

THERMAL SHOCKS IN SOLAR BOILER TUBES AND MECHANICAL TOLERANCE TO HEATING VELOCITY

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The boiler circular cross-section tubes are cooled by an internal flow and are subjected to a non uniform heat flux around their outer circumference that changes very rapidly with time. Thus thermal shocks can develop in the thickness of tube walls and may cause brittle fracture or fatigue damage.

In a first step we solve the corresponding thermoelastic problem. The determination of temperature distribution through the wall thickness requires the solution of one-dimensional transient heat equation obtained by performing a Fourier expansion in the angular variable. For each harmonic, Galerkin's method with respect to the radial coordinate together with a finite difference scheme with respect to time permit to completely discretize the associated equation.

The mechanical problem is set in terms of stresses only, by expressing the conditions for displacement single-valuedness on the inner part of the boundary of the doubly connected tube cross-section, without using an Airy stress function : for each harmonic, a closed form expression can be derived which directly leads to the stress values, completely avoiding the computation of derivatives which would heavily impair the accuracy of numerical results. This formulation permits to split the stress field into two basic parts :

- a "pseudo-transient" part, essentially due to thermal shock, whose initial value is zero and which tends to zero after a sufficient time.
- an "instantaneous pseudo-stationary" part, mainly due to the conduction that would subsist in steady state, that is the stationary field that would be obtained if the values of temperature and its gradient on the inner boundary ceased to vary at the considered instant.

In a second step we study the mechanical tolerance to heating intensity of the boiler tubes. The maximum equivalent stress (related to Mises' criterion) attained during operation is expressed in terms of the characteristic parameters of the tube and the maximum flux. Many interesting features can be deduced with a simple and accurate numerical scheme, notably : the evolution of stresses with time at every point, the relative contributions of the two parts of the stress field, the equivalent stress distribution through wall thickness at every instant.

This study should be applicable to a wide range of exchanger tubes subjected to similar thermal loading conditions.

Introduction

In solar central receiver electric power plants the concentrated heat flux is received by circular cross section tubes in which some cooling fluid flows. The operation involves transient working conditions due to start up, cloud passage, ... It is worthy to assess the value of stresses obtained during these critical stages. The first step of such a study necessarily consists of a thermoelastic analysis. In the present work the tube thickness is arbitrary. KETTLEBOROUGH [1] treated a related problem with convection boundary conditions on both internal and external surfaces. An angular variation of the external heat transfer coefficient was taken into account. His formulation leads to solving the transient heat equation together with a biharmonic equation, then computing numerical derivatives and adjusting some single-valuedness conditions on the inner boundary. We recently were made aware of a work by GUPTA et alii [2], who consider a thick tube subjected to a heat flux varying with the angular coordinate on its inner surface. The problem is set in terms of displacement Fourier components. The thermoelastic solution is not completely in closed form and obtaining stresses requires the computation of some derivatives.

On the other hand we sought a fully explicit solution capable of displaying some qualitative properties of the transient thermoelastic phenomenon, that can be given a physical significance.

2 Problem formulation

2-1 Basic assumptions

A long circular cylindrical tube is referred to a cylindrical coordinate system ; (r, θ, x) respectively denote the radial, angular and longitudinal coordinates.

* The outer tube surface is subjected to a heat flux of the form :

$$\begin{aligned} \phi(\theta, t) &= \phi_0 H(t) \langle \cos \theta \rangle \text{ where } H \text{ is the Heaviside unit step function and} \\ \langle \cos \theta \rangle &= \cos \theta \quad \text{if } 0 < \theta < \pi/2 \quad \text{or} \quad 3\pi/2 < \theta < 2\pi \\ &= 0 \quad \pi/2 < \theta < 3\pi/2, \end{aligned}$$

t denotes the time.

* In the initial state, temperature is uniform.

* The bulk temperature of the fluid T_b does not vary with time.

* The tube is very long and no axial temperature variation occurs.

* According to the way in which the tube is held, several deformation states are to be considered : all of these can readily be deduced from the plane strain solution.

* We neglect other external loads and fluid pressure.

* The material is assumed to be purely thermoelastic.

2-2 Equations

The temperature and the prescribed heat flux are expanded in Fourier series with respect to the angular coordinate : $T(r, \theta, t) = \sum_{p=0}^{\infty} T^{(p)}(r, t) \cos p\theta$; $\Phi(\theta, t) = \sum_{p=0}^{\infty} \Phi^{(p)}(t) \cos p\theta$.

Each harmonic $T^{(p)}$ solves a distinct independent problem [3] :

$$\left\{ \begin{aligned} \Delta T^{(p)} &= \frac{\rho c}{k} \frac{\partial T^{(p)}}{\partial t} \\ k (T_b^{(p)} - T^{(p)}) &= -k \frac{\partial T^{(p)}}{\partial r} && \text{at } r = R_a \text{ (internal radius) eq. (1)} \\ \Phi^{(p)}(t) &= -k \frac{\partial T^{(p)}}{\partial r} && \text{at } r = R_b \text{ (external radius)} \end{aligned} \right.$$

where $T_b^{(0)} = T_b$; $T_b^{(p)} = 0$ if $p > 0$, and h, k, c, ρ respectively denote the heat transfer coefficient, thermal conductivity coefficient, specific heat and mass density of the material.

The elastic problem is set in terms of stresses only, without using any Airy stress function so as to avoid any derivative of temperature in the stress expressions.

Let $\sigma_r, \sigma_\theta, \sigma_{r\theta}$ denote the usual stress components in the cross-section plane. We assume

$$\sigma_r(\pi, \theta) = \sum_{p=0}^{\infty} \sigma_r^{(p)}(\pi) \cos p\theta, \quad \sigma_\theta(\pi, \theta) = \sum_{p=0}^{\infty} \sigma_\theta^{(p)}(\pi) \cos p\theta, \\ \sigma_{r\theta}(\pi, \theta) = \sum_{p=1}^{\infty} \sigma_{r\theta}^{(p)}(\pi) \sin p\theta,$$

where the variable t is understated. For each harmonic p the governing equations are - equilibrium equations :

$$\begin{cases} \frac{d\sigma_r^{(p)}}{d\pi} + \frac{p}{\pi} \sigma_{r\theta}^{(p)} + \frac{\sigma_r^{(p)} - \sigma_\theta^{(p)}}{\pi} = 0 \\ \frac{d\sigma_{r\theta}^{(p)}}{d\pi} - \frac{p}{\pi} \sigma_\theta^{(p)} + 2 \frac{\sigma_{r\theta}^{(p)}}{\pi} = 0 \end{cases} \quad \text{eq. (2)}$$

- compatibility equation :

$$\frac{d^2 \sigma_r^{(p)}}{d\pi^2} + \frac{1}{\pi} \frac{d\sigma_r^{(p)}}{d\pi} - \frac{p^2}{\pi^2} \sigma_r^{(p)} = 0 \quad \text{with } \sigma^{(p)} = \sigma_r^{(p)} + \sigma_\theta^{(p)} + \frac{E\alpha}{1-\nu} T^{(p)} \quad \text{eq. (3)}$$

where E denotes Young's modulus and ν is Poisson's ratio.

- single-valuedness of displacements on the inner tube boundary :

$$\begin{cases} \pi \frac{d\sigma_r^{(0)}}{d\pi} (1-\nu) - \pi \nu \frac{d\sigma_r^{(0)}}{d\pi} + \pi \alpha E \frac{dT^{(0)}}{d\pi} + \sigma_\theta^{(0)} - \sigma_r^{(0)} = 0 \\ \pi \frac{d\sigma_\theta^{(1)}}{d\pi} (1-\nu) - \pi \nu \frac{d\sigma_\theta^{(1)}}{d\pi} + \pi \alpha E \frac{dT^{(1)}}{d\pi} - (1-\nu) \sigma_r^{(1)} + \nu \sigma_\theta^{(1)} - \alpha E T^{(1)} - 2 \sigma_{r\theta}^{(1)} = 0 \end{cases}$$

- traction-free boundary conditions :

$$\begin{cases} \sigma_r^{(p)} = \sigma_{r\theta}^{(p)} = 0 & \text{at } \pi = R_a \\ \sigma_r^{(p)} = \sigma_{r\theta}^{(p)} = 0 & \text{at } \pi = R_b \end{cases}$$

3 Solution method

3-1 Temperature

The problem defined by system (1) possesses complicated closed form solutions but we found it more rewarding to use a purely numerical method. After temperature has been expanded in a Fourier series, we first discretize the heat equation with respect to the variable r through a weighted residual procedure (which amounts to a Galerkin method here [4]). We used eight uniformly spaced nodes over the tube thickness and linear interpolation functions. Then discretization with respect to time is achieved through the Crank-Nicholson finite difference scheme ([5], [6]).

3-2 Stresses

For each harmonic, the key idea is to split the stress field into two distinct parts :

- the first contribution is obtained by putting $\Delta T = 0$ in the compatibility condition while retaining the temperature terms in the single-valuedness conditions : at each instant the corresponding temperature field is harmonic : the pertaining stress field is said to be the "instantaneous pseudo-stationary" part of the solution (P.S.F.) ; it is denoted by $(\sigma_r^1, \sigma_\theta^1, \sigma_{r\theta}^1)$.
- the second contribution is obtained by cancelling the temperature terms in the single-valuedness conditions while retaining the full compatibility equation : the pertaining stress field is termed the "pseudo-transient" part of the solution (P.T.F.) and denoted by

$(\sigma_r'', \sigma_\theta'', \sigma_{r\theta}'')$; it is mainly due to the "thermal shock" aspect of the phenomenon.

The single-valuedness conditions only involve the harmonics of order zero and one ; we are thus led to investigate the three cases $p = 0$, $p = 1$ and $p > 1$ separately. In all three cases, we start by solving equation (3) which yields σ_θ in terms of σ_r . Afterwards the stress $\sigma_{r\theta}$ obtained from the first equation of system (2) is substituted in the second equation of this system, which results in a single second-order differential equation for the stress σ_r only.

When $p = 0$, we get :

$$\begin{cases} \sigma_r^{(0)} = A_0' \text{Log } r + B_0' + \frac{C_0'}{r^2} ; \sigma_\theta^{(0)} = A_0' \text{Log } r + B_0' - \sigma_r^{(0)} ; \sigma_{r\theta}^{(0)} = 0 . \\ \sigma_r'' = A_0'' \text{Log } r + B_0'' + \frac{C_0''}{r^2} - \frac{E\alpha}{r^2} \int_{R_a}^r \frac{T^{(0)}}{1-\nu} r \, dr ; \\ \sigma_\theta'' = A_0'' \text{Log } r + B_0'' - \frac{E\alpha T^{(0)}}{1-\nu} - \sigma_r'' ; \sigma_{r\theta}'' = 0 . \end{cases}$$

For each field the three integration constants are determined from the two boundary conditions involving σ_r and the single-valuedness condition.

When $p = 1$, we get :

$$\begin{cases} \sigma_r^{(1)} = \frac{A_1'}{4} r + \frac{C_1'}{r} + \frac{D_1'}{r^3} ; \sigma_\theta^{(1)} = A_1' r + \frac{B_1'}{r} - \sigma_r^{(1)} ; \\ \sigma_{r\theta}^{(1)} = \sigma_\theta'^{(1)} - \sigma_r'^{(1)} - r \frac{d\sigma_r'^{(1)}}{dr} . \\ \sigma_r'' = \frac{1}{r} \left[\int_{R_a}^r \frac{b}{2} r^2 \, dr + C_1'' \right] + \frac{1}{r^3} \left[- \int_{R_a}^r \frac{b}{2} r^4 \, dr + D_1'' \right] \\ \sigma_\theta'' = A_1'' r + \frac{B_1''}{r} - \sigma_r'' - \frac{E\alpha T^{(1)}}{1-\nu} ; \sigma_{r\theta}'' = \sigma_\theta'' - \sigma_r'' - r \frac{d\sigma_r''}{dr} . \end{cases}$$

with $b = \frac{2A_1''}{r} - \frac{E\alpha T^{(1)}}{(1-\nu)r^2} - \frac{E\alpha}{(1-\nu)r} \frac{dT^{(1)}}{dr}$.

For each field the four integration constants are determined from the four boundary conditions (which lead to three independent relations only) and the single-valuedness condition.

When $p > 1$, we get :

The PSF is zero

$$\begin{cases} \sigma_r^{(p)} = r^{p-2} \left[\int_{R_a}^r \frac{b}{2p} \frac{dr}{r^{p-3}} + C_p'' \right] + \frac{1}{r^{p+2}} \left[- \int_{R_a}^r \frac{b}{2p} r^{p+3} \, dr + D_p'' \right] ; \\ \sigma_\theta^{(p)} = A_p'' r^p + \frac{B_p''}{r^p} - \sigma_r^{(p)} - \frac{E\alpha T^{(p)}}{1-\nu} ; \sigma_{r\theta}^{(p)} = \frac{\sigma_\theta''^{(p)} - \sigma_r''^{(p)}}{p} - \frac{r}{p} \frac{d\sigma_r''^{(p)}}{dr} . \end{cases}$$

with $b = A_p'' r^{p-2} (2+p-p^2) + \frac{B_p''}{r^{p+2}} (2-p-p^2) - \frac{E\alpha T^{(p)} (2-p^2)}{r^2 (1-\nu)} - \frac{E\alpha}{r(1-\nu)} \frac{dT^{(p)}}{dr}$

The four constants are determined with the help of the four boundary conditions involving σ_r and $\sigma_{r\theta}$.

The total stress field is then obtained as the sum of the PSF and the PTF for each harmonic. Finally the Fourier series can be broken off after the eight first terms.

As regards the axial stress σ_x , distinction has to be made between four problems :

- plane strain ($\epsilon_x = 0$) - We have : $\sigma_x = \nu (\sigma_r + \sigma_\theta) - \alpha ET$ with $\sigma_x(r, \theta) = \sum_{p=0}^{10} \sigma_x^{(p)}(r) \cos p\theta$
 - case where the tube is free to expand along its axis while remaining straight. Let R be the normal stress resultant on the cross-section for the previous problem : $R = 2\pi \int_{R_a}^{R_b} \sigma_x^{(0)} r dr$.
 One just has to add the tensile loading of intensity $\sigma_x = -R/S$, where S is the cross-section area.

- case where the tube is axially constrained but free to bend. Let M be the moment of normal stresses over the cross-section : $M = -\pi \int_{R_a}^{R_b} \sigma_x^{(1)} r^2 dr$. One has to add the normal stress distribution : $\sigma_x = +\frac{Mr}{I} \cos \theta$ where I is the moment of inertia of the section with respect to the centroidal axis.

- case where the tube is free to expand along its axis and to bend : one has to add the normal stress distribution $\sigma_x = -R/S + Mr \cos \theta / I$.

Now we are in a position to calculate the equivalent stress σ^* according to Mises' criterion in each of the above cases :

$$\sigma^* = \sqrt{(\sigma_r + \sigma_\theta + \sigma_x)^2 - 3(\sigma_r \sigma_\theta + \sigma_\theta \sigma_x + \sigma_x \sigma_r - \sigma_r \sigma_\theta^2)}$$

4 Results

4-1 Qualitative Properties :

We may display some characteristic features of the two PTF and PSF components :

- when the thermal steady state is achieved, one has $\Delta T = 0$ which means that the PTF is zero. Thus the PTF starts from zero and tends to zero after a sufficient time.
- the PSF only depends on the zero and first order harmonics of the temperature, as far as the stresses σ_r , σ_θ and $\sigma_{r\theta}$ only are concerned (which is a classical result ; see [7]).
- at each instant the PSF represents the stress field which would be attained if the values of temperature and temperature gradient on the inner boundary ceased to vary with time.
- during working incidents, the PTF needs looking after, as it might lead to stress values higher than those obtained in the steady state.

4-2 Numerical results

At each instant, at each node and for any value of the angular variable, the computer program gives non-dimensional values of σ_r , σ_θ , $\sigma_{r\theta}$, σ_x , PTF, PSF and the equivalent stress as functions of two non-dimensional parameters :

$$B = \frac{hR}{k} \quad \text{and} \quad \frac{R_b}{R_a}$$

The example considered involves a thick steel tube whose geometrical, thermal and mechanical properties are as follows : internal radii $R = 10$ mm ; external radius $R = 15$ mm ; specific heat $C = 0,55$ J/g/°C ; mass density $\rho = 0,0078$ g/mm³ ; heat transfer coefficient $h = 0,00225$ W/mm²/°C ; thermal conductivity $k = 0,025$ W/mm/°C ; Poisson's ratio $\nu = 0,3$; Young's modulus $E = 190$ GPa ; coefficient of thermal expansion $\alpha = 1,8 \times 10^{-5}$ °C⁻¹ ; maximum flux $\phi_0 = 0,81$ W/mm². For the considered steel, the elastic limit is close to 135 MPa at a temperature of about 500°C.

Values of temperature, stresses and equivalent stress have been plotted against time in figures 1 to 5 and the equivalent stress distribution through the tube thickness at several instants is given in figure 6: the tube is supposed free to expand along its axis and to bend.

It can be noted that the tangential stress is very sensitive to transient temperature distribution, which can lead to plastic deformation on the outer tube surface at ($\theta = 0$)

(Fig 1, 2). Figure 6 also shows that plastic deformation occurs at the early stage of heating.

5 Conclusion

It can be seen that transient temperature distributions do not induce a very marked maximum of stress values, which shows that the steady state plays a prevailing role. Moreover for a thin tube, the radial stress σ_r and the tangential stress $\sigma_{r\theta}$ are indeed small compared with other stress components, whereas σ_θ is not negligible, which suggests that a one-dimensional analysis would lead to inaccurate results whereas a shell theory seems to be well adapted.

The method presented can be applied to many similar problems : steam generators, heat exchangers, nuclear reactors... We hope it can readily be extended to a creep analysis, by making suitable modifications in the compatibility equation and single-valuedness conditions.

References

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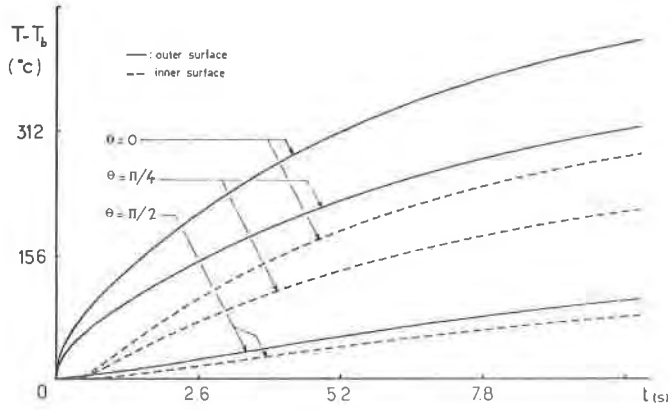


FIG 1 : VARIATION OF TEMPERATURE WITH TIME

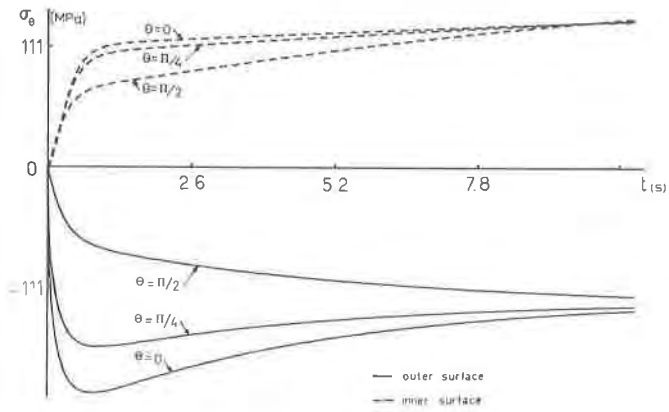


FIG 2 : VARIATION OF TANGENTIAL STRESS WITH TIME

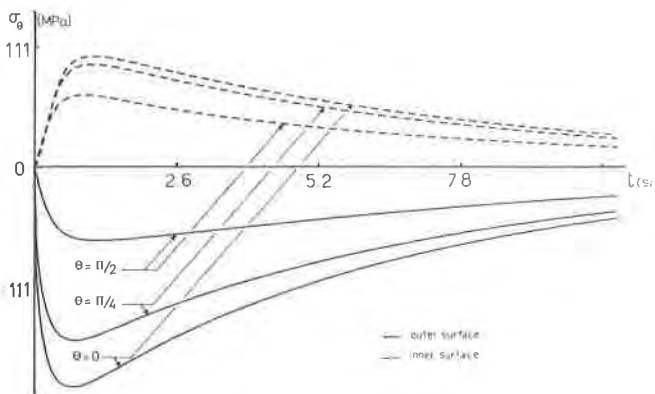


FIG 3 : VARIATION OF PSEUDO TRANSIENT PART OF TANGENTIAL STRESS WITH TIME

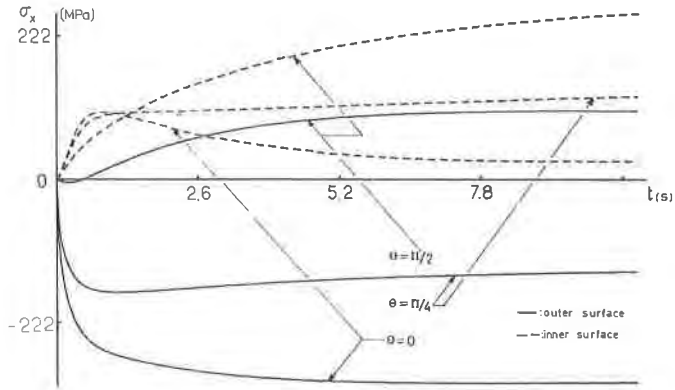


FIG 4 : VARIATION OF AXIAL STRESS WITH TIME

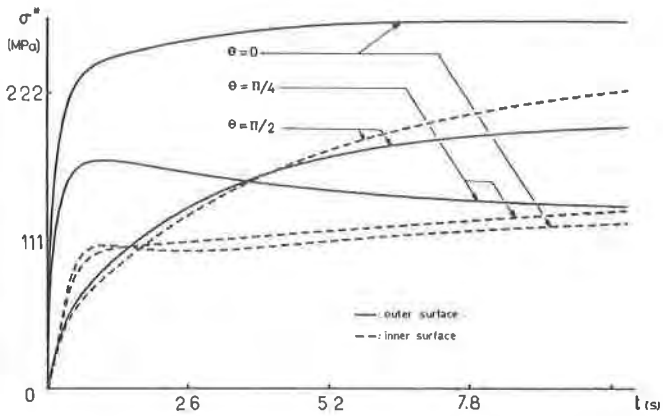


FIG 5 : VARIATION OF MISES EQUIVALENT STRESS WITH TIME

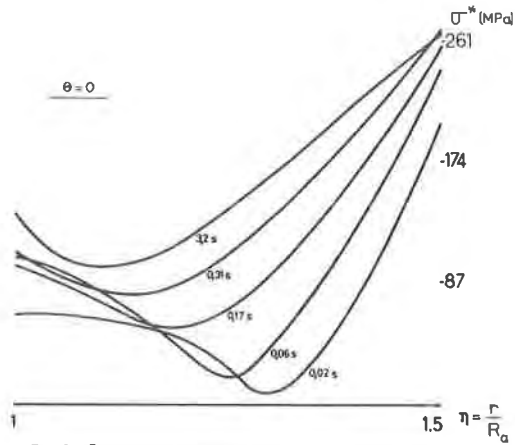


FIG 6 : EQUIVALENT STRESS DISTRIBUTION THROUGH WALL THICKNESS AT VARIOUS TIMES