Transactions of the 15th International Conference on
Structural Mechanics in Reactor Technology (SMiRT-15)
Seoul, Korea, August 15-20, 1999 K12/5

Forced Response of Piping System with Elasto-Plastic Damper

Shigeru Aoki[1] and Takeshi Watanabe[2]

1) Tokyo Metropolitan College of Technology, Japan
2) Yamanashi University, Japan

ABSTRACT This paper deals with steady-state response of a piping system with elasto-plastic support (elasto-plastic damper) having hysteresis loop characteristics. An analytical method of approximate solution for the continuous system with elasto-plastic damper is presented. Some numerical results of the approximate solution are shown. The results of the system with hysteresis loop characteristics are compared with those of the system with no hysteresis loop characteristics (elastic support). The peak of the resonance curve of the piping system with the elasto-plastic damper is reduced. The resonant frequency of the system becomes lower than that of the system with elastic support.

INTRODUCTION

The forced response of the continuous system such as piping system with nonlinear boundary conditions is of great importance for several engineering applications[1]-[3]. The piping systems are subjected to many vibrational load such as thermal load, flow induced vibration and other external load. The piping system is failed when the system is subjected to excess load. In such a case, reduction of the response is important for design and construction of the piping system. Some dampers are developed for reduction of the response of the piping system[4]. An elasto-plastic damper is one of the vibration absorbers in which energy is absorbed by elesto-plastic deformation of the damper. Many studies on the response of the system with elasto-plastic characteristics have been published [5],[6]. In many papers, single- or multi-degree-of-freedom system is used as an analytical model. For the design of continuous system such as the piping system, continuous model such as beam should be used. As elasto-plastic characteristics, many models·such as perfectly-elasto-plastic, bilinear, trilinear, stiffness degrading models are used.

In this paper, considering above mentioned points, an analytical method for steady-state response of continuous system with elasto-plastic support having hysteresis loop characteristics (elasto-plastic damper) is proposed. And effect of elasto-plastic damper on the forced response of continuous system is examined. For elasto-plastic characteristics, the perfectly-elasto-plastic restoring force-deformation relation is dealt with. As an analytical model, a beam simply supported at one end, with elasto-plastic damper at the other end is used. Nonlinear restoring force is expanded into the Fourier series. Considering only fundamental terms of the Fourier series, an approximate method is proposed. Some numerical results are shown. The results obtained for the continuous system with the elasto-plastic damper are compared with those of the continuous system with no damper (elastic support).

ANALYTICAL METHOD FOR THE SYSTEM WITH ELASTO-PLASTIC DAMPER

A simplified dynamical model of the continuous system with elasto-plastic damper is shown in Fig.1. Namely, this model consists of a beam simply supported at one end, with elasto-plastic damper having hysteresis loop characteristics (Fig.2) at the other end.

Let ρ be the mass density, A the cross-sectional area, and EI the modulus of flexural rigidity. The equation for transverse free vibration of a beam can be written as follows:

$$\frac{\partial^2 y}{\partial t^2} + \frac{EI}{\rho A}\frac{\partial^4 y}{\partial x^4} = 0 \qquad (1)$$

The relations between y and z as shown in Fig.1 are given by

$$y = z + y_0 \cos\omega t \qquad (2)$$

Hence we have

$$\frac{\partial^2 z}{\partial t^2} + \frac{EI}{\rho A}\frac{\partial^4 z}{\partial x^4} = y_0 \omega^2 \cos\omega t \qquad (3)$$

Assuming the solution for Eq.(1) as

$$y = \sum_{n=1}^{\infty} X_n(x) \cos n\omega t \qquad (4)$$

a formal solution of Eq.(3) can be expressed as follows:

$$z = -y_0 \cos\omega t + \sum_{n=1}^{\infty} (A_n \cosh\lambda_n x + B_n \sinh\lambda_n x + C_n \cos\lambda_n x + D_n \sin\lambda_n x) \cos n\omega t \qquad (5)$$

where A_n, B_n, C_n, D_n are constants to be determined in each particular case from the boundary conditions of the beam, where

$$\lambda_n^4 l^4 = Z_1^4 n^2 \Omega_1^2, \quad \Omega_1 = \omega/\omega_1 \qquad (6)$$

and

$$\omega_1 = \frac{Z_1^2}{l^2}\sqrt{\frac{EI}{\rho A}}, \quad Z_1 = 3.927 \qquad (7)$$

The boundary conditions for this case are as follows:

$$1)\ x = 0,\ z = 0 \qquad (8)$$

$$2)\ x=0,\ \frac{d^2z}{dx^2}=0 \tag{9}$$

$$3)\ x=l,\ \frac{d^2z}{dx^2}=0 \tag{10}$$

$$4)\ x=l,\ EI\frac{d^3z}{dx^3}=f(z_l,\dot{z}_l) \tag{11}$$

where z_l is the transverse displacement at the beam end ($x=l$) and the nonlinear shearing force $f(z_l,\dot{z}_l)$ is defined by the following piecewise-linear characteristics as shown in Fig.2:

$$\left.\begin{array}{lll} f(z_l,\dot{z}_l)=K_1 z_y & ;\ z_{l3}\leqq z_l\leqq z_{lmax},\ \dot{z}_l\geqq 0 & (\mathrm{I}) \\ f(z_l,\dot{z}_l)=K_1(z_l-z_{lmax}+z_y) & ;\ -z_{l3}\leqq z_l\leqq z_{lmaxx},\ \dot{z}_l\leqq 0 & (\mathrm{II}) \\ f(z_l,\dot{z}_l)=-K_1 z_y & ;\ -z_{lmax}\leqq z_l\leqq -z_{l3},\ \dot{z}_l\leqq 0 & (\mathrm{III}) \\ f(z_l,\dot{z}_l)=K_1(z_l+z_{1max}-z_y) & ;\ -z_{lmax}\leqq z_l\leqq z_{l3},\ \dot{z}_l\geqq 0 & (\mathrm{IV}) \end{array}\right\} \tag{12}$$

where K_1 is the spring constants of the absorber in elastic range as shown in Fig.2 and z_y is yield displacement. Displacement at the beam end of the interval (IV) z_{l3} is written as follows:

$$z_{l3}=-z_{lmax}+2z_y \tag{13}$$

where z_{lmax} denotes the maximum displacement at the beam end.

From Eq.(5) with the equations of boundary conditions (8) and (9), the following equations are obtained.

$$C_1=A_1=y_0/2\ (n=1),\quad C_n=A_n=0\ (n=2,3,4,.....) \tag{14}$$

Substituting Eq.(14) into Eq.(5),

$$z=y_0\left(\frac{1}{2}\cosh\lambda_1 x+\frac{1}{2}\cos\lambda_1 x-1\right)\cos\omega t+\sum_{n=1}^{\infty}(B_n\sinh\lambda_1 x+D_n\sin\lambda_1 x)\cos n\omega t \tag{15}$$

And then, from Eq.(15) with the boundary condition Eq.(10), the following equations will be obtained:

$$B_1\sinh\lambda_1 l-D_1\sin\lambda_1 l=y_0\left(\frac{1}{2}\cosh\lambda_1 l+\frac{1}{2}\cos\lambda_1 l\right)\quad (n=1) \tag{16}$$

$$B_n\sinh\lambda_n l-D_n\sin\lambda_n l=0,\qquad (n=2,3,4,.....) \tag{17}$$

In this paper, the steady-state vibration is dealt with for the case in which the beam end collides with the nonlinear support once in one period of its vibration. Once the impact vibration becomes steady and periodic, the nonlinear shearing force $f(z_l,z_l)$ becomes also periodic and can be represented as a periodic function $g(\theta)$ of θ with the period 2π. And θ is defined by the following equation:

$$\theta=\omega t-\alpha \tag{18}$$

where α is the phase lag angle.

This periodic function $g(\theta)$ must satisfy the conditions of the given characteristics of the nonlinear shearing force Eq.(12), which is, in this case, to be written as the following equations:

$$\left.\begin{array}{llll} f(z_l,\dot{z}_l) = K_1 z_y & ; & -\theta_1 \leqq \theta \leqq 0 & (\mathrm{I}) \\ f(z_l,\dot{z}_l) = K_1 (z_l - z_{l\,max} + z_y) & ; & 0 \leqq \theta \leqq \theta_2 = \pi - \theta_1 & (\mathrm{II}) \\ f(z_l,\dot{z}_l) = -K_1 z_y & ; & \theta_2 \leqq \theta \leqq \pi & (\mathrm{III}) \\ f(z_l,\dot{z}_l) = K_1 (z_l + z_{l\,max} - z_y) & ; & \pi \leqq \theta \leqq 2\pi - \theta_1 & (\mathrm{IV}) \end{array}\right\} \tag{19}$$

where θ_1 and θ_2 denote the range of the phase angle θ during the period of the contact of the beam end with the nonlinear support. In the foregoing, one period 2π of the resulting vibration is divided into four intervals. During the first and the third intervals of length θ_1, response of the absorber is in the plastic range. During the second and the fourth intervals of length and θ_2, the response of the absorber is in the elastic range.

In this paper, let the function $g(\theta)$ be approximated by

$$g(\theta) = a_1 \cos\theta + b_1 \sin\theta \tag{20}$$

As shown in reference[7], when $g(\theta)$ is approximated by only the fundamental term of the Fourier expansion, approximate solution agree with exact solution for relatively low ratio of nonlinear parameter (K_1/k) used in this paper. And k is equivalent stiffness of the beam defined by Eq.(26)

Thus, from Eq.(15) with the last equation of boundary condition Eq.(11), the following equation is derived:

$$\begin{aligned} \frac{d^3 z}{dx^3} &= y_0 \lambda_1^3 \left(\frac{1}{2}\sinh\lambda_1 x + \frac{1}{2}\sin\lambda_1 x\right)\cos(\theta+\alpha) \\ &\quad + \lambda_1^3 (B_1 \cosh\lambda_1 x - D_1 \cos\lambda_1 x)\cos(\theta+\alpha) \end{aligned} \tag{21}$$

and with Eqs.(11),(16) and (20),

$$\begin{aligned} B_1 = &\frac{a_1\cos\theta + b_1 \sin\theta}{EI\lambda_1^3 \Delta_1 \cos(\theta+\alpha)} \sin\lambda_1 l \\ &- \frac{1}{2\Delta_1} y_0 (1 + \sinh\lambda_1 l \sin\lambda_1 l + \cosh\lambda_1 l \cos\lambda_1 l) \end{aligned} \tag{22}$$

$$\begin{aligned} D_1 = &\frac{a_1\cos\theta + b_1 \sin\theta}{EI\lambda_1^3 \Delta_1 \cos(\theta+\alpha)} \sinh\lambda_1 l \\ &- \frac{1}{2\Delta_1} y_0 (\sinh\lambda_1 l \sin\lambda_1 l + \cosh\lambda_1 l \cos\lambda_1 l - 1) \end{aligned} \tag{23}$$

where

$$\Delta_1 = \cosh\lambda_1 l \sin\lambda_1 l - \sinh\lambda_1 l \cos\lambda_1 l \tag{24}$$

Then, displacement of beam end z_l is

$$z_l = y_0 N_l \cos(\theta+\alpha) + M_1 \frac{a_1}{k}\cos\theta + M_1 \frac{b_1}{k}\sin\theta \tag{25}$$

where

$$k=\frac{3EI}{l^3} \tag{26}$$

$$N_l=\frac{\sin\lambda_1 l(1-\cosh\lambda_1 l)+\sinh\lambda_1 l(\cosh\lambda_1 l-1)}{\cosh\lambda_1 l\sin\lambda_1 l-\sinh\lambda_1 l\cos\lambda_1 l} \tag{27}$$

$$M_1=\frac{6\lambda_1 l\ (\sinh\lambda_1 l\sin\lambda_1 l)}{Z_1{}^4\Omega_1{}^2(\cosh\lambda_1 l\sin\lambda_1 l-\sinh\lambda_1 l\cos\lambda_1 l)} \tag{28}$$

Meanwhile the switching-over conditions from one to another of the three intervals (I),(II),(III) and (IV) are expressed by

$$\theta=-\theta_1,\ z_l=z_{l3}=-z_{lmax}+2z_y \qquad (\text{IV}\rightarrow\text{I}) \tag{29}$$
$$\theta=0,\ \dot{z}_l=0,\ z_l=z_{lmax} \qquad (\text{I}\rightarrow\text{II}) \tag{30}$$
$$\theta=\theta_2,\ z_l=-z_{l3} \qquad (\text{II}\rightarrow\text{III}) \tag{31}$$

Equation (25) can be written as follows:

$$z_l=(y_0N_l\cos\alpha+M_1\frac{a_1}{k})\cos\theta \tag{32}$$

Introducing Γ as

$$\Gamma=y_0N_l\cos\alpha+M_1\frac{a_1}{k}=\frac{e_0}{\cos\theta_1} \tag{33}$$

Equation (32) is to be reduced to

$$z_l=\Gamma\cos\theta \tag{34}$$

And the nondimensional Fourier coefficients are defined as follows:

$$x_1=\frac{a_1}{k\Gamma},\ y_1=\frac{b_1}{k\Gamma} \tag{35}$$

From Eqs. (33), (34), (35) and (29)-(31), the amplitude of the beam end Γ, the phase lag angle α and the amplitude y_0 of sinusoidal excitation are determined as:

$$\frac{\Gamma}{z_y}=\frac{y_0}{z_y}\frac{N_l}{\sqrt{(1-M_1x_1)^2+(M_1y_1)^2}} \tag{36}$$

$$\alpha=\tan^{-1}\left(\frac{M_1y_1}{1-M_1x_1}\right) \tag{37}$$

$$\frac{y_0}{z_y}=\frac{2\sqrt{(1-M_1x_1)^2+(M_1y_1)^2}}{(1+\cos\theta_1)N_l} \tag{38}$$

Since the function $g(\theta)$ in Eq. (19) has been approximated by Eq. (20), the piecewise-linear characteristics expressed by Eq. (19) of nonlinear shearing force, combined with Eq. (34), can be approximately written as follows:

$$\left.\begin{aligned}
\frac{g(\theta)}{k\Gamma} &= a_1\cos\theta + b_1\sin\theta \\
&= \frac{K_1}{k}\,\frac{1+\cos\theta_1}{2} \qquad ; -\theta_1 \leqq \theta \leqq 0 \\
\frac{g(\theta)}{k\Gamma} &= a_1\cos\theta + b_1\sin\theta \\
&= \frac{K_1}{k}\left(\cos\theta - \frac{1}{2} + \frac{\cos\theta_1}{2}\right); \; 0 \leqq \theta \leqq \theta_2
\end{aligned}\right\} \tag{39}$$

Applying a technique similar to that for determining Fourier coefficients, namely, multiplying both sides of Eq.(39) by $\cos\theta$ and $\sin\theta$ and integrating through the whole period of 2π, the nondimensional coefficients x_1 and y_1 are obtained as follows:

$$x_1 = \frac{1}{\pi}\frac{K_1}{k}(\pi - \theta_1 + \sin\theta_1\cos\theta_1) \tag{40}$$

$$y_1 = \frac{1}{\pi}\frac{K_1}{k}(\cos^2\theta_1 - 1) \tag{41}$$

Fixing the parameter Ω_1 and y_0/z_y, θ_1 can be obtained by iteration method with Eqs. (38), (40) and (41). Then, Γ/z_y can be obtained, utilizing Eq.(36).

NUMERICAL EXAMPLES

Figures 4(a) and 4(b) show resonance curves of the system with no hysteresis loop characteristics (elastic support) with the amplitude Γ/e_0 of the beam end versus the frequency ratio Ω_1 obtained by the method of approximate solution in which several exciting ratios y_0/z_y are taken to be constant. Figures 5(a) and 5(b) show resonance curves for y_0/z_y=1.0 and 0.5 of the system with elasto-plastic damper of nonlinear parameter K_1/k=3. Figure 6(a) and 6(b) show resonance curves of the system with high ratio of nonlinear parameter K_1/k=10.

From Figures 4, 5 and 6, the peak of resonance curve of the piping system with elasto-plastic damper is reduced. The resonant frequency of the system becomes lower than that of the system with elastic support.

CONCLUSIONS

An approximate solution for the steady-state response of continuous system simply supported at one end and supported with elasto-plastic damper having hysteresis loop characteristics at the other end is proposed. Resonance curves are obtained for several exciting ratios y_0/z_y of the systems with low and high ratio of nonlinear parameter K_1/k by applying the approximate solution. The peak of resonance curve of the piping system with elasto-plastic damper is reduced and the resonant frequency of the system becomes lower than that of the system with elastic support having no hysteresis loop characteristics.

Finally, the authors would like to acknowledge their deep appreciation for the valuable suggestions given to this work by Professor Emeritus Heki Shibata of the University of Tokyo.

REFERENCES

1. Watanabe,T., "Forced vibration of continuous system with nonlinear boundary conditions," *Trans. of ASME, Journal of Mechanical Design*, Vol.100, 1978, pp.487-491.
2. Moon,F.C. and Shaw,S.W., "Chaotic vibrations of a beam with nonl-inear boundary conditions," *International Journal of Non-linear Mechanics*,Vol.18, No.6, 1983, pp.465-477.
3. Shaw,S.W., "Forced vibrations of a beam with one-sided amplitude constraint:Theory and experiment," *Journal of Sound and Vibration*, 99-2, 1985, pp.199-212.
4. Soong,T.T. and Dargush,G.F., *Passive Energy Dissipation Systems in Structural Engineering*, John Wiley & Sons, 1988
5. Shibata,H., et. al., "Development of the Elasto-Plastic Damper and Its Application to the Piping System in Nuclear Power Plants," *Transactions of the 11th International Conference on Structural Mechanics in Reactor Technology*, K2, 1991, pp.487-492
6. Ohmata,K., Sugiura,W. and Taniguchi,T., Elastoplastic Damper Utilizing Plastic Torsion of a Metal Rod or Plate, *Proceedings of the Third International Conference on Motion and Vibration Control*, Vol.3, 1996, pp.1-5
7. Watanabe,T. and Shibata.H., "On nonlinear vibration of a beam —Response of a beam with a gap at one end—," *Report of the Institute of Industrial Science The University of Tokyo*, Vol.24, No.187, 1981, pp.1-25.

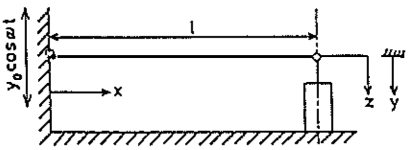


Fig.1 Analytical model of continuous system with elasto-plastic damper

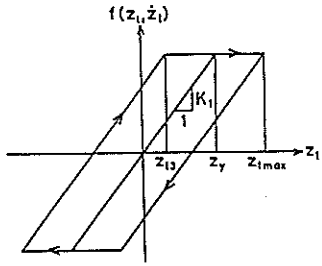
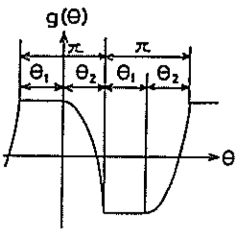
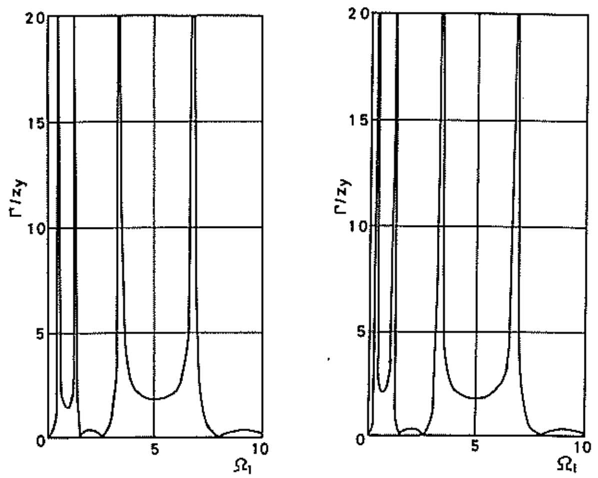


Fig.2 Hysteresis loop characteristics of restoring force at the beam end

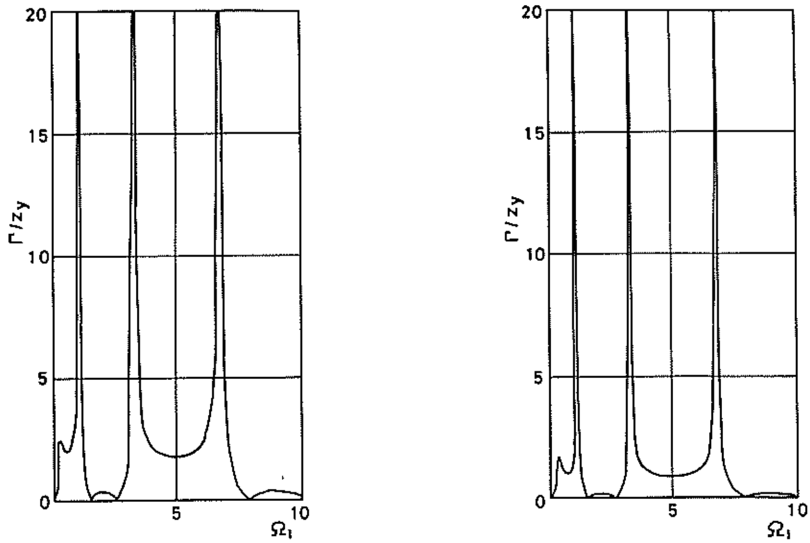


g.3 Waveform of restoring force at beam end



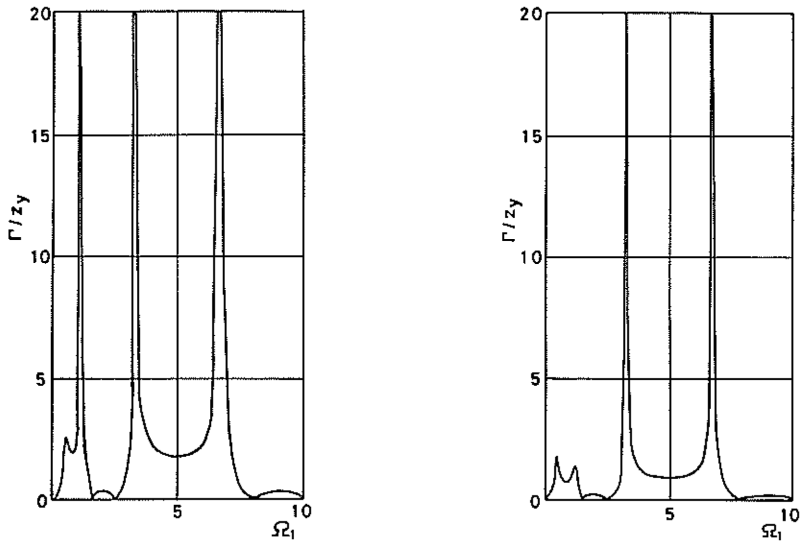
(a) $K_1/k=3$ (b) $K_1/k=10$

Fig.4 Resonance curve of continuous system with no damper (elastic support) ($y_0/z_y=1.0$)



(a) y_0/z_y=1.0 (b) y_0/z_y=0.5

Fig.5 Resonance curve of continuous system with elasto-plastic damper (K_1/k=3)



(a) y_0/z_y=1.0 (b) y_0/z_y=0.5

Fig.6 Resonance curve of continuous system with elasto-plastic damper (K_1/k=10)