RESPONSE OF UNDERSEA NUCLEAR POWER PLANTS DURING EARTHQUAKES

N. AKKAŞ
Department of Civil Engineering,
Middle East Technical University, Ankara, Turkey

SUMMARY

A complete study of structures built under water must include the dynamic analysis, because these structures will inevitably be under the effects of some dynamic forces. For this reason, in the present work, a random forcing function representing a seismic excitation is chosen as a dynamic force.

When a structure is partially or totally submerged in a fluid medium, its dynamic characteristics will be substantially different from those due to its vibration in absence of the medium. A very important aspect of the problem is the solid-fluid interaction. Hydrodynamic pressures are generated by the vibrating structure which in turn modifies the pressures causing them. Therefore, what one is actually faced with is a "coupled" or "elastic-hydrodynamic" problem.

The mathematical model of the containment structure of an underwater nuclear power plant used in this investigation is a clamped hemispherical shell. In the derivation of the theory the following assumptions are made: (a) The shell is thin and elastic, (b) a linear analysis is sufficient, (c) the shell is in deep water and so the effects of the surface waves can be ignored, (d) the water is inviscid and irrotational. These assumptions reduce the problem to the analysis of forced motion of the shell in a semi-infinite fluid medium.

The differential equations governing the behaviour of the shell are:

$$[L]\begin{vmatrix} u \\ v \\ w \end{vmatrix} - c\begin{vmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ -\dot{\bar{w}} \end{vmatrix} - \begin{vmatrix} \ddot{\bar{u}} \\ \ddot{\bar{v}} \\ -\ddot{\bar{w}} \end{vmatrix} = -\begin{vmatrix} 0 \\ 0 \\ q_r \end{vmatrix} - \begin{vmatrix} 0 \\ 0 \\ f\dot{\Phi} \end{vmatrix} \quad (1)$$

The wave equation:

$$\nabla^2 \Phi = \frac{1}{s^2}\ddot{\Phi}. \quad (2)$$

In these equations (u,v,w) and $(\bar{u},\bar{v},\bar{w})$ are absolute and relative displacement components, respectively. The dots represent time derivatives, $[L]$ is a (3×3) linear matrix differential operator. Moreover, q_r=external pressure in radial direction, c=damping coefficient, f, s=constants of fluid medium, ∇^2;=Laplacian operator in spherical coordinates.

In equations (1) the absolute displacements are expressed in terms of the relative displacements. These relations insert the ground acceleration function into the equations. All dependent variables are expanded in a sine or cosine series in terms of the circumferential coordinate. The problem is completed by specifying the boundary, apex, kinematic boundary, and initial conditions. The conclusions that can be drawn from this work can be summarized as follows:

(a) The wave equation is always coupled with the other three equations through velocity potential term.
(b) For mode number $n\geq 2$ the problem is a free-vibration problem. These circumferential modes are not set into motion by the horizontal ground motion.
(c) The ground motion does not excite the axisymmetric mode, either. Since it is a horizontal ground motion, only the $n = 1$ mode is excited thereby and the shell responds with only a cantilever beam-type motion.

The equations given above are solved by finite difference techniques, using central difference formulas for space derivatives and Houbolt's backward difference formulas for time derivatives. In the solution of the difference equations Potters' method is employed. Some numerical results are presented.

1. Introduction

Nuclear power plants will eventually be built under water for safety purposes, if for no other reason. In densely populated coastal areas, since the availability of land is limited, the underwater construction of plants may prove to be an economical solution. Moreover, for the undersea cities of the future energy will most probably be supplied by undersea power plants. In fact, in recent literature some proposals, such as the ones by Hromadik and Breckenridge [1] , have been made regarding the design of this type of structures.

Nuclear power plants are sometimes built on sites where earthquake activity has been known to occur.Hence, it is necessary to include the seismic analysis in the general design analysis of the plant. Actually,the problem of nuclear reactor safety against earthquakes is one particular aspect of the more general safety problem.

The seismic analysis of nuclear power plants in a fluid medium is substantially different from that in absence of a medium. A very important aspect of the problem is the solid-fluid interaction. Hydrodynamic pressures are generated by the vibrating structure.These pressures modify the deformations which in turn modify the hydrodynamic pressures causing them.Therefore, what one is actually faced with is a "coupled" or elasto-hydrodynamic problem.

The seismic response of nuclear containment vessels in vacuo has received considerable attention in the literature. Lin [2] studied the earthquake response of the vessel modeling it as a fixed-free thin cylindrical shell. In his analysis he uses the linear shell theory. His solution indicates that when subjected to horizontal ground motion, the vessel vibrates purely as a cantilever beam. This conclusion had previously been obtained numerically by Kalnins [3] . The nonlinear seismic response of thin reactor containment vessels subjected to a horizontal acceleration at the base was examined by Citerley and Ball [4,5] . Their model is a cylindrical shell with a spherical head.They conclude that the nonlinear effects are negligible and motion in the ovalling mode of the vessel is insignificant.

The dynamic interaction between a shell and a fluid medium has mostly been investigated as an acoustics problem. Most of the investigations of this nature available in the literature are on cylindrical shells. The investigations on the forced vibration of a spherical shell in an acoustic medium are not very many. Junger [6] discusses the steady state response of a spherical shell in an infinite acoustic medium, and the equations are derived on the basis of the extensional theory only. The problem of forced motion of a spherical shell in an acoustic medium under a concentrated force was studied by Hayek [7] . Lou and Klosner [8] investigated the transient response of a submerged, ring-stiffened spherical shell to a pressure increase in the surrounding acoustic medium. In references [7] and [8] the equations for axisymmetric, nontorsional vibration of spherical shells have been used. To the author's knowledge the seismic response of spherical shells in a fluid medium has not been investigated so far.

It is the purpose of this paper to study the response of undersea nuclear power plant containment vessels during earthquakes. The vessel is modeled as a clamped hemispherical shell. The earthquake is simulated by a random horizontal acceleration. The governing equations are solved numerically and some numerical results are presented.

2. Theory

The mathematical model of the containment structure of an underwater nuclear power plant used in this investigation is a clamped hemispherical shell as shown in Fig.1. The shell is assumed to be thin, and the shell material is linearly elastic, homogeneous, and isotropic. The containment structure is in deep water and so the effects of the surface waves are ignored. The water is inviscid and irrotational. Only those aspects of fluid loading that are essentially a reaction of the fluid to motion of the body are taken into consideration. Forces due to fluid motion, such as turbulent boundary layers, on bodies are not considered. No approximate relations are introduced in the fluid-shell phenomena in this study; hence, the results obtained are exact within the scope of the linear thin shell theory, and the approximation errors introduced by the numerical techniques.

Under the assumptions given above the equations of motion for the shell are given, in nondimensional form, as follows :

$$[L]\begin{Bmatrix}\bar{u}\\ \bar{v}\\ \bar{w}\end{Bmatrix} = (1-\nu^2)\,\mathrm{Sin}\,\varphi\begin{Bmatrix}\partial^2\bar{u}/\partial\tau^2 + c\,\partial u/\partial\tau\\ \partial^2\bar{v}/\partial\tau^2 + c\,\partial v/\partial\tau\\ q - \partial^2\bar{w}/\partial\tau^2 - c\,\partial w/\partial\tau\end{Bmatrix} \qquad (1)$$

in which [L] is a 3x3 linear matrix differential operator whose elements are given in the Appendix. Equations (1) are similar to the ones given by Flügge [9] . The nondimensional quantities appearing in eqs. (1) are related to the corresponding physical quantities through the following relations:

$$k = \frac{1}{12}\left(\frac{h}{a}\right)^2 , \quad \bar{u} = \bar{U}/a , \quad \bar{v} = \bar{V}/a , \quad \bar{w} = \bar{W}/a$$

$$u = U/a , \quad v = V/a , \quad w = W/a \qquad (2)$$

$$c = \frac{aC}{h\sqrt{\rho_s E}} , \quad \tau = \left(\frac{t}{a}\right)\sqrt{\frac{E}{\rho_s}} , \quad q = \frac{ap}{Eh}$$

In the foregoing expressions a and h are, respectively, the radius and the thickness of the shell. $\bar{U},\bar{V},\bar{W}$ are the absolute meridional,circumferential, and radial displacement components, respectively; and U,V,W are the corresponding relative displacement components. The mass density and the modulus of elasticity of the shell material are denoted by ρ_s and E, respectively. The damping coefficient is C, t denotes the physical time, and p is the pressure in radial direction. Finally, Poisson's ratio is denoted by ν , φ and θ are meridional and circumferential coordinates.

The motion of an inviscid and irrotational fluid undergoing small oscillations is governed by the wave equation. In spherical coordinates its nondimensional form is

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\operatorname{Cot} \xi}{r^2}\frac{\partial \Phi}{\partial \xi} + \frac{1}{r^2 \operatorname{Sin}^2 \xi}\frac{\partial^2 \Phi}{\partial \theta^2} = \frac{1}{s^2}\frac{\partial^2 \Phi}{\partial \tau^2} \qquad (3)$$

in which the nondimensional quantities are related to the physical quantities through the following relations :

$$\Phi = \frac{\bar{\Phi}}{a c_s} \quad , \quad r = \frac{R}{a} \quad , \quad s = \frac{c_f}{c_s} \qquad (4)$$

Here, c_f and c_s are the speeds of sound in the fluid and the shell, respectively. R is radial coordinate and ξ is described in Fig.1. The velocity potential is denoted by $\bar{\Phi}$.

The absolute and relative displacements of a point on the hemispherical shell are related through

$$\begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} + X \begin{Bmatrix} -\operatorname{Cos}\theta \operatorname{Cos}\varphi \\ \operatorname{Sin}\theta \\ -\operatorname{Cos}\theta \operatorname{Sin}\varphi \end{Bmatrix} \qquad (5)$$

in which X=x/a is the nondimensional horizontal displacement of the base of the shell and x is the actual ground displacement.

To represent the ground displacement a random acceleration function of the form

$$d^2x / dt^2 = 0 \qquad \text{for } t < 0$$

$$d^2x / dt^2 = \sum_{j=1}^{J} \xi_j \, t \, e^{-\zeta_j t} \operatorname{Cos}(\omega_j t + \psi_j) \qquad \text{for } t \geqslant 0 \qquad (6)$$

will be used. This random acceleration function is suggested by Bogdanoff et al. [10] and it is employed by Parmelee et al. [11] in their seismic analysis of structure-foundation systems. In eq.(6) ξ_j and ζ_j are real positive numbers, with $\omega_1 < \omega_2 < \dots < \omega_J$, and $\psi_1, \psi_2, \dots \psi_J$ are J independent real random variables uniformly distributed over the interval 0 to 2π. Following Parmelee et al. [11], in this study it is assumed that ξ_j and ζ_j are constants and equal to 0.50 and 0.333, respectively. Moreover, J is taken to be ten and ω_j and ψ_j for j = 1,2,....10 are given in reference[11]. The resulting earthquake acceleration is shown in Fig.2.

Substituting eqs.(5) into eqs.(1) one obtains

$$[L]\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = (1-\nu^2)\sin\varphi \begin{Bmatrix} \partial^2 u/\partial\tau^2 + c\,\partial u/\partial\tau - \cos\theta\cos\varphi\, d^2X/d\tau^2 \\ \partial^2 v/\partial\tau^2 + c\,\partial v/\partial\tau + \sin\theta\, d^2X/d\tau^2 \\ q - \partial^2 w/\partial\tau^2 - c\,\partial w/\partial\tau + \cos\theta\sin\varphi\, d^2X/d\tau^2 \end{Bmatrix} \quad (7)$$

The radial pressure q consists of three components:

$$q = q_i - q_e - f\,\frac{\partial \Phi}{\partial \tau} \quad (8)$$

in which q_i and q_e are internal and external pressures, respectively, and the third term is the pressure in the fluid. The nondimensional fluid-shell interaction parameter f is

$$f = \frac{a\rho_f}{h\rho_s} \quad (9)$$

in which ρ_f is the mass density of the fluid.

Assuming that the earthquake forcing function is applied at $\theta = 0^\circ$, the response of the shell will be symmetric with respect to $\theta = 0, \pi$ plane. Therefore, one can let

$$(u,v,w,\Phi) = \sum_{n=0}^{\infty} (u_n\cos n\theta,\ v_n\sin n\theta,\ w_n\cos n\theta,\ \Phi_n\cos n\theta) \quad (10)$$

in which n is the circumferential mode number. Substituting eqs.(8) and(10) into eqs.(7) and (3) one obtains

$$[L_n]\begin{Bmatrix} u_n \\ v_n \\ w_n \end{Bmatrix} = (1-\nu^2)\sin\varphi \begin{Bmatrix} \partial^2 u_n/\partial\tau^2 + c\,\partial u_n/\partial\tau - \cos\varphi\, d^2X/d\tau^2\,\delta_1 \\ \partial^2 v_n/\partial\tau^2 + c\,\partial v_n/\partial\tau + d^2X/d\tau^2\,\delta_1 \\ -\partial^2 w_n/\partial\tau^2 - c\,\partial w_n/\partial\tau + \sin\varphi\, d^2X/d\tau^2\,\delta_1 \\ +(q_i - q_e)\delta_0 - f\,\partial\Phi_n/\partial\tau \end{Bmatrix} \quad (11)$$

$$\frac{\partial^2\Phi_n}{\partial r^2} + \frac{2}{r}\frac{\partial\Phi_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi_n}{\partial\xi^2} + \frac{\cot\xi}{r^2}\frac{\partial\Phi_n}{\partial\xi} - \frac{n^2\Phi_n}{r^2\sin^2\xi} = \frac{1}{s^2}\frac{\partial^2\Phi_n}{\partial\tau^2} \quad (12)$$

in which

$$\delta_i = \begin{cases} 1 & \text{for} \quad i = n \\ 0 & \text{for} \quad i \neq n \end{cases} \tag{13}$$

The matrix differential operator $[L_n]$ is obtained from $[L]$ by substituting corresponding n's for the derivatives with respect to circumferential coordinate θ .

The linear behavior of a hemispherical shell in a fluid medium subjected to a horizontal earthquake motion is governed by eqs.(11) and (12). The problem is completed by specifying the apex, boundary, initial, and kinematic boundary conditions. The kinematic boundary condition is given as follows :

$$\partial \Phi_n / \partial r = \partial w_n / \partial \tau - \delta_1 \operatorname{Sin} \varphi \, dX / d\tau \tag{14}$$

at r = 1. This implies that the radial velocities of the shell and fluid are equal for all φ and τ . The initial conditions used in this work are

$$u_n = v_n = w_n = \frac{\partial u_n}{\partial \tau} = \frac{\partial v_n}{\partial \tau} = \frac{\partial w_n}{\partial \tau} = \Phi_n = \frac{\partial \Phi_n}{\partial \tau} = 0 \tag{15}$$

at $\tau = 0$. Moreover, as $r \longrightarrow \infty$, $\Phi_n \longrightarrow 0$.

Referring to eqs.(11) and (12) it is seen that the wave equation (12) is always coupled with the other three equations through the $\partial \Phi_n / \partial \tau$ term. For $n \geqslant 2$ the problem is a free-vibration problem. The circumferential modes with $n \geqslant 2$ are not set into motion by the horizontal ground motion. This type of ground motion does not excite the axisymmetric mode (n = 0) , either. Therefore, only n = 1 mode is excited by the horizontal ground motion, and the shell responds with only a cantilever beam-type motion.This conclusion is obtained also by Lin [2] .

The total response of the containment vessel of the nuclear power plant consists of the seismic response and the static response of the vessel due to an internally applied pressure, if there is any, and the external hydrostatic pressure. For the static response only the mode n = 0 needs to be considered, and for the seismic response only n = 1 mode is to be considered These responses are then added linearly to obtain the total response. In the following section the numerical results will be presented only for n=1 mode.

3. Numerical Procedure and Results

The coupled linear differential equations (11) and (12) governing the behavior of hemispherical shell under water subjected to a horizontal ground motion are solved numerically by finite difference techniques. The meridional and radial derivatives are replaced by the conventional central finite difference approximations. However, at the apex of the shell the forward finite difference approximations are used. The inertial terms that appear in the equations are approximated by Houbolt's [12] backward differencing scheme. Accordingly, the second time derivative of the nondimensional velocity potential Φ , for instance, is approximated as follows :

$$\frac{\partial^2 \Phi}{\partial \tau^2}\Big|_{I,J}^{k} = \frac{1}{(\Delta \tau)^2}\left(2\,\Phi_{I,J}^{k} - 5\,\Phi_{I,J}^{k-1} + 4\,\Phi_{I,J}^{k-2} - \Phi_{I,J}^{k-3}\right) \qquad (16)$$

in which $\Delta\tau$ is the nondimensional time interval and k denotes the time step. Stations along meridional and radial directions are denoted by I and J, respectively. For the first time derivative of the ground displacement function X the following approximation is used :

$$\frac{dX}{d\tau}\Big|^{k} = \frac{1}{\Delta\tau}\left(X^{k} - X^{k-1}\right) + \frac{\Delta\tau}{2}\frac{d^2X}{d\tau^2}\Big|^{k} \qquad (17)$$

Using the finite difference approximations mentioned the governing equations are reduced to sets of algebraic equations which are, then,solved using Potters' [13] form of Gaussian elimination.

For the finite difference approximations the shell meridian in one quadrant is divided into nine equal increments; in other words, the meridional increment is taken to be ten degrees. The size of the computer available limited the number of equal increments along the radial direction to five. The finite difference scheme is shown in Fig.1. The physical time increment Δt is taken to be 0.20 sec.

For the numerical calculations it is assumed that the containment vessel is made of steel and the surrounding medium is water. The physical dimensions and properties used are as follows :

a = 600 in. , h = 3 in.,

$\nu = 0.30$, $E = 30 \times 10^6$ psi ,

$\rho_s = 0.7347 \times 10^{-3}$ lb - sec^2/in^4 ,

$\rho_f = 0.9582 \times 10^{-4}$ lb - sec^2/in^4 ,

$c_f = 6 \times 10^4$ in/sec,

Since, in the numerical calculations, only n = 1 mode is analyzed, the adequacy of the shell with the dimensions given in resisting the hydrostatic pressure present is not considered in this investigation.

The nondimensional displacements u_1,v_1, and w_1 are plotted in Figs.3-5 as functions of time t. They are plotted only for I=5 ; however, the conclusions are the same for the other mesh points. A comparison of Figs.2-5 reveals that the displacements are also random functions of time. The effect of damping is also shown in Figs.3-5. The nondimensional damping coefficient c is taken to be 10. Damping is generally reducing the amplitude of the vibrations, although at some time stations the displacements of the damped system are larger than those of the undamped system. One final remark about Figs.3-5 is the fact that the radial and meridional displacements u_1 and w_1 are in phase, but the circumferential displacement v_1 has a 0.2 seconds phase difference. Fig.6 presents the velocity potential Φ_1, at I=4 , J=1 as a function of time t. The figure is self-explanatory.

In Fig.7 the nondimensional displacements u_1,v_1, and w_1 are plotted as functions of meridional mesh point I at a fixed time t=2.20 sec. The effect of damping for c=10 is also shown in the figure. Finally, Figs.8 and 9 give the nondimensional velocity potential Φ_1 as functions of radial mesh point J and meridional mesh point I, respectively at t=2.20sec. Damping coefficient c is taken to be zero.

References

[1] Hromadik,J.J.,Breckenridge,R.A.,"Construction concepts for the deep ocean," Civil Engineering in the Oceans,ASCE Conference,713(1967).

[2] Lin,C.W.,"Seismic response of nuclear reactor containment vessels," Nucl.Eng.Design 9,234(1969).

[3] Kalnins,A.,"Calculation of the response of a shell to earthquake excitation,"Unpublished Manuscript (1967).

[4] Citerley,R.L.,Ball,R.E.,"Nonlinear seismic response of a reinforced concrete nuclear reactor containment vessel,"Nucl.Eng.Design 16,341 (1971).

[5] Citerley,R.L.,Ball,R.E.,"Nonlinear seismic analysis of thin reactor containment vessels," J.Eng.for Industry, 925 (1972).

[6] Junger,M.C., "Vibration of elastic shells in a fluid medium and the associated radiation of sound," J.App.Mech. 74,439 (1952).

[7] Hayek,S., "Vibration of a spherical shell in an acoustic medium," J.Acoust.Soc.Am. 40,342 (1966).

[8] Lou,Y.K., Klosner,J.M., "Dynamics of a submerged ring-stiffened spherical shell," J.App.Mech., 408 (1971).

[9] Flügge,W., Stresses in Shells, 3rd printing, Springer-Verlag,US(1966).

[10] Bogdanoff,J.L., Goldberg,J.E., Bernard,M.C.,"Response of a simple structure to a random earthquake-type disturbance," Bull.Seism.Soc.Am. 51,293 (1961).

[11] Parmelee,R.A., Perelman,D.S., Lee,S.L., Keer,L.M.,"Seismic response of structure-foundation systems," J.Eng.Mech.Div.ASCE 94,(1968).

[12] Houbolt,J.C., "A recurrence matrix solution for the dynamic response of aircraft in gusts," NACA Report, 1010 (1951).

[13] Potters,M.L., "A matrix method for the solution of a second order difference equation in two variables,"Math.Cent.Amsterdam,MR19 (1955).

APPENDIX

$$L_{11} : (1+k)\left(\sin\varphi \frac{\partial^2}{\partial\varphi^2} + \cos\varphi \frac{\partial}{\partial\varphi} - \frac{\cos^2\varphi + \nu\sin^2\varphi}{\sin\varphi} + \frac{1-\nu}{2\sin\varphi} \frac{\partial^2}{\partial\theta^2}\right)$$

$$L_{12} : (1+k)\left(\frac{1+\nu}{2} \frac{\partial^2}{\partial\theta\partial\varphi} - \frac{3-\nu}{2} \cot\varphi \frac{\partial}{\partial\theta}\right)$$

$$L_{13} : (1+k)(1+\nu)\sin\varphi \frac{\partial}{\partial\varphi} - k\left[\sin\varphi \frac{\partial^3}{\partial\varphi^3} + \cos\varphi \frac{\partial^2}{\partial\varphi^2} + (1-\cot^2\varphi)\sin\varphi \frac{\partial}{\partial\varphi} + \frac{1}{\sin\varphi} \frac{\partial^3}{\partial\theta^2\partial\varphi} - \frac{2\cos\varphi}{\sin^2\varphi} \frac{\partial^2}{\partial\theta^2}\right]$$

$$L_{21} : (1+k)\frac{(1+\nu)}{2} \frac{\partial^2}{\partial\theta\partial\varphi} + (1+k)\frac{(3-\nu)}{2} \cot\varphi \frac{\partial}{\partial\theta}$$

$$L_{22} : (1+k)\left\{\frac{(1-\nu)}{2}\left[\sin\varphi \frac{\partial^2}{\partial\varphi^2} + \cos\varphi \frac{\partial}{\partial\varphi} - \sin\varphi(\cot^2\varphi - 1)\right] + \frac{1}{\sin\varphi} \frac{\partial^2}{\partial\theta^2}\right\}$$

$$L_{23} : (1+k)(1+\nu)\frac{\partial}{\partial\theta} - k\left(\frac{\partial^3}{\partial\theta\partial\varphi^2} + \cot\varphi \frac{\partial^2}{\partial\theta\partial\varphi} + 2\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\varphi} \frac{\partial^3}{\partial\theta^3}\right)$$

$$L_{31} : (1+k)(1+\nu)\left(\sin\varphi \frac{\partial}{\partial\varphi} + \cos\varphi\right) - k\left[\sin\varphi \frac{\partial^3}{\partial\varphi^3} + 2\cos\varphi \frac{\partial^2}{\partial\varphi^2} - \frac{\cos^2\varphi}{\sin\varphi} \frac{\partial}{\partial\varphi} + (3+\cot^2\varphi)\cos\varphi + \frac{1}{\sin\varphi} \frac{\partial^3}{\partial\varphi\partial\theta^2} + \frac{\cos\varphi}{\sin^2\varphi} \frac{\partial^2}{\partial\theta^2}\right]$$

$$L_{32} : (1+k)(1+\nu)\frac{\partial}{\partial\theta} - k\left[\frac{\partial^3}{\partial\theta\partial\varphi^2} - \cot\varphi \frac{\partial^2}{\partial\theta\partial\varphi} + (3+\cot^2\varphi)\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\varphi} \frac{\partial^3}{\partial\theta^3}\right]$$

$$L_{33} : 2(1+k)(1+\nu)\sin\varphi + k\left[\sin\varphi \frac{\partial^4}{\partial\varphi^4} + 2\cos\varphi \frac{\partial^3}{\partial\varphi^3} - (1+\nu+\cot^2\varphi)\sin\varphi \frac{\partial^2}{\partial\varphi^2} + (2-\nu+\cot^2\varphi)\cos\varphi \frac{\partial}{\partial\varphi} - 2(1+\nu)\sin\varphi + \frac{2}{\sin\varphi} \frac{\partial^4}{\partial\theta^2\partial\varphi^2} - 2\frac{\cos\varphi}{\sin^2\varphi} \frac{\partial^3}{\partial\theta^2\partial\varphi} + \frac{(3-\nu+4\cot^2\varphi)}{\sin\varphi} \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^3\varphi} \frac{\partial^4}{\partial\theta^4}\right]$$

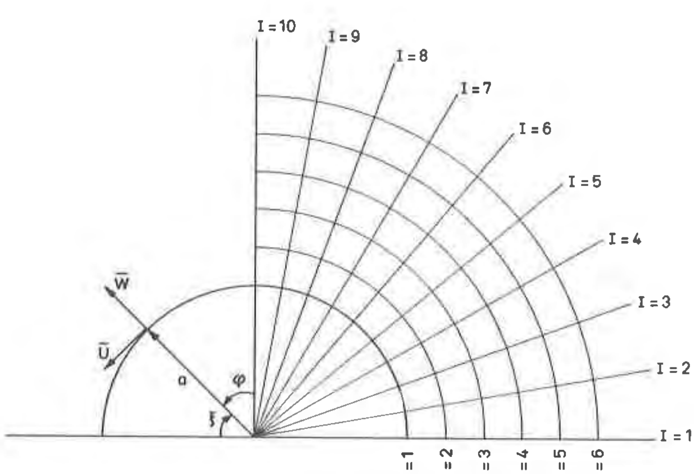

Fig.1 Geometry of the shell

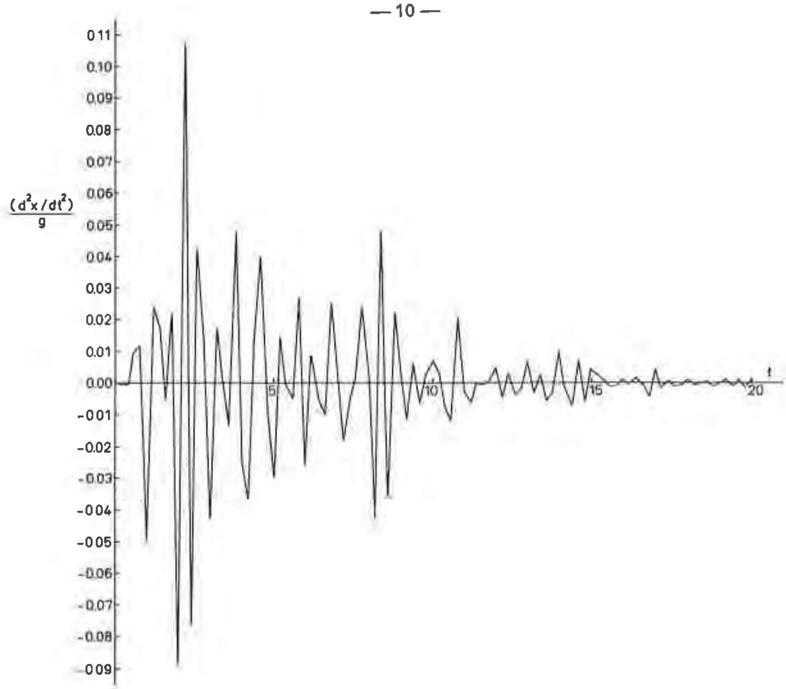

Fig.2 Earthquake acceleration function.

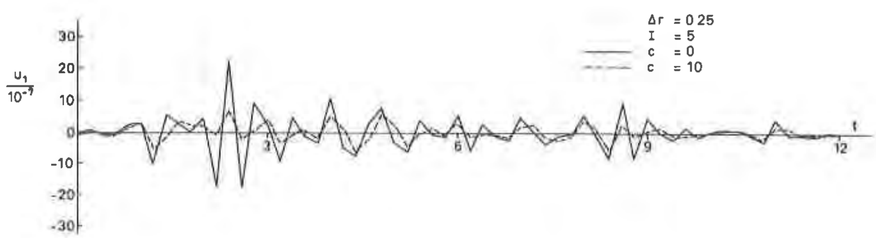

Fig.3 Meridional displacement u_1 as function of time.

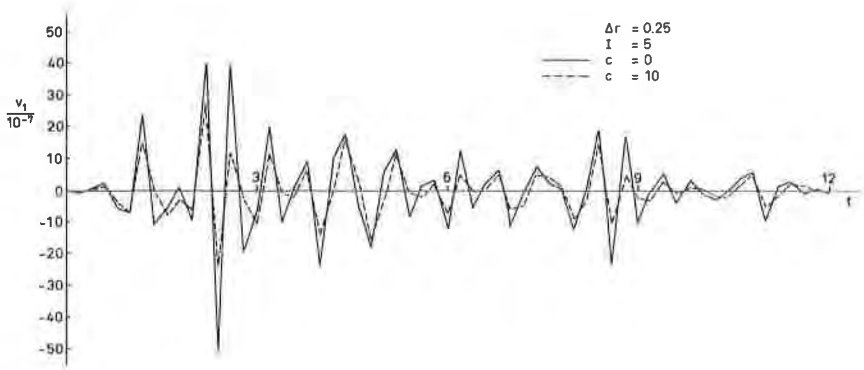

Fig.4 Circumferential displacement v_1 as function of time.

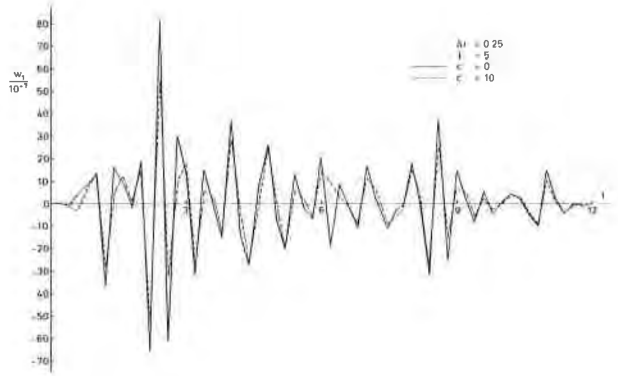

Fig.5 Radial displacement w_1 as function of time.

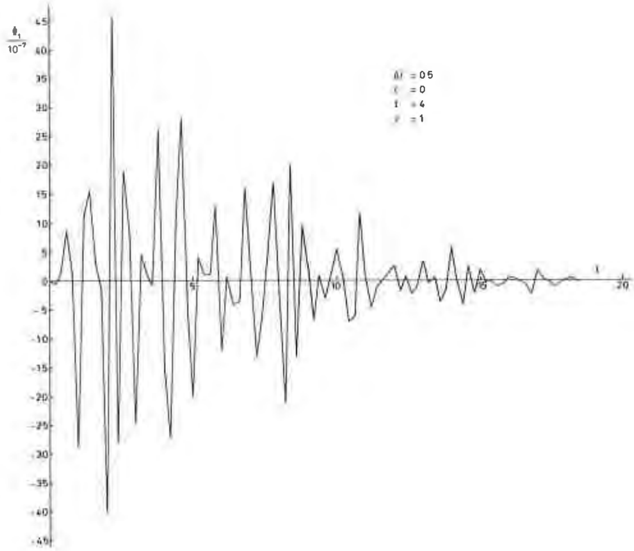

Fig.6 Velocity potential Φ_1 as function of time.

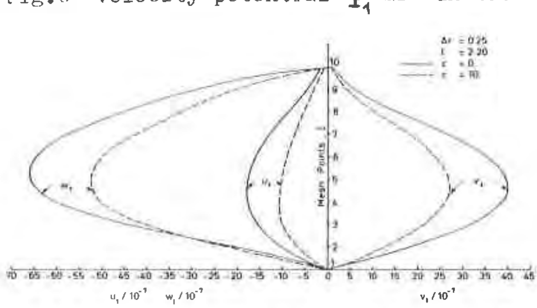

Fig.7 Displacements as functions of meridional mesh point

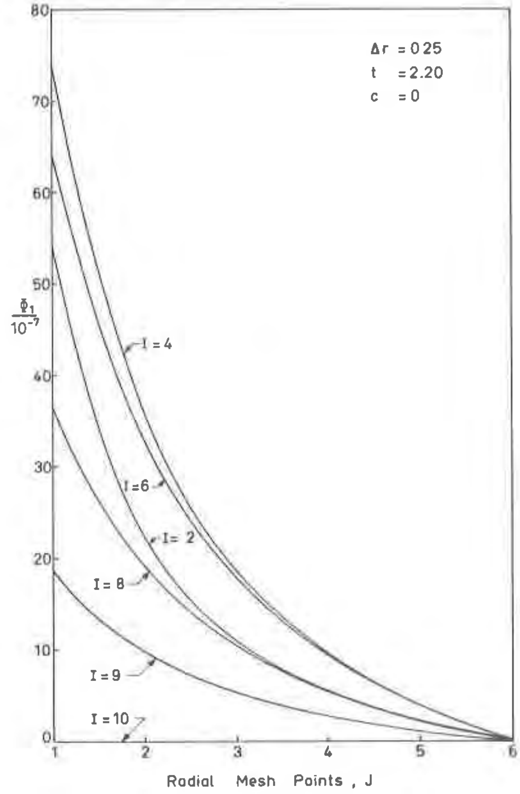

Fig.8 Velocity potential Φ_1 as function of radial mesh point

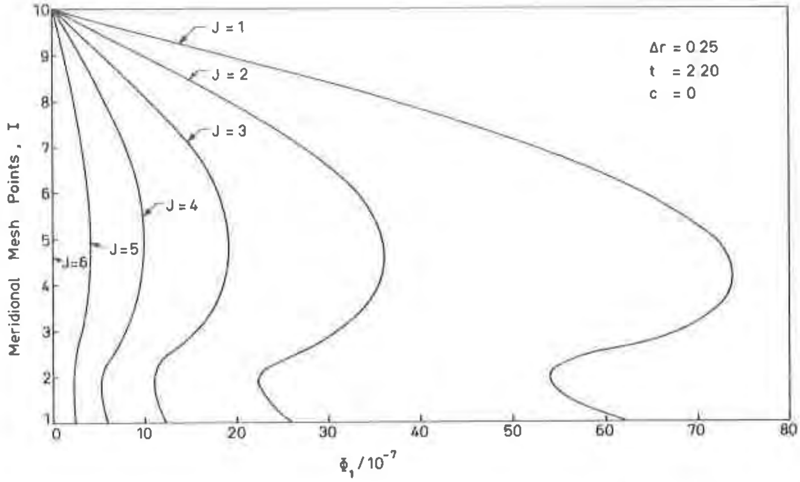

Fig.9 Velocity potential Φ_1 as function of meridional mesh point .