EXPERIMENTAL AND ANALYTICAL INVESTIGATION OF AN MHD CHANNEL WINDOW FRAME

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SUMMARY

A common design of a magnetohydrodynamic (MHD) generator channel emphasizes a series of mutually connected window frames providing the support for electrodes and containing the jet of ionized gases. Structurally, a window frame is a rectangular frame with a cross section in the form of a quadrilateral weakened by one or more holes forming the cooling system. Window frames are joined together to make a beam with box cross section by means of shear pins.

A typical window frame is subjected to a rather severe environment characterized by large pressure and high temperatures. The main sources of stresses and strains would include static and dynamic gas pressures and thermal fields characterized by large gradients across the thickness of frame members.

Somewhat less significant are electromagnetic (Cauchy and Lorentz) forces, coolant pressures, dead weight, and interface forces.

In the sequel we will consider only the stresses associated with thermal fields. The actual problem is still a very complex one involving analysis of the heat conduction through the electrode-frame system heated by the gas and cooled by the coolant fluid and determination of stresses (elastic and plastic) in a frame with an irregular cross section.

It is quite apparent that the actual stress analysis could easily evolve into an expensive and time consuming effort. This is especially true since the operation can under certain conditions be cyclic in nature. In order to obtain a reasonable qualitative and quantitative insight into the phenomenon, three distinctly different approaches have been used: a large elasto-plastic finite element program was used to determine the stresses in the actual frame, an experiment was performed to establish the stress concentrations around holes needed for the passage of coolant, and a two-bar model was used in order to assess the time dependent behavior of the structure for various loading conditions.

The approximation of the longer frame member by a two-bar model is justified by the geometry of the frame and the particular thermal field characterized by two regions (separated by the cooling duct) with distinctly different temperatures. This rather simple model can be readily analyzed even in the case of rather complex constitutive laws (including the fact that material parameters are dependent on temperature). In particular, the material is assumed to follow a creep law including elastic deformation, instantaneous plastic deformation and the creep deformation. The creep law is written in such a way that it can be used for tension and compression.

In addition, the bar in tension is assumed to deteriorate in time. The degree of deterioration is defined through the so-called damage factor relating the damaged part of the cross section to the original cross section. The coupling of the original creep law and the equation relating the damage and the stress defines the damage-creep interaction. The approach can be further extended in order to include other effects which are particularly important in the case of cyclic temperature applications.

Using such a two-bar model three particular problems were investigated: operational mode characterized by a time independent temperature field and stresses below the yield limit, a case in which after a single "short" thermal pulse the inside bar is at yield in tension, and a mode characterized by a cyclic application of thermal pulses.

As expected, the damage of the structure is minimal in the first case (the structure is almost in pure relaxation) and somewhat larger (but still not dramatic) in the second case. The damage in the third case is dependent on the length of thermal pulses and cool-off periods.

1. Introduction

A typical design of a magnetohydrodynamic (MHD) power generator consists of a combustor, a nozzle, a channel (placed in a strong magnetic field), and a diffuser. In the present paper we will focus attention on the channel. In particular, we will consider a channel design emphasizing a stack of "window" frames mutually connected by means of bolts to form a hollow duct. The channel conducts a jet of heated ionized gas at high velocities through a magnetic field of an externally placed magnet.

Structurally the channel represents a continuous beam-column with a cross section in the form of a hollow rectangle (Fig. 1). A typical window frame (Fig. 1) is usually designed in the form of a plane trapezoidal frame. It houses electrodes on two opposite sides. Each member of a frame is hollowed in order to provide necessary passage for the coolant. Each frame is connected to adjacent frames by means of bolts. A layer of insulation is interposed between adjacent frames. The joint is made leak-proof by means of O-ring seals.

Electrical, magnetic, and thermal considerations dictate the use of a material such as silver bearing copper for making the window frames. The dimensions of load carrying part of the structure (i.e., wall thickness) is governed by the necessary plasma volume from one and the magnet clearance (interior) from the other side.

2. Loads on Channel

The loading on a channel consists of

- -gas pressure
- -axial thrust from gas jet
- -coolant pressure
- -electromagnetic forces and
- -dead weight

In many cases at substantial mass flows the gas velocity in MHD channels is supersonic. Consequently a shock is generated in the nozzle and moves down the channel into the diffuser. Diffuser slows down the gas speed to subsonic values. The large pressure difference across the shock would cause undesirable vibration of the channel walls [1]. In the present design the gas velocity is very close to the sonic value and is only slightly supersonic in a small portion of the channel.

The gas jet from the combustor is characterized by a mean static pressure and random dynamic fluctuations about that mean pressure. The static pressure varies along the length of the channel with a maximum of about 2 atm. The pressure in the diffuser drops to a small negative value. The peak values of the fluctuations in the combustor are of the order of 20% of the mean value. The frequency spectrum is very wide ranging from 10 to 10^4 Hz.

The axial jet thrust loading varies along the length and depends on the variation of velocity, pressure, density, and duct cross sectional area in the channel. It reaches a maximum value of 6700 lb in the present case.

The coolant pressure in the hollows of the frame members is considered to be static. This has a maximum value of 235 psi.

The electromagnetic forces on the channel arise from the current flow in the window frames and the Hall current in the gas, in the presence of the magnetic field. The current in the frame members gives rise to body forces directed perpendicular to the plane of the window frame. The component of this force along the axis of the channel tends to pull the frames downstream. The Hall current in the hot gas sets up a transverse pressure gradient in the gas. This pressure differential between opposite sides of the frame is treated as a transverse force and is added to the transverse component of the body forces in the frame. In the channel being considered here the transverse component is assumed uniformly distributed with a value of 411 lb/ft. The total axial component is computed to be 8400 lb.

The dead weight of the channel incorporates also the weights of various mountings, electrodes, etc.

In addition to the above mechanical loads a window frame is exposed to a severe thermal ambient in the region between the hot inner surface and the cooling water passage. The maximum temperature in the frame is about 400°F. Due to the complexity of the geometry (presence of cooling passage, electrodes and insulation) temperature across the wall can be determined accurately only by means of numerical techniques.

3. Structural Response

The response of the channel can be roughly classified into two distinctly different modes:

- the beam-column mode
- the transverse plane frame mode

3.1 Beam-Column Mode

Along the length of the channel it is supported only intermittently. The transverse loads such as the magnetic load component and the dead weight induce shear forces and bending moments because of the continuous beam type behavior of the channel. The dynamic fluctuations in gas pressure could also cause flexural vibrations in this mode. The axial loads such as the jet thrust and the magnetic load component gives rise to normal and shear stresses in the connecting bolts. The shear stresses arise because the planes of the frames are not normal to the axis of the channel.

The main problem in determining the flexural response is the determination of the bending rigidity of the stack of frames and insulating layers connected by means of bolts. In the particular case of the channel being considered the supports are close enough and the numbers of bolts at any cross section numerous enough to render the bending and shear stresses rather insignificant. The main area of concern in this mode is the possibly high concentration of bearing stresses around the bolts.

3.2 Frame Mode

The frame type of response consists essentially of the following, somewhat loosely related problems

- The bending of the frame as a result of static pressure from gas
- The dynamic response of the frame as a result of gas pressure fluctuations
- The response to thermal gradient and cycling
- The determination of the peak stresses (stress concentration) at corners, near where the coolant passage holes intersect

The bending of an elastic plane frame subjected to internal gas pressure is an elementary problem which will not be discussed any further. The stresses are usually well within allowed limits and the response is strictly elastic.

3.2.1 Dynamic Gas Pressure

The fluctuations in gas pressure originating from the combustor are random in character and are not defined in the channel. A rigorous analysis for response to random loading would not be meaningful in the absence of load characterization. Therefore, an approximate check for stresses for an assumed peak amplitude and forcing frequency is made and a test for random fatigue damage is proposed.

The check for stresses under an assumed peak amplitude and frequency involves the calculation of a dynamic load factor. An equivalent-one-degree freedom system for the window frame is determined using the method given by Biggs [2]. The problem is reduced to computing the three quantities: equivalent mass, equivalent force, and equivalent stiffness. The equivalent mass is the same as that of some significant point of the structure, say the midspan of the longer side of the frame. The assumed deflected shape of the frame is the same as that resulting from the static application of the dynamic loads. The dynamic load factor thus determined is multiplied by the assumed peak amplitude of gas pressure fluctuation to yield an equivalent static load on the frame. The stresses under this static pressure are obtained by proportioning the results of the static analysis. For the present channel, the stresses thus determined are found to be less than those due to static gas pressure.

In order to determined approximately the safety against fatigue failure, the following test procedure is proposed. A relatively short test duration that includes start-up, steady state operation and shut-down will be chosen. The stress-time history from each strain-gage (mounted at highly stressed points) will be recorded during this duration. From this record the peak amplitude range will be determined. Then, the number of cycles each with a range of 100%, 90%, 80%...10% of the peak amplitude range would be counted. For a stationary, Gaussian proces the probability distribution of the cycle ranges is a Rayleigh distribution. In the absence of extensive test results the distribution of peak amplitudes can be assumed to be a Rayleigh distribution [3]. With the knowledge of the design fatigue curve for the silver bearing copper, the cumulative damage index will be calculated as per Miner's criterion, i.e.,

$$\sum \frac{n_i}{N_i} \leq c$$

where n_i is the number of cycles with a certain amplitude range proportioned for the entire operating duration, N_i is the number of cycles to failure at this particular amplitude range (obtained from design fatigue curve), and c is a constant taken to be unity, though smaller values have been proposed by some authors [4].

3.2.2 Thermal Effects

The stresses associated with the rather severe thermal gradients are in excess of yield limits of the material. Thus a suitable plastic analysis is appropriate in the given case. A finite element program, CREEP-PLAST was chosen to perform this analysis. CREEP-PLAST is a two-dimensional finite element computer program for creep and plasticity analysis of metal structures operating at high temperature. The program considers instantaneous time-independent elastic-plastic and time-dependent creep simultaneously using an incremental procedure. The method of calculation is based on the assumption that the total strain at any instant consists of elastic, plastic and creep parts with no interdependence between the plastic and creep components.

The application of the program to the present problem involved certain approximations. It was necessary to neglect the three-dimensional aspects of the geometry and consider the cross section of frame members to be solid rectangle. Since a separate creep-rupture analysis was made, only instantaneous plastic deformations were considered. A temperature distribution that varies only across the depth of the cross section was assumed. The variation of α, the coefficient of thermal expansion, E, the Young's Modulus and σ_y, the yield stress with temperature was assumed to be linear. The computation was limited to instantaneous loading, the effect of cyclic loading being considered separately.

The results of this analysis show that plastic deformation first occurs when the highest temperatue reaches about 70 percent of its maximum value. As may be expected, the "hottest" fibers are the first to yield. At the peak load, the entire part of the section between the hot surface and the cooling passage becomes plastic. There is a sharp increase in plastic strains of almost 65 percent at the corner. This increase was comparable to the elastic stress concentration factor at the corner.

Even though part of the frame becomes plastic under thermal loading the code provisions permit such a design if there is no cyclic strain accumulation as a result of repeated start-ups and shut-downs. In order to assess the effect of the cyclic mode of operation on the structure the frame member is idealized by a simple two-bar model subjected to thermal load only.

A two-bar model consists of two bars of different cross sections constrained such that their elongations remain equal at all times. The bars are subjected to different temperatures. The temperatures vary as step functions in time. At the end of the first half cycle, i.e., after application of thermal load, the hotter bar yields and strains plastically while the colder bar remains elastic. During the second half cycle, i.e., removal of thermal load, the process is assumed to be elastic. The final stress and residual strains at the end of the first cycle are determined by superimposing the results of the two half cycles. Such an analysis shows that subsequent to the first full cycle, the stresses will remain elastic under further thermal cycling and that the plastic strain in the "hot" bar does not increase after the first cycle. In other words, the frame responds in the shakedown mode.

Next, the same two-bar model is used for creep analysis. In addition to conventional creep, the analysis takes into account the incremental accumulation of damage while a bar is in tension. We use the theory of brittle fracture proposed by Kachanov [5] and modified by Odqvist [6].

Two separate temperature histories are considered. In the first case, each bar is subjected to a temperature pulse in the form of a step function. The temperatures of two bars are different. The initial stresses and strains are obtained from the thermal analysis described before. We write the governing equations for the two bar model as follows:

Equilibrium:

$$\sigma_a A_a + \sigma_b A_b = 0 \tag{1}$$

Creep law and compatibility:

$$\frac{\dot{\sigma}_a}{E_a} + \left(\frac{\sigma_a}{\sigma_a^o}\right)^{n_a} = \frac{\dot{\sigma}_b}{E_b} - \left(\frac{-\sigma_b}{\sigma_b^o}\right)^{n_b} \tag{2}$$

Damage law:

$$\dot{\omega} = C\left\{\frac{a}{1-\omega}\right\}^{\nu} \tag{3}$$

where σ is the nominal stress, A is the area of cross section of bar, E the Young's Modulus, σ^o and n are temperature-dependent material constants, and C and ν, temperature-independent material constants. The subscripts a and b denote the two bars with the initial stresses satisfying the condition: $\sigma_a > 0$, $\sigma_b < 0$. The dot, ($^\bullet$), denotes differentiation with respect to time. ω is the damage factor. Initially it is taken to be zero and as damage increases, ω increases with rupture occurring when $\omega = 1$. We can easily eliminate σ_b and $\dot{\sigma}_b$ from Eq. (2) by using Eq. (1). This leads to a system of two non-linear partial differential equations. The unknown functions of time σ_a and ω are then determined numerically. The numerical results show that even though the damage factor increases with time, the rate of increase is very small. Further, the nominal tensile stress in the "hot" bar, decreases with time.

For example, after 100 hours, the damage factor

$$\omega = 0.8 \times 10^{-4} << 1$$

The rate of increase of ω decreases with time. This means that the time required for ω to approach unity is much longer than the duration of continuous operation. The stresses in the bars decrease to 40% of their initial values after 100 hours. From the above results we conclude that brittle fracture due to creep appears to be an unlikely mode of failure.

In the second case, a different initial condition is assumed. The temperatures are applied as a step load and removed instantaneously after a short time. The instant at which the temperatures are removed is taken as the initial time. The governing equations for this case are obtained by interchanging a and b in equations (2) and (3) since $\sigma_a < 0$ and $\sigma_b > 0$ at the initial time.

Since the initial tensile stress in bar b is large compared to the tension in bar a of the previous case, the rate of increase in damage factor is larger even though ω still remains small. For instance, after 100 hours,

$$\omega = 0.18 \times 10^{-2} << 1$$

As in the previous case the rate of increase of ω decreases with time. This is because the stresses decrease at a relatively fast rate. After 100 hours, the stresses are only 17% of the initial values. Consequently, the time required for ω to approach unity is very large in this case too. Therefore, we arrive at the same conclusion as in the first case.

Finally, considered is the case of the cyclic thermal load. Since the test in question is scheduled to last approximately 100 hours, the duration of cycle is assumed to be 200 hours (implying 100 hours in operation and another 100 hours of hold time at the room temperature). The results of computation indicate again strong relaxation of stresses and very slow build-up of damage factor.

Regardless of the small number of thermal cycles involved in the test program, the major area of concern is obviously the possibility of occurrence of ratcheting or alternate plasticity leading ultimately to collapse. The situation is complicated by the fact that the elevated-temperature code [7] screening criterion

$$(P_L + \frac{P_b}{K_f})_{max} + (Q_R)_{max} \leq S_y$$

is not satisfied as a result of large thermal stress. Hence, there is no a priori assurance that plastic ratcheting will not occur. The dependence of material parameters (such as the yield stress and elastic modulus) on the temperature adds to the complexity of the problem.

Since the response of the frame subjected to internal gas pressure cannot be satisfactorily approximated by the two-bar model, the analysis becomes increasingly complex and time consuming. However, in a typical case the stresses due to gas pressure are much lower than those associated with thermal effects, and the number of cycles is rather modest. Thus, as a first, rather crude but conservative approximation, it is possible to retain the two-bar element and consider the cyclic primary bending stress as a sustained primary membrane stress. On the basis of such an admittedly rough approximation it is possible to show that the response of the frame member will most probably be in the shakedown mode. This conclusion was later corroborated by results computed from the finite element model subjected to several thermal and pressure cycles.

3.2.3 Stress Concentration at Corner

All the approximate stress analyses discussed above do not take into account the stress concentration at the re-entrant corner of the frame. The presence of intersecting cooling water passages further complicates this problem. Since the complex three-dimensional geometry at the corner makes even numerical solutions difficult, it was decided to subject frame corners to a simple loading test. Two typical, full sized frame corners were loaded in a tensile testing machine as shown in Fig. 2. The stresses at various points in the frame corner were measured by means of strain gages. The test results were compared with the stresses calculated by the Strength of Materials method. As expected, the largest stress concentration was observed at the inner surface of the re-entrant corner. Since the experimental values exceeded the calculated values at this point by about 80%, a stress concentration factor of 1.8 is assumed. Even though this stress concentration factor is valid only for static elastic situations, the same value is used to raise the thermal and dynamic stresses at this point in the absence of reliable methods to determine the exact factors.

4. Conclusion

The intent of the paper is to outline some of the problems of the stress analysis of MHD channels which have not as yet attracted serious attention.

The mechanical analysis of the problem is rendered complex by

- the unusual type of the structure (being a stack of "window" frames connected by bolts) and
- the loading characterized by severe thermal ambient, significant randomly fluctuating gas pressures, jet thrust and magnetic forces.

It is safe to say that a more-or-less "exact" solution of the problem would necessitate rather extensive and certainly expensive computer analyses.

In order to proportion the structure and reduce the computational effort and expense it is necessary to use engineering judgment and rather simple analyses such as analysis of a rectangular frame, two-bar model etc. In the case of the considered channel, these approximate analyses lead to a structure proved safe by more sophisticated models and tests.

Acknowledgment

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Figure 2. Test Loading of Frame Corner

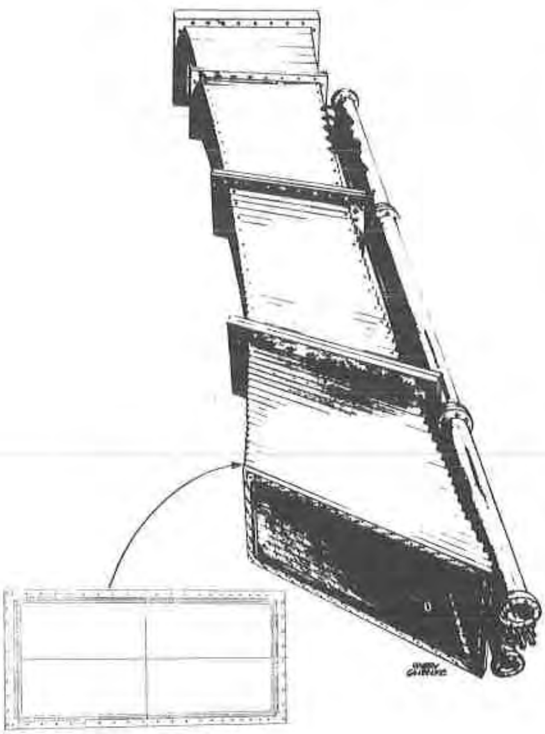

Figure 1. MHD Channel with Details of a Window Frame