investigated by numerical integration of the equations of motion and with the FE method. For the numerical integration the classical fourth-order Runge-Kutta method and for the FE analyses the program Abaqus was used. The obtained results for a rectangular block with an aspect ratio of 2 for models with geometric variations, different contact conditions and different properties of the ground result in the following conclusions especially for the coefficients of restitution:

- Rigid FE models without additional damping are not suitable because they produce an unrealistic bouncing and jumping. For 2D or 3D FE models the usage of infinite elements is recommended.
- There are no relevant differences between the coefficients of restitution for an impact on a half-infinite ground and horizontal infinite plate with a finite thickness.
- All variations from the ideal geometry and contact conditions cause larger coefficients of restitution than the theoretical value 0.7 of the Housner model. The maximum values for the coefficients are about 0.9. The consideration of plasticity gives slightly higher values as in the elastic case. It is recommended to use a value of at least 0.82 for theoretical calculations.
- With the exception of the model with the anti-slip pad the agreement between results from FE analyses and the numerical integration of the equations of motion is very good, if the coefficients of restitution are based on best estimate values from FE analyses.
- Anti-slip pads of linear elastic material lead to greater coefficients of restitution. The coefficients of restitution strongly depend on the amplitude of the rocking motion. Therefore, the theoretical model is not usable. The usage of anti-slip pads can only be recommended if overturning is not a problem.
- In all cases there is a large loss of kinetic energy by the impact. For the models without a pad the loss of kinetic energy exceeds 30% and for the model with a pad it is higher than 17%. This loss is caused by the radiation damping of the ground. This damping is much higher than the material damping of the ground.
- A resonance like for a harmonic oscillator does not occur. Nevertheless, the rotation amplitude increases during the first cycles by a factor of about 2.5 for a pure sine excitation.

First investigations using 3D models with multiaxial excitation show additional effects like a rolling off around the edges. These effects will be studied in future analyses.

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