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FORMULATION OF EQUIPMENT-STRUCTURE INTERACTION FOR EQUIPMENT WITH MULTIPLE SUPPORTS

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ABSTRACT

Usage of 3D FEM models of building at Nuclear Plant has been paid attention recently to evaluate seismic influence of equipment. It is known that, in the 3D FEM model, large local deformation may be obtained due to out-of-plane modes of floors and walls; the equipment responses should be evaluated properly when support locations have such large local deformation.

When dynamic building responses are applied to the equipment, quite conservative design results are often obtained when equipment and building are at the resonance condition. It is anticipated that an appropriate evaluation method be constructed considering equipment-structure interaction (ESI) effects which could reduce the building responses.

It is important to take an advantage of ESI effects to obtain a rational design input motion of the building for dynamic analysis of equipment. The simplest way to consider the ESI effect is incorporating equipment analysis model to the 3D FEM model of building at Nuclear Plant. However, it may be unsuitable to incorporate the equipment analysis model into the 3D FEM model; the 3D FEM model is so large that it is complicated to alter equipment configurations in the trial-and-error design process.

Although the ESI formulation for a single degree of freedom system has been established, it is not applicable to the equipment with multiple supports, which may be subjected to different seismic excitations. The new ESI formulation is developed in a mathematically rigorous way for a multiple degree freedom system with multiple supports employing the transfer function synthetic method with two substructures of equipment and building: the building compliance functions at support locations and equipment response transfer function are used.

In this paper, the computational procedures are described in a step-by-step way and a numerical analysis example of simple piping system is presented. It is confirmed that the newly developed formula provides the correct results numerically by comparing with results of combined analysis of equipment and building.

Introduction

Usage of 3D FEM model of building at Nuclear Plant has been paid attention recently to evaluate seismic influence of equipment. The usage of 3D FEM model may lead large local deformation caused by out-of-plane modes of floors and walls; equipment responses should be evaluated properly when support locations have such large deformation.

The application of dynamic building response for equipment often leads to excessive design conservatisms, especially when the equipment frequencies coincide with one of the natural frequencies of the building (at the resonance condition). Proper seismic evaluation can be made by considering equipment-structure interaction (ESI) in the seismic response analysis of equipment.

It is important to consider ESI effects to obtain a rational design input motion of the building for dynamic analysis of equipment. The simplest way to consider the ESI effect is incorporating equipment analysis model into the 3D FEM model of building at Nuclear Plant; however, it may be complicated to incorporate the equipment analysis model, because the 3D FEM model is too large to alter equipment configurations in trial-and-error design process.

Although the ESI formulation for a single degree of freedom (SDOF) system has been established (Tseng 89, Scott 17) , it is not applicable to the equipment, such as piping system, with multiple supports which may be subjected to different seismic excitations. The ESI formulation is newly derived for equipment of multiple degree of freedom (MDOF) system with multiple supports employing the transfer function synthetic method with two substructures of equipment and building. The ESI effects are materialized by the building compliance (displacement due to harmonic unit force) functions at support locations and the equipment response transfer function.

In this paper, the computational procedures are described in a step-by-step way and a numerical analysis example of simple piping system is presented. It is confirmed that the newly developed formula provides the correct results numerically by comparing with results of combined analysis of equipment and building.

FORMULATION

Overview

For ESI formulation for the equipment of MDOF system with multiple supports, the transfer function synthetic method (Nagamatsu 84) is applied using two substructures as shown in Figure 1; one is equipment and other is building.

When the equipment and the building are not connected, relation of the building acceleration $\{\ddot{U}_s\}$ and the equipment acceleration $\{\ddot{U}_e\}$ and relation of the seismic excitation for the building $\ddot{Z}_g$ and the building acceleration $\{\ddot{U}_s\}$ are expressed as,

$$\{\ddot{U}_e\} = [H_{ea}]\{\ddot{U}_s\} \quad (1)$$

$$\{\ddot{U}_s\} = \{H_{sa}\}\ddot{Z}_g \quad (2)$$

where $[H_{ea}]$ and $\{H_{sa}\}$ are the response transfer functions of equipment and building, respectively. Please note that vectors and matrices are used for accelerations and transfer functions corresponding to MDOF system of the equipment.

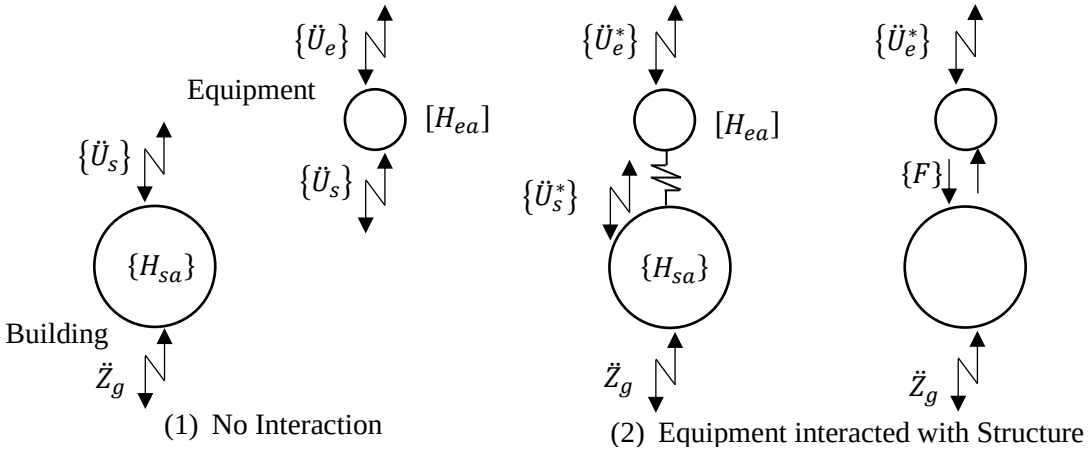

Figure 1 Overview of Formulation

When the equipment and the building is connected, building acceleration with ESI $\{\ddot{U}_s^*\}$ can be expressed, including reactions $\{F\}$ produced by springs connecting the equipment and the building, as,

$$\{\ddot{U}_s^*\} = \{H_{sa}\}\ddot{Z}_g + [G_{sa}]\{F\} \tag{3}$$

where $[G_{sa}]$ is the accelerance function of the building at support locations. The term of $[G_{sa}]\{F\}$ represents relation between $\{F\}$ and $\{\ddot{U}_s^*\}$ properly; the building accelerations at the support locations are produced by the reactions not only from the own supports but also from other supports.

Using Equations (1) to (3), the ESI transfer function $[H_{esi}]$ between $\{\ddot{U}_s\}$ and $\{\ddot{U}_s^*\}$ is to be derived hereinafter; the ESI transfer function $[H_{esi}]$ is composed of the equipment information and the local building information. Once $\{\ddot{U}_s^*\}$ is obtained, the sectional forces of equipment with ESI $\{F_e^*\}$ can be easily obtained using the equipment sectional force transfer function $[H_{ef}]$.

Equipment Response Transfer Function $[H_{ea}]$

For a MDOF system representing an equipment having mass $[M_e]$, damping $[C_e]$ and stiffness $[K_e]$, the equation of motion subjected to the time history seismic excitations of velocity $\{\dot{u}_s\}$ and displacement $\{u_s\}$ at support locations can be written as,

$$[M_e]\{\ddot{u}_e\} + [C_e]\{\dot{u}_e\} + [K_e]\{u_e\} = -[B_{ec}]([C_c]\{\dot{u}_s\} + [K_c]\{u_s\}) \tag{4}$$

where $\{\ddot{u}_e\}$, $\{\dot{u}_e\}$ and $\{u_e\}$ are absolute acceleration, velocity, and displacement of the MDOF system, respectively. $[C_c]$ and $[K_c]$ are diagonal matrix for damping and stiffness of spring connecting the equipment and the building. Hereinafter, it is assumed that each node has one degree of freedom for simple explanation; one node actually has six degrees of freedom. $[B_{ec}]$ is the matrix, with $n_e \times n_c$, where n_e is number of the equipment nodes and n_c is the number of the supports. $[B_{ec}]$ assigns the seismic excitations at supports to proper equipment nodes; the element of i-th row and j-th column of $[B_{ec}]$ equals one if i-th equipment node is connected to j -th support and it equals 0 otherwise. (Please refer to **SUPPLEMENTARY EXPLANATION** which provides further explanation of $[B_{ec}]$ using a simple example.) In Equation (4), absolute accelerations are introduced deliberately to enable individual seismic excitations at multiple support locations; adoption of relative accelerations allows only one seismic motion applied to the multiple support locations.

Equation (4) can be rewritten in the frequency domain as follows,

$$(-\omega^2[M_e] + i\omega[C_e] + [K_e])\{U_e\} = [B_{ec}](i\omega[C_c] + [K_c])\{U_s\} \quad (5)$$

or

$$(-\omega^2[M_e] + i\omega[C_e] + [K_e])\{\ddot{U}_e\} = [B_{ec}](i\omega[C_c] + [K_c])\{\ddot{U}_s\} \quad (6)$$

where $\{\ddot{U}_e(\omega)\}$ and $\{\ddot{U}_s(\omega)\}$ are the Fourier Transforms of $\{\ddot{u}_e(t)\}$ and $\{\ddot{u}_s(t)\}$. By multiplying inverse matrix of the coefficient of $\{\ddot{U}_e\}$ to both sides of Equation (6), it can be rewritten as,

$$\{\ddot{U}_e\} = (-\omega^2[M_e] + i\omega[C_e] + [K_e])^{-1}[B_{ec}](i\omega[C_c] + [K_c])\{\ddot{U}_s\}$$

Thus, Equipment Response Transfer function $[H_{ea}]$ in Equation (1) is defined as,

$$\{\ddot{U}_e\} = [H_{ea}]\{\ddot{U}_s\} \quad \text{Reappeared (1)}$$

$$[H_{ea}] = (-\omega^2[M_e] + i\omega[C_e] + [K_e])^{-1}[B_{ec}](i\omega[C_c] + [K_c]) \quad (7)$$

$[H_{ea}]$ generates equipment acceleration without ESI $\{\ddot{U}_e\}$ when building acceleration without ESI $\{\ddot{U}_s\}$ is used as an input (Equation 1). Similarly, $[H_{ea}]$ generates equipment acceleration $\{\ddot{U}_e^*\}$ when building acceleration $\{\ddot{U}_s^*\}$ is used as an input as shown Figure 1.

$$\{\ddot{U}_e^*\} = [H_{ea}]\{\ddot{U}_s^*\} \quad (8)$$

ESI Transfer Function $[\boldsymbol{H_{esi}}]$

As introduced previously, the building acceleration without ESI $\{\ddot{U}_s\}$ and building acceleration with ESI $\{\ddot{U}_s^*\}$ are expressed as,

$$\{\ddot{U}_s\} = \{H_{sa}\}\ddot{Z}_g \quad \text{Reappeared (2)}$$

$$\{\ddot{U}_s^*\} = \{H_{sa}\}\ddot{Z}_g + [G_{sa}]\{F\} \quad \text{Reappeared (3)}$$

Reactions $\{F\}$, produced by springs connecting the equipment and the building, is expressed as,

$$\{F\} = (i\omega[C_c] + [K_c])([B_{ec}]^T\{U_e^*\} - \{U_s^*\}) \quad (9)$$

where the term of $(i\omega[C_c] + [K_c])$ is the support damping and stiffness and the term of $([B_{ec}]^T\{U_e^*\} - \{U_s^*\})$ is the relative displacement between the equipment and the building at the support locations. Please note that the term of $[B_{ec}]^T\{U_e^*\}$ is the absolute displacements of the equipment nodes where the supports are connected.

By substituting Equations (2) and (9) to Equation (3) and considering $\{U_e^*\} = -\{\ddot{U}_e^*\}/\omega^2$ and $\{U_s^*\} = -\{\ddot{U}_s^*\}/\omega^2$, Equation (3) can be rewritten as,

$$\{\ddot{U}_s^*\} = \{\ddot{U}_s\} - \frac{[G_{sa}]}{\omega^2}(i\omega[C_c] + [K_c])\left([B_{ec}]^T\{\ddot{U}_e^*\} - \{\ddot{U}_s^*\}\right) \quad (10)$$

By considering $[G_{sa}] = -\omega^2[G_{sd}]$ ($[G_{sd}]$ is compliance functions), and $\{\ddot{U}_e^*\} = [H_{ea}]\{\ddot{U}_s^*\}$ (Equation (8)), Equation (10) can be further rewritten as,

$$\{\ddot{U}_s^*\} = \{\ddot{U}_s\} + [G_{sd}](i\omega[C_c] + [K_c])\left([B_{ec}]^T[H_{ea}]\{\ddot{U}_s^*\} - \{\ddot{U}_s^*\}\right) \quad (11)$$

Please note that, in Equation (11), the information of the entire building, $\{H_{sa}\}$ and $\ddot{Z}_g$, is replaced with the local information of the building, $[G_{sd}]$ and $\{\ddot{U}_s\}$. $[G_{sd}]$ is a matrix with $n_c \times n_c$ and the element of i-th row and j-th column is corresponding to the displacement of i-th support location subjected to harmonic unit force at j-th support location.

After several times of multiple matrix manipulations, Equation (11) becomes as,

$$\{\ddot{U}_s^*\} = \left([I] + [G_{sd}](i\omega[C_c] + [K_c])([I] - [B_{ec}]^T[H_{ea}])\right)^{-1}\{\ddot{U}_s\}$$

Thus, the ESI transfer function $[H_{esi}]$ is now defined as,

$$\{\ddot{U}_s^*\} = [H_{esi}]\{\ddot{U}_s\} \tag{12}$$

$$[H_{esi}] = \left([I] + [G_{sd}](i\omega[C_c] + [K_c])([I] - [B_{ec}]^T[H_{ea}])\right)^{-1} \tag{13}$$

It is noted that $\{\ddot{U}_s\}$ can be converted to $\{\ddot{U}_s^*\}$ with $[G_{sd}]$, $[H_{ea}]$ and $(i\omega[C_c] + [K_c])$. Once $\{\ddot{U}_s^*\}$ is obtained, the equipment sectional forces with ESI $\{F_e^*\}$ can be easily obtained by using the equipment sectional force transfer function $[H_{ef}]$ as,

$$\{F_e^*\} = [H_{ef}]\{\ddot{U}_s^*\} \tag{14}$$

COMPUTATIONAL PROCEDURE

Based on the ESI formulation presented in the previous section, the building acceleration with ESI $\{\ddot{U}_s^*\}$ and equipment sectional forces with ESI $\{F_e^*\}$ can be generated by the procedure listed in Table 1. The column of Code shows the computer code used in the numerical analysis example in this paper.

NUMERICAL ANALYSIS EXAMPLE

Table 2 shows a numerical analysis example of the analysis with the ESI formula. The analysis model is shown in Sketch of Table 2; the lumped mass stick model with dark blue dots is building floor and the lumped mass stick model with light blue dots is piping system with two supports.

Three analyses are carried out. The first is the analysis of the building floor only to obtain the responses of the building floor without ESI as an input for analysis with the ESI formula. The second is the analysis with ESI formula following the computational procedure listed in Table 1. The third is the analysis of the combined model of the piping system and the building floor which provides the numerically correct results of ESI. The results obtained from the first and second analyses together are compared with that of third analysis to confirm the validity of the ESI formula presented in this paper.

Natural frequencies are 9.2 Hz and 9.8 Hz for the piping system and the building floor respectively; they are at the resonance condition. The damping ratios are 1 % and 5 % for the piping system and building floor respectively. The mass ratio of the piping system to the building floor is 0.07.

The left end node of the piping system, Node 1, and the left support of the building floor, Support 1, are selected to show the time histories in Table 2. It is confirmed that ESI formula provides the exact results numerically, by comparing results of analysis with ESI and those of the combined analysis, for the building floor acceleration at Support 1 $\ddot{U}_{s\cdot1}^*$ and piping acceleration at Node 1 $\ddot{U}_{e\cdot1}^*$.

Table 1: Computational Procedure

Step	Explanation	Code
1	Conduct Fourier Transform of the time history seismic excitations at the support locations $\{\ddot{u}_s(t)\}$to obtain $\{\ddot{U}_s(\omega)\}$.	Matlab®
2	For given equipment support locations in the building, compute the building compliance functions $[G_{sd}]$, using the standard frequency domain dynamic analysis procedure to determine the steady-state displacement response of the building subjected to unit harmonic force. The element of i-th row and j-th column of $[G_{sd}]$ is the displacement of i-th support location subjected to the harmonic unit force at j-th support location.	Nastran®
3	Compute the equipment response transfer function $[H_{ea}]$ by Equation (7).	Matlab®
4	Construct diagonal matrix $[K_c]$ and $[C_c]$ based on support springs connecting equipment and building.	Matlab®
5	Construct $[B_{ec}]$. The element of i-th row and j-th column of $[B_{ec}]$ equals one if i-th equipment node has connected to j-th support and it equals 0 otherwise.	Matlab®
6	Compute the ESI Transfer function $[H_{esi}]$ by applying Equation (13).	Matlab®
7	Compute the building acceleration with ESI $\{\ddot{U}_s^*\}$ by applying Equation (12).	Matlab®
8	Compute the equipment sectional force transfer function $[H_{ef}]$, using the standard frequency domain dynamic analysis; the element of i-th row and j-th column of $[H_{ef}]$ is the steady-state sectional force response of i-th equipment node subjected to harmonic unit acceleration excitations at the j-th support.	Nastran®
9	Obtain the sectional forces with ESI $\{F_e^*\}$ by performing frequency domain convolution of $[H_{ef}]$ with $\{\ddot{U}_s^*\}$. Obtain the time history of the sectional force with ESI $\{f_e^*(t)\}$ by performing inverse Fourier Transform of $\{F_e^*(\omega)\}$.	Matlab®

To examine ESI effects in detail, the transfer functions $[H_{ea}]$ and $[H_{esi}]$ produced in the above-mentioned numerical analysis example are presented here. Equations (1) and (12) can be rewritten considering numbers of equipment nodes and supports,

$$\begin{Bmatrix} \ddot{U}_{e\cdot1} \\ \ddot{U}_{e\cdot2} \\ \ddot{U}_{e\cdot3} \\ \ddot{U}_{e\cdot4} \\ \ddot{U}_{e\cdot5} \end{Bmatrix} = \begin{bmatrix} H_{ea\cdot11} & H_{ea\cdot12} \\ H_{ea\cdot21} & H_{ea\cdot22} \\ H_{ea\cdot31} & H_{ea\cdot32} \\ H_{ea\cdot41} & H_{ea\cdot42} \\ H_{ea\cdot51} & H_{ea\cdot52} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{s\cdot1} \\ \ddot{U}_{s\cdot2} \end{Bmatrix}$$

$$\begin{Bmatrix} \ddot{U}_{s\cdot1}^* \\ \ddot{U}_{s\cdot2}^* \end{Bmatrix} = \begin{bmatrix} H_{esi\cdot11} & H_{esi\cdot12} \\ H_{esi\cdot21} & H_{esi\cdot22} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{s\cdot1} \\ \ddot{U}_{s\cdot2} \end{Bmatrix}$$

In Figure 2, the graphs of functions $H_{ea\cdot11}$ and $H_{esi\cdot11}$ are shown. Function $H_{ea\cdot11}$ has very sharp peak at the natural frequency of the equipment, 9.2 Hz, as expected. Function $H_{esi\cdot11}$ has concave shape from 8.8 Hz to 10 Hz with the magnification factor of 0.4 and two peaks at 8.2 Hz and 11 Hz with magnification factors of 2.0 and 1.5 respectively. Owing to ESI effects, building floor acceleration is reduced around the natural frequency of the piping system.

Figure 3 shows the building floor accelerations $\ddot{U}_{s\cdot1}$ and $\ddot{U}_{s\cdot1}^*$. $\ddot{U}_{s\cdot1}(\omega)$ is Fourier Transform of $\ddot{u}_{s\cdot1}(t)$ obtained by the analysis of the building floor alone. $\ddot{U}_{s\cdot1}^*$ is converted from $\{\ddot{U}_s\}$ by applying Equation (12).

Figure 3 demonstrates that, at the resonance range of the piping system and the building floor, from 8.8 Hz to 10 Hz, $\ddot{U}^*_{s\cdot 1}$ is dramatically decreased compared with $\ddot{U}_{s\cdot 1}$ though two new peaks appear at 8.2 Hz and 11 Hz.

Figure 4 shows the piping accelerations with ESI $\ddot{U}^*_{e\cdot 1}$ obtained by Equation (8) and those without ESI $\ddot{U}_{e\cdot 1}$ by Equation (1). It is revealed that the response of the piping system can be reduced dramatically as one-third by taking account ESI.

CONCLUSIONS

The equipment-structure interaction formulation for equipment with multiple supports, which may be subjected to different seismic excitation, is newly developed in a mathematically rigorous way. The transfer function synthetic method is employed with two substructures; equipment and building.

In the equation of motion for a MDOF system, the absolute accelerations are introduced deliberately to enable different seismic excitations at multiple support locations. (Adoption of relative accelerations allows only single seismic motion for multiple support locations.)

The computational procedures are also described as step-by-step procedure. Finally, the numerical analysis example of piping system applying ESI formula is presented together with results of the combined analysis of the piping system and the building floor; the combined analysis provides the exact results considering ESI effects. It is confirmed that analysis with ESI formula can produce the right results. The response reduction effects by ESI are also confirmed by the numerical analysis example.

Up to now, equipment analysis model must be incorporated into 3D FEM model of building at Nuclear Plant to consider ESI effects. It should be emphasized that the of ESI effects can be evaluated relatively easily with only the limited local information of building, such as responses and compliance functions at support locations, by employing the ESI formulation presented in this paper.

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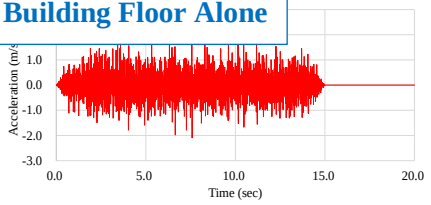
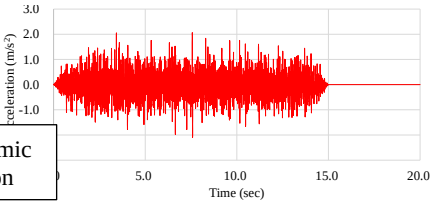
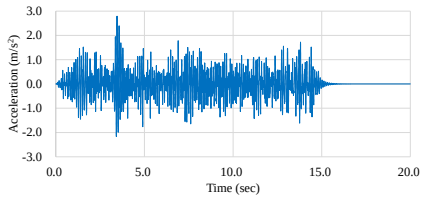
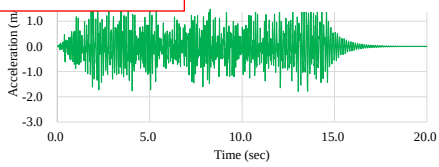
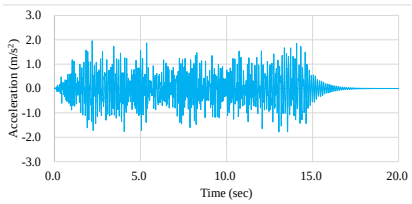
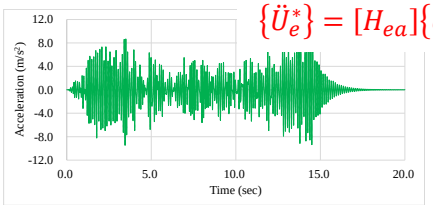
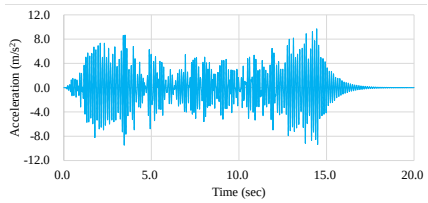
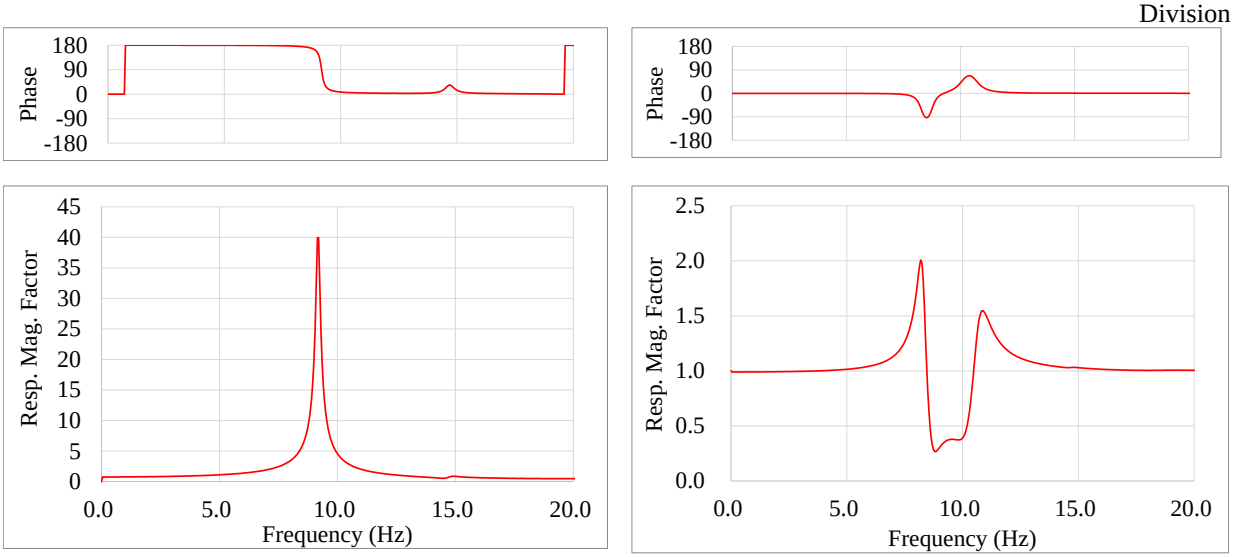
Table 2: Numerical Analysis Example

	(1) Building Floor Alone (2) ESI Formula	(2) Piping System-Building Floor Combined
Sketch	$\ddot{U}^*_{e\cdot1}$ $f = 9.2\text{Hz}, h = 1\%$ Node 1 2 3 4 5 Support 1 Support 2 $\ddot{U}^*_{s\cdot1}$ $\ddot{U}^*_{s\cdot2}$ $\{\ddot{U}^*_s\} = [H_{esi}]\{\ddot{U}_s\}$ $\ddot{U}_{s\cdot1}$ $\ddot{U}_{s\cdot2}$ $\ddot{Z}_g$ $f = 9.8\text{Hz}, h = 5\%$ $\ddot{Z}_g$	$\ddot{U}^*_{e\cdot1}$ Mass Ratio: 0.07 Node 1 2 3 4 5 Support 1 Support 2 $\ddot{Z}_g$ $\ddot{U}^*_{s\cdot1}$ $\ddot{U}^*_{s\cdot2}$ $\ddot{Z}_g$
$\ddot{Z}_g$	**(1) Building Floor Alone** 	Same Seismic Excitation 
$\ddot{U}_{s\cdot1}$		**(3) Combined Analysis**
$\ddot{U}^*_{s\cdot1}$	**(2) ESI Formula** $\{\ddot{U}^*_s\} = [H_{esi}]\{\ddot{U}_s\}$ 	
$\ddot{U}^*_{e\cdot1}$	$\{\ddot{U}^*_e\} = [H_{ea}]\{\ddot{U}^*_s\}$ 	

$\ddot{Z}_g$: Seismic Excitation for Building Floor
$\ddot{U}_{s\cdot1}$: Building Floor Acceleration at Support Location 1 without ESI
$\ddot{U}^*_{s\cdot1}$: Building Floor Acceleration at Support Location 1 with ESI
$\ddot{U}^*_{e\cdot1}$: Piping Response at Node 1 with ESI



(1) $H_{ea\cdot 11}$ (2) $H_{esi\cdot 11}$

Figure 2 Transfer Functions of Numerical Analysis Example

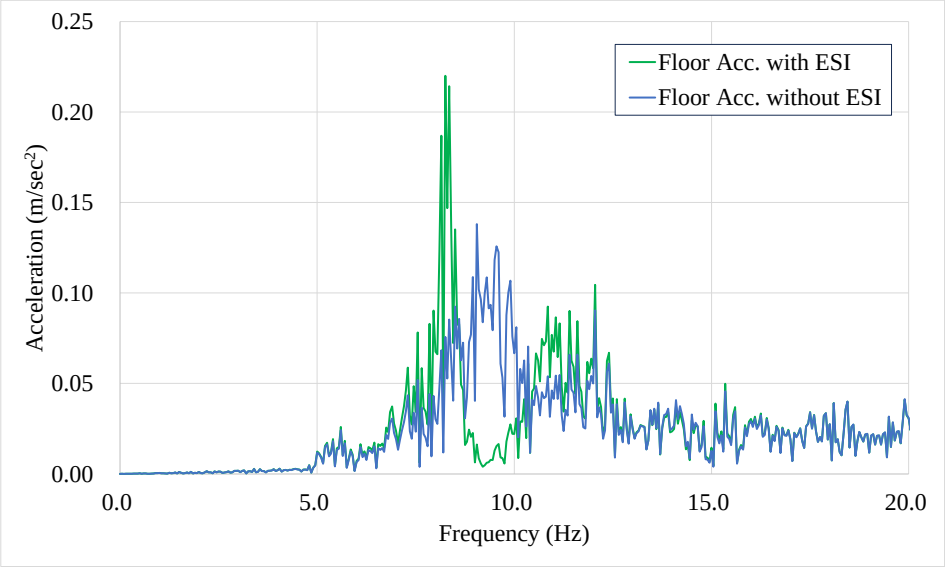


Figure 3 Building Floor Accelerations $\ddot{U}_{s\cdot 1}$ and $\ddot{U}^{*}_{s\cdot 1}$ of Numerical Analysis Example

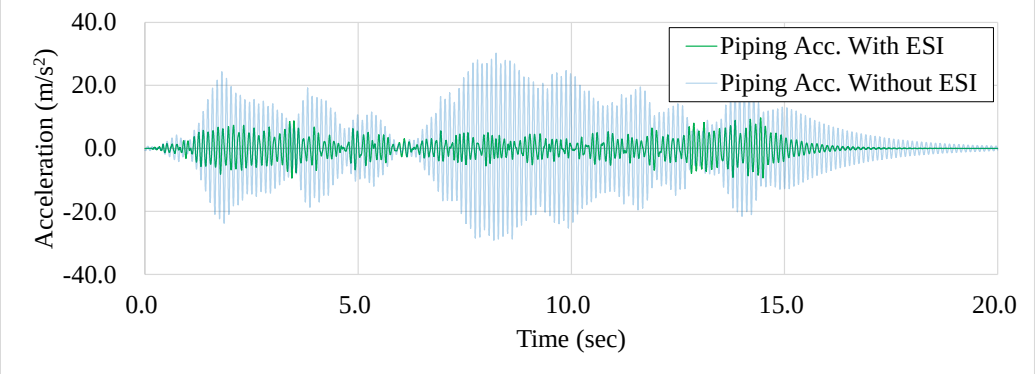


Figure 4 Piping Acceleration $\ddot{U}_{e\cdot 1}$ and $\ddot{U}^{*}_{e\cdot 1}$of Numerical Analysis Example

SUPPLEMENTARY EXPLANATION

As shown Figure S1, a simple piping system with three lumped mass and two supports are introduced here to facilitate the understanding $[B_{ec}]$ and $[K_c]$in Equation (4). By neglecting damping and assuming that each node has one degree of freedom, the equilibrium at each node is expressed as,

$$m_1\ddot{u}_{e1} + k_{e1}(u_{e1} - u_{e2}) + k_{c1}(u_{e1} - u_{s1}) = 0 \quad \text{(S1)}$$
$$m_2\ddot{u}_{e2} + k_{e1}(u_{e2} - u_{e1}) + k_{e2}(u_{e2} - u_{e3}) = 0 \quad \text{(S2)}$$
$$m_3\ddot{u}_{e3} + k_{e2}(u_{e3} - u_{e2}) + k_{c2}(u_{e3} - u_{s2}) = 0 \quad \text{(S3)}$$

where u_{s1} and u_{s2} are displacement excitations at the building floor. Equations (S1) to (S3) can be rewritten in matrix form as,

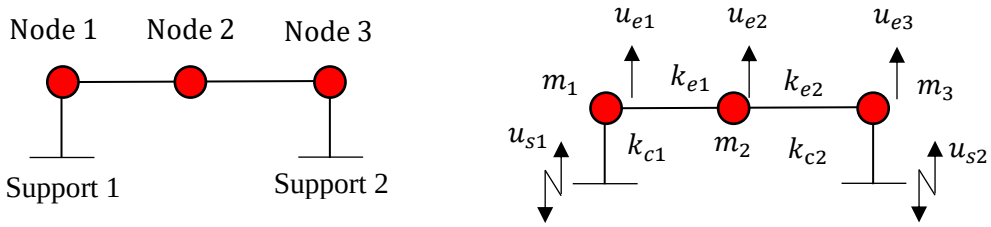


Figure S1 Simple Piping System with Two Supports

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{u}_{e1} \\ \ddot{u}_{e2} \\ \ddot{u}_{e3} \end{Bmatrix} + \begin{bmatrix} k_{e1}+k_{c1} & -k_{e1} & 0 \\ -k_{e1} & k_{e1}+k_{e2} & -k_{e2} \\ 0 & -k_{e2} & k_{e2}+k_{c2} \end{bmatrix}\begin{Bmatrix} u_{e1} \\ u_{e2} \\ u_{e3} \end{Bmatrix} = \begin{Bmatrix} k_{c1}u_{s1} \\ 0 \\ k_{c2}u_{s2} \end{Bmatrix} \quad \text{(S4)}$$

Equation (S4) can be further rewritten as,

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{u}_{e1} \\ \ddot{u}_{e2} \\ \ddot{u}_{e3} \end{Bmatrix} + \begin{bmatrix} k_{e1}+k_{c1} & -k_{e1} & 0 \\ -k_{e1} & k_{e1}+k_{e2} & -k_{e2} \\ 0 & -k_{e2} & k_{e2}+k_{c2} \end{bmatrix}\begin{Bmatrix} u_{e1} \\ u_{e2} \\ u_{e3} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} k_{c1} & 0 \\ 0 & k_{c2} \end{bmatrix}\begin{Bmatrix} u_{s1} \\ u_{s2} \end{Bmatrix}$$

$$[M_e]\{\ddot{u}_e\} + [K_e]\{u_e\} = -[B_{ec}][K_c]\{u_s\} \quad \text{(S5)}$$

In Equation (S5), $[B_{ec}]$ and $[K_c]$ are defined as,

$$[B_{ec}] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$
$$[K_c] = \begin{bmatrix} k_{c1} & 0 \\ 0 & k_{c2} \end{bmatrix}$$

$[B_{ec}]$ is the matrix to assign the seismic excitation at supports to proper equipment nodes; the i-th row and j-th column of $[B_{ec}]$ equals one if i-th equipment node has connected to j-th support and it equals 0 otherwise. $B_{ec\cdot 11}$ and $B_{ec\cdot 32}$ equal one because Piping Node 1 is connected to Support 1 and Piping Node 3 is connected to Support 2, respectively. $[K_c]$ is diagonal stiffness matrix for spring connecting the piping system and the building floor. Please note that the support stiffnesses k_{c1} and k_{c2} are appeared not only in $[K_c]$ but also in $[K_e]$.