On Determining Inspection Times for Turbine Discs Subject to Stress Corrosion Cracking — an Example of the Use of PRA in Structural Mechanics

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Abstract

Two methods can be used to calculate the inspection intervals of discs subject to stress corrosion cracking. The first is the traditional deterministic approach. The second, probabilistic method, seeks to make explicit and quantitative that which is only implicit in the traditional approach; namely, our degree of confidence that we will avoid a wheel burst. This is certainly a commendable objective, however, there exists a certain numerical problem in its implementation. A similar problem arises in all applications of PRA to structural mechanics. This paper points out this problem and discusses implications for decision-making.

1. Introduction

Turbine discs subject to stress corrosion cracking present a classic case of a cost/risk trade-off problem; for, on the one hand, it is very costly to open and inspect a turbine and on the other, if the crack is allowed to reach a size at which the wheel bursts we are then faced with a ruined turbine and a disastrously long plant outage. Accordingly, procedures are being developed to come to grips with this decision problem in a quantitative way.

Among these procedures two basic approaches can be discerned, the first being the traditional factor of safety approach and the second being a probabilistic approach which the work of Clark, Seth, and Schaffer[1] is an excellent published example. In the present paper we shall review these two approaches. We shall point out that the bottom line number calculated in the probabilistic approach is essentially numerically unstable. We point out further that a similar instability can occur in many or all structural mechanics risk problems and explain why it tends not to occur in ordinary nuclear plant risk calculations.

2. Deterministic or Factor of Safety Method

Figure 1 is a diagram showing the deterministic approach to finding the inspection time for a particular specific turbine disc. At time t_1 a crack of size a_1 is measured. Using the growth rate, g, the size is projected into the future and crosses the critical size, a_{cr}, at t_b, at which time, in the deterministic model, brittle rupture of the disc would take place. Therefore, in this approach one divides a_{cr} by a safety factor, typically two or four, and requires an inspection before that size is reached.

3. Probabilistic Method

The probabilistic method shown in Figure 2 repeats the deterministic process but includes a quantification of all uncertainties. Thus, the initial size-number a_1 is replaced by a probability curve, which reflects the uncertainty in the ultrasonic detection process.

Similarly, since the value of the growth rate is not known with certainty, the previous growth line now becomes a growth "cone," i.e., a set of possible growth lines, each with a probability assigned.*

Actually, even the assertion that the growth rate is constant in time is subject to considerable uncertainty. One could argue that the growth should accelerate with time, i.e., as the crack approaches critical size. On the other hand, because of branching, one could argue that the growth rate decelerates. Thus, the uncertainty in the crack growth in Figure 2 is represented by the "fan" emanating from the initial crack. The fan when "cut" at a particular time by a vertical line yields a probability curve showing our current state of knowledge about the crack size at that future time.

Finally, we must acknowledge that we do not know the critical crack size with certainty. We therefore express what we do and do not know about that with a probability curve, shown on the left axis, against a_{cr}.

Now for any instant of future time, our probability that the wheel will have burst by then is given by:

$$P_b(t) = \int_0^\infty p_a(a_{cr}) \int_{a_{cr}}^\infty p_a(a|t)\, da\, da_{cr} \tag{1}$$

This burst probability may then be added to the diagram, plotted against the right hand scale as shown in Figure 2.

4. Probabilistic Criterion

With the curve $P_b(t)$ established, one can now simply pick some value, e.g., 10^{-4}, and say "I do not want to bear a risk of burst greater than 10^{-4};" therefore, one would simply enter the figure at 10^{-4} and pick off the inspection time as shown. That is, inspection must be done no later than t_{insp}.

5. Numerical Instability

The above probabilistic criterion sounds very simple and sensible, and indeed it is. In using it however one must be aware that the numerical value from Equation (1) results primarily from the overlap of the tails of the p_a and p_c curves. A small change in the way these tails are drawn could change P_b by easily a factor of 10 or more. But these tails are poorly "known," that is, they are not well dictated by the evidence available; therefore, the final number P_b obtained from the calculation is very malleable.

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*Using here the discrete probability distribution idea [2] applied to the quantity g.

6. Structural Mechanics Risk and Ordinary PRA

A similar situation arises in any structural mechanics problem where the probability of failure of a structure comes from the overlap of tails of a "load" curve and a "strength" curve, as in Figure 3. The resulting number is malleable, especially when it is small, in the range of 10^{-4} or less.

Ordinary nuclear PRA also deals with numbers in this range. For example, a typical core melt frequency, ϕ_{cm}, is $\approx 10^{-4}$ per year. However, although ϕ_{cm} is small, it is not obtained from an overlap of tails. Basically, ϕ_{cm} is obtained from a process like that in Figure 4. Here the failure rate of the system, λ_s, is the product of the failure rates λ_1, λ_2, λ_3, of the components. By putting many redundant components in parallel (and guarding against common cause failures) λ_s can be made as small as one likes, yet it is not numerically unstable. Uncertainty in the λ_1 is, of course, transmitted through to uncertainty in λ_s. This uncertainty may be large percentagewise, when λ_s is a small number, but this is a real uncertainty, not a tails phenomenon as in Figure 3.

To say this another way, if in Figure 3 we changed the vertical axis from p to ϕ, and thus defined the curves as frequency distributions (i.e., population variability curves) rather than probability curves, one then could sensibly ask what is the uncertainty in those frequency distributions. One could express the uncertainty, using the DPD idea, by putting forth two families of possible frequency distributions, one family for "load" and one family for "strength." From the intersections of these two families one could calculate a probability curve like Figure 5 against the frequency of structural failure.

In this way of looking at it, the notion of numerical instability would show up in the form of the broadness of the Figure 5 curve.

7. Implications for Decision

We have suggested above that the probabilities of failure, calculated in structural mechanics problems are numerically very sensitive to the tails of the input distributions. This is in contrast to the frequencies of core melt calculated in ordinary PRA where only the tails are sensitive to the tails. What are the implications of this for structural mechanics decision-making?

Does it mean we should not calculate probabilities of failure for structural problems? No, by all means we should calculate them, but we should not believe the calculations in an uncritical way. We should not simply look at the number P_b and adopt a "pass/fail" regulatory type mentality. Rather, we need to look holistically at the entire diagram of Figure 2 and use good judgment. We should make sure that the probability curves p_c and p_a reflect our true state of uncertainty, and therefore we may want to intuitively broaden the curves suggested by the growth rate data and the fracture mechanics theory as suggested in Figure 6. This gives a "real" P_b, i.e., a value for probability of burst which truly expresses our degree of uncertainty about the matter.

As an aid to this latter thinking process we might even adopt the "probability of frequency" approach described in the last paragraph of Section 6. We would then obtain a probability curve, against structural failure frequency, like that of Figure 5, which would explicitly display the degree of numerical instability. Study of this figure together with study of Figure 2, or 3, and the background information going into these figures, provides a fuller basis for making the tough judgments and decisions that need to be made about structural safety.

References

/ 1 / Clark, W. G., Jr., B. B. Seth, and D. H. Shaffer, "Procedures for Estimating the Probability of Steam Turbine Disc Rupture from Stress Corrosion Cracking," Presented at the Joint ASME/IEEE Power Generation Conference, St. Louis, Missouri, October 4-8, 1981.

/ 2 / Kaplan, S., "On the Method of Discrete Probability Distributions in Risk and Reliability Calculations--Application to Seismic Risk Assessment," Risk Analysis, Vol. 1, No. 3, September 1981.

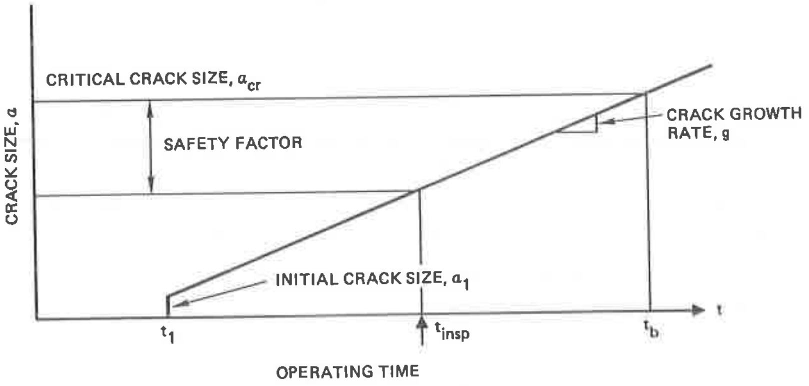


Figure 1. Determination of Inspection Time-Deterministic Method.

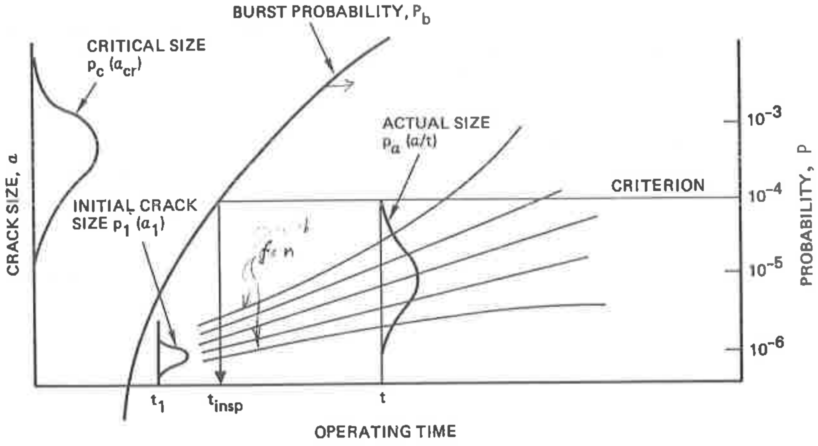


Figure 2. Determination of Inspection Time-Probabilistic Method.

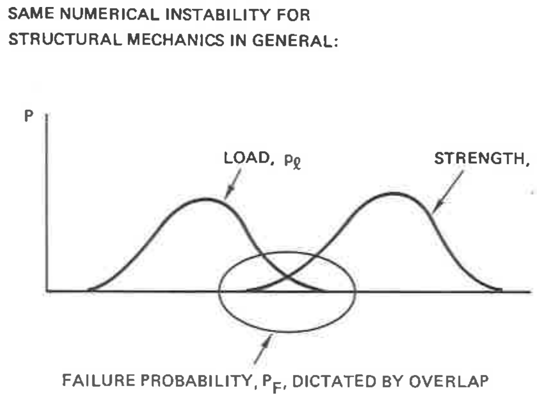
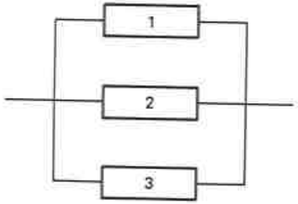


Figure 3. Numerical Instability for Structural Mechanics in General.



$$\lambda_s = \lambda_1 \times \lambda_2 \times \lambda_3$$

$$\lambda_i \approx 10^{-2}$$

$$\Rightarrow \lambda_s \approx 10^{-6}$$

Figure 4. Relation of System Failure Rate, λ_s, to Component Failure Rates, λ_i.

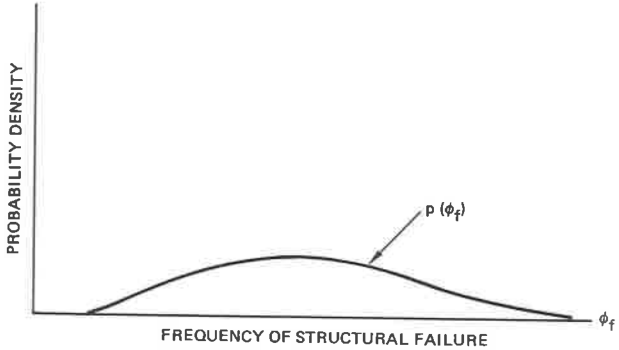


Figure 5. Probability of Structural Failure Frequency, ϕ_f, Showing the Numerical Instability in the Calculation by the Broadness of the Curve.

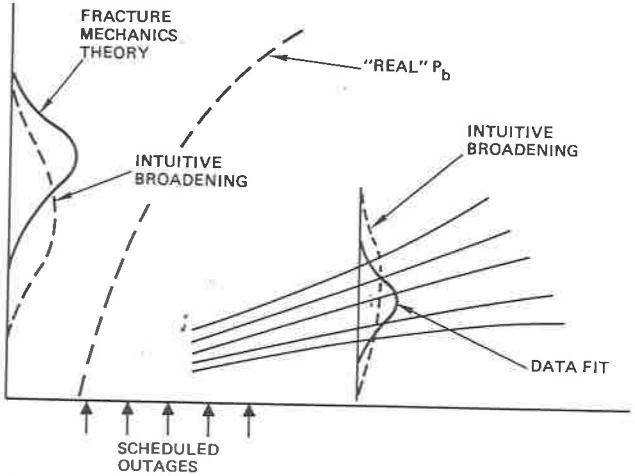


Figure 6. Inspection Interval Planning Diagram, Showing Intuitive Broadening of Probability Curves.