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SPRING EFFECT AND PRIMARY STRESS

R.L. Roche

ABSTRACT

In order to avoid overstraining a fraction of a secondary stress should be added to the primary stress . For well designed components, this fraction is very low and may be neglected, but it is a good practice to check if such a simplification is valid. The strong parts of a componant could act like a spring on the weakest ones which are overstrained. Conventional elastic computations cannot take this spring effect into account, but this can be done by methods based on the spring factor **r**.These methods are exposed in this paper , then a special attention is given to the action of the primary stress on the spring effect (in order to be clear, a typical case is considered).

1. INTRODUCTION

11. Elastic Computations and Codes Requirements

Assuming a linear elastic behavior of the material is the most popular way to perform structural analysis. The method is very simple and there is a lot of computer programs able to perform such an analysis at very low cost. Unfortunatly, the results of elastic computations cannot be used straightforwardly and must be traited before used. To prevent dangerous failures the materials of nuclear components must be ductile, able to be plastically deformed without noticeable dammage; their behavior is not elastic, it is the reason why elastic computations have to be treated before used.

The current practice for considering the stress redistribution by plastic deformation is *dividing the (elastic computed) stress into different categories*. This *"stress classification"* is required by design codes [1],[2],[3],[4]. A good explanation of this need can be found in [5]: "A calculated value of stress means little until it is associated with its location and distribution in the structure and with the type of loading which produced it. Different types of stress have different degrees of significance and must, therefore, be assignated different allowable values...The setting of allowable stress values required dividing stresses into categories ...".

12. Difficulty of the Stress Classification

Only two categories of stresses will be considered in this paper: primary stresses and secondary ones. Their definitions can be found in the design codes. It is written in [1] that "Primary stress is any stress developed by an imposed loading which is necessary to satisfy the laws of equilibrium. The basic characteristic of a primary stress is that it is not self-limitating.Secondarystress is a stress developed by the constraint of adjacent material or by self-constraint of the structure. The basic characteristic of a secondary stress is that it is self-limiting."

Every designer know that dividing calculated stress into categories is not an easy task. This difficulty is acknowledged by the writers of the codes: "Many cases arise in which it is not obvious which category a stress should be placed in, and considerable judgement is required."[5]. A first step is to remove the peak stresses which are localised in very small regions (regarding the thickness). this step is not too difficult and the main problem is discriminating between primary and secondary stresses.

The definition of primary stress show the best way to find it: *Limit Analysis. The Uper bound theorem* allows to find conservative fields of primary stress.This way is authorized by design codes, for instance,[1] allows using limit analysis instead of primary stress limits (NB 3228-1).Hence it could be concluded that stress classification would be easy if limit analysis were more popular. Such a conclusion is almost true, but some attention must be given to the limited ductility of engineering materials, this is pointed out in NB 3228 of [1]: "The use of...may result in permanent strains...when used...the effects of plastic strain...must be considered". *The aim of this paper is to estimate the staining caused by the relaxation of secondary stresses in order to avoid a too large exhaustion of the material ductility. For doing that, the spring effect (or elastic follow-up) must be discuted.*

2. SPRING EFFECT AND "SECONDARY STRESS"

21. Straining caused by stress relaxation

By definition, the secondary stress is necessary to satisfy the equations of compatibility (material continuity). Its basic characteristic is that it is self-limitating, it is to say that it may disappear as a result of inelastic deformation. However it is obvious these inelastic deformations must not be so large that the material straining could be damaging by ductility exhaustion.

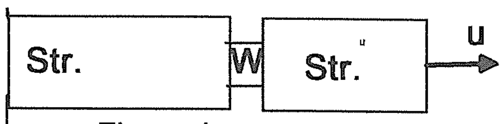


Figure 1

The elastically computed strain (stress value divided by Young'modulus) is not significant. Due to the inelastic deformation needed for relaxing deformation-controlled stresses, the actual strain field is very different of the elastic one. Generally, actual strain is larger in the parts exhibiting high stress (weakest parts) and smaller in thoses exhibiting low values (strongest ones). It is not wrong to say the strongest part of a component acts as a spring whith the small part of said component. This effect is the spring effect (or elastic follow-up). It is very important when the weakest region is small and the majority of the structure acts as a spring with it.

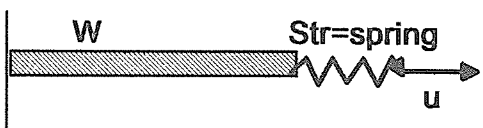


Figure 2

A exemple is given on figure 1 where a bar is submited to an imposed elongation **u**. The inelastic strain is concentrated in the part W, the parts **Str** behaving like a spring. Such a situation can be schematised in substituting a spring to the strong parts **Str** (Figure 2)

22. Spring Factor and Evaluation of Actual Strain

When the stresses **Q** are relaxable, actual strains can be evaluated with the help of a non-dimen sional coefficient **r** (the spring factor). This method is explained in several publications [6],[7], [8],[9], therefore a short explanation is sufficient here. It can be shown the actual stress **S** and strain **ε** are on the straight line given by the equation:

$$E.\varepsilon - Q = r.(Q-S)$$

(see figure 3) where **r** is the spring factor. Knowing the elastically computed stress **Q**, the value of the spring factor **r** in a section and the material tensile curve allows to determine the strain ε in this section. In writing the tensile curve equation $\varepsilon(\sigma) = (\sigma/E)+e(\sigma)$ where **e** is the plastic strain, **S** and ε are given by $E.e(S) = (1+r).(Q-S)$

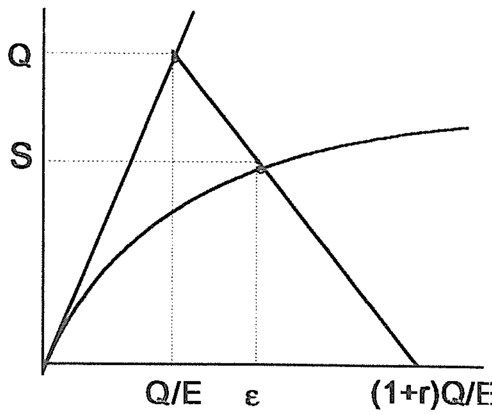


Figure 3

The numerical value of r in a section can be evaluated as follows:

- in each section of the structure compute the local $m = e(Q)/(Q/E)$,
- the global M of the structure is $M =\{ \int m.Q^2.dv\}/\{\int Q^2.dv\}$
- in a section $r = (m/M)-1$ where **m** is the value in the section

It could be funny to applied this rule to the structure shown on figure 2. There are two small difficulties. First one, the spring is elastic only and its stiffness is K. Second one $\int Q^2$ dv is the elastic energy multiplied by 2E . Knowing that, m=Ee(Q)/Q and (A is the cross section area)

$M=(Ee(Q)LA)/\{Q^2LA+(QA)^2/K\}= [Ee(Q)/Q]/[1+(AE/KL)]$ The rule give:

$r=(m/M)-1=AE/KL$ (verification is obvious)

23. Primary Part of a Conventional "Secondary Stress"

Secondary stresses beeing self limiting, they are assignated larger allowable values than primary stresses (three times, for their range). The distortion caused by their relaxation is not considered (with the exception of what is called "local membrane stress"). It is quoted in [5] that " thermal stresses which can produce distortion of the structure are placed in the secondary category".

Such are the requirements of the codes [1],[2] for structures operating below the creep range Unfortunatly the ones [3],[4] for structures operating at elevated temperature do not sing the same song, for instance in [4] it is stated that "unless otherwis justified, any stress with elastic follow up should be included as primary stress ..". Taking spring effect (elastic follow up) into account for stress classification is a very intricate question. It is the aim of this section to try giving an answer. A first point is why this question is only concerning the creep range, is it because ductility exhaustion is one consequence of creep?

Dividing the elasticaly primary stress into different categories allows to take advantage of material ductility. The concept of secondary stress is without meaning for glass or cast iron, the strain needed for stress relaxation beeing too damaging for these materials. Therefore the classification between primary stress and secondary one is depending onto the straining (wich can be evaluated with the help of the spring factor) and the material ductility.

The most significant measure of the material ductility is the percentage reduction area RA in a tensile test. The strain at fracture ε_u is equal to -ln(1-RA) and the stress value is almost equal to the ultimate stress S_u. A stress-strain state reaching these values causes the fracture in the same way a primary stress equal to S_u.

For ductile materials ε_u is so large that relaxable stresses are considered as secondary, it is the definition of conventional secondary stress **Q.** For materials with poor ductility, this is not true any longer, relaxable stresses can cause excessive straining and rupture, in other words, a conventional secondary stress act partially as a primary stress. Evaluating the part **pQ** acting as a primary stress can be made on the base of the spring factor r and the properties of the material S_u and ε_u.

The rule is simple: *a conventional secondary stress Q causing the critical state S_u, ε_u exhibits the same effect as a primary stress equal to S_u, therefore its primary part is $p=S_u/Q$.*

On figure 4, it is easy to see that this critical value of Q is given by the equation

$S_u = (1+r)Q - E\varepsilon_u$, therefore

$$p = (1+r) / \{(E\varepsilon_u/S_u)-1\}$$

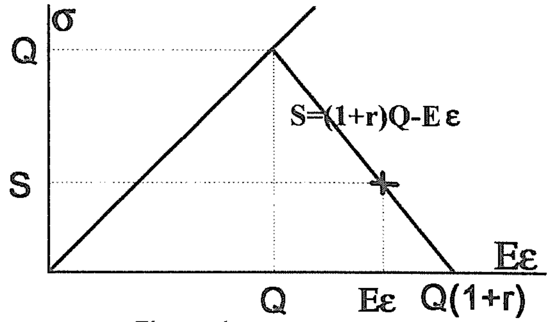


Figure 4

For exemple, for a steel with S_u =690 MPa, E=200000MPa, ε_u=40%, if the factor r is equal to 2, the primary part is negligeable (2,6%). In case of ductility exhaustion by thermal aging or by fast neutron irradiation leading to ε_u=1,5%, the primary part of a conventional secondary stress become 0,896 , it almost fully primary!

3. PRIMARY STRESS AND "SIMILAR SPRING EFFECT"

31. Strain Needed for Relaxation and Primary Stress

All the preceding studies on the spring effect are concerning structures without primary stress. As the cause of the spring effect is a non-uniform distribution of secondary stress, it would be useful to examine if a non-uniform distribution of stress can cause a behavior similar to the spring effect. Such a discussion is the main aim of this paper: are primary stresses able to enhence the strain needed for the relaxation of secondary stresses?

To avoid too theoretical explanations, the discussion will be made on a typical structure, the well known three bars structure. For begining, the case wihout any spring effect will be examined.

32. Three Bars Structure without Spring Effect

The structure is shown on figure 5. The area of the cross section of the middle bar is A, twice this of the lateral bars. As its length is only L-u , the fitting of the structure product secondary stresses (tension in the middle bar, compression in the others). A primary stress is added by the force F.

The tensile curve of the material is $\varepsilon(\sigma)=(\sigma/\varepsilon)+e(\sigma)$ where e is the plastic strain and E the modulus.

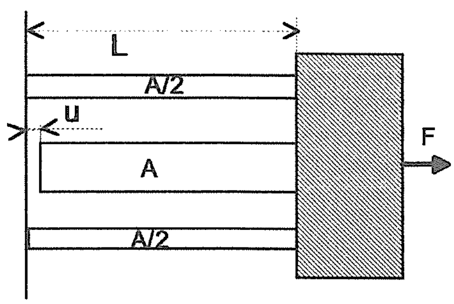


Figure 5

The primary stress is the same in the three bars and equal to $P = F/A$

To this stress is added a tensile stress S in the middle bar and a compression one (-S) in the two others. In elastic computations S is the conventional secondary stress Q.

The critical bar is the central one (where the stress is higher). Its strain is

$\varepsilon = e(P+S) + (P+S)/E$ larger than the computed one $(P+Q)/E$, it is conveniant to write

$$E\varepsilon - (P+Q) = Ee(P+S) - (Q-S)$$

which is (multipied by E) the strain increment due to pastic relaxation. This stress relaxation can by obtained by comparing the expressions of u

in elastic computation u = [(P+Q)L/E] -[(P-Q)L/E] = 2QL/E

in actual state u = {[(P+S)/E]+e(P+S)}L - {[(P-S)/E]+e(P-S)}L

u/L = (2S/E) +e(P+S) - e(P-S)

eliminating u give the stress relaxation Q-S = [e(P+S) - e(P-S)]/2

A factor **a** *similar at the spring factor may be introduced, ratio of the overstrain to the stress relaxation:* **E.ε - (P+Q) = a . {(P+Q) - (P+S)}** its value is: **a = [e(P+S)+e(P-S)]/[e(P+S)-e(P-S)]**

33. Discussion

This result seems very different from the one for spring effect. The spring factor r does not depend on the stress S, but the factor a is depending on the stress as can be seen on figure 6 (n is the exponent in Ramberg Osgood law). For low values of P (or P/S) , its value is going to zero (no primary stress, no effect of it). For high values it is going to the infinite (a primary stress is not relaxable).

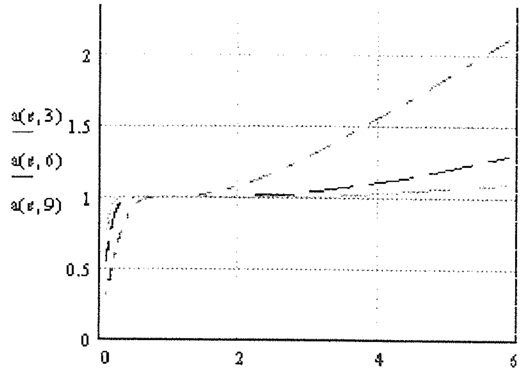


Figure 6 . a versus P/S and n

At the present time, it seems that the question of a good engineering use of this result must be kept open. Nevertheless it is convenient to make a suggestion: it can be seen the value of a is near one for P/S between 0,1 and 2, it is to say for almost all the practical cases. Accepting a=1 is conservative, it is exaggerating the effect of primary stresses and considering the material is without strain hardening.

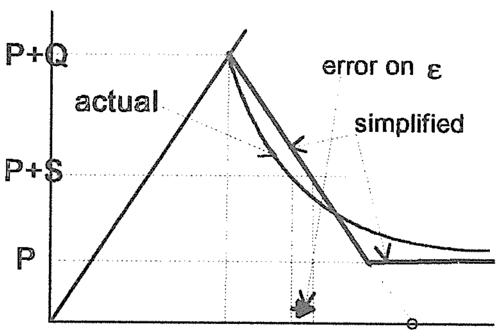


Figure 7

The difference between this proposal (a simplified rule) and the actual behavior is illustrated on the figure 7. It can be seen the strain needed for secondary stress relaxation is overestimated by the proposed rule, but not to much for engineering application.

The bold line (a=1 for S>0 and a= ∝ for S=0) show the behavior ot a material perfectly plastic (without any strain-hardening). For such a material, all the needed strain is concentrated in the higher stress region and the simplified value a_s is the ratio of the volume of the higher stress region to the one of the others.

34. Interaction with the spring effect

The figure 8 shows the modification to the preceeding structure (figure 5). The secondary stress is caused by the extension of a spring placed between the central bar and the support

Without the force F, it is the case of figure 2, the lengh of the bar beeing 2L. Therefore the spring effect is **r** equal to 1 - (AK/2EL) , where K is the spring stiffness

There are few changes in the computations made in the paragraph 32.

- in elastic Eu/2L=Q+(AK/2EL)(P+Q) = Q+r(P+Q)
- in real. Eu/2L=S+r(P+S)+E[e(P+S)-e(P-S)]/2

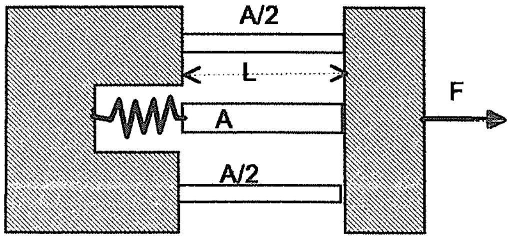
eliminating Eu/2L give
Q-S=[e(P+S)-e(P-S)]/[2(1+r)] and the *spring factor* $\mathbf{r_p}$ *modified by the primary stress*

$$r_p = [Ee-(P+S)] / [(P+Q)-(P+S)]$$

equal to {[2(1+r)e(P+S)]/[e(P+S)-e(P-S)]}-1 or

$$r_p+1 = (r+1).(a+1)$$

obviously $r_p = r$ for a =0 (no primary stress) and $r_p = a$ for r = 0 (no spring effect). But what is interessing is how taking primary stress under consideration when evaluating the spring effect.



Spring effect & Primary Stress
Figure 8

4. CONCLUDING REMARKS

- Straining is needed for secondary stresses relaxation
- This strain must be low enough for avoiding material ductility exhaustion
- If it is not the case a part of the secondary stress must be added to the primary stress
- Such a situation can occur when the spring effect (elastic follow-up) is significant
- When primary stresses are negligeable, the strain can be evaluated with the help of the spring factor r
- A good appraisal of the value of r can be made (based on Kachanov approximation)
- It must be pointed out that r-value can be high for materials with low strain-hardening
- Non negligeable primary stress can enhance the strain needed for the relaxation of the secondary stress.
- By examinating a three bars structure, it has been shown that a conservative simplification allows to evaluate the effect of a primary stress in the same way that the spring effect in using a new factor a. The interaction of a and r has been presented.
- In this paper, the primary stress effect on relaxation has only been examined in a typical case in order to illustrate a simplification. Future work will show that this simplification could be the base of a practical evaluation of a and r_p.

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