The Computer Design of Piping Systems Using the Force Method

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Abstract

A computer programme for static and dynamic linear analyses of piping structures was developed. The reasons for the choice of the force method are discussed, particularly the types of supports and connections commonly used in piping structures and the advantages of this method in dynamic studies. Algorithms were developed to deal with some particular problems, such as inclined supports and eigenvalue determinations. The programme is described with flow charts for static and dynamic analyses.

1. Introduction

Piping systems are complex structures with a high degree of redundancy. Both static and dynamic linear analyses are commonly available in several computer programmes. The dynamic analysis is required not only for seismic studies, but also to assess the influence of other disturbing forces such as wind effects or rotating equipment connected to the structure.

The dynamic analysis consists of the determination of eignfrequencies and vibration eigenmodes. The latter are always superimposed and form a global displacement vector. This paper describes the basis for a computer programme developed for static and dynamic linear analyses of piping systems, using the force method previously described by the authors. [1]

2. Static Analysis

2.1 The displacement method.

The solution consists of the nodal displacement vector, measured at selected points - or nodes - of the structure:

$$[K] \cdot \underset{\sim}{\delta} = \underset{\sim}{F} \qquad \text{eq. (1)}$$

where $[K]$ is the global stiffness matrix, $\underset{\sim}{\delta}$ is the nodal displacement vector, and $\underset{\sim}{F}$ is the force vector, so that uniformly distributed forces must be replaced by their equivalent nodal force system.

The displacement method is extensively used and well-suited to computer programming. However, the stress calculations for piping systems using the displacement method depend on a previous $\underset{\sim}{\delta}$ calculation, and some dynamic models are better suited to the force method. Thus, we note the interest in developing a computer programme which would use the force method.

2.2 The force method.

This is dual of the displacement method. The unknowns consist of substitution forces on redundant connections which were suppressed, thereby transforming the structure into a statically-determined system. The elastic behaviour of the statically-equivalent system loaded by known forces and unknown substitution forces is identical to the original redundant structure. The number of excessive connections is the degree of redundancy of the structure. The solution vector consists of the substitution forces on hangers, supports, and other connection systems. The general equation for this method is:

$$[C]\cdot\underset{\sim}{F} = \underset{\sim}{\delta} \qquad \text{eq. (2)}$$

where $[C]$ is the flexibility matrix for the modified structure, $\underset{\sim}{F}$ is the unknown constraint forces, and $\underset{\sim}{\delta}$ is the displacement vector of the modified connections when the statically-equivalent system is subjected to external actions such as thermal effects, discreet or continuous forces, or support distortions. Many statically-equivalent systems are possible, all leading to a correct solution. However, for the pipework presented in Fig. 1a, the system in Fig. 1b could symplify the problem, using an automatic computer routine. The "tree-shaped" structure in Fig. 1b is always possible in piping systems, since at least one fully-restrained connection, such as a rigid flange, is always available.

A reference system must also be considered. If some boundary conditions are not parallel to the reference axis for the structure, such as the skew support in Fig. 2, it is necessary to introduce suitable coordinated transformation matrixes. Considering Fig. 2, matrix $[C]$ is obtained on the global axis system, where skew supports are considered as fully-restrained joints. The same procedure is applied to vector $\underset{\sim}{\delta}$ of Equation 2.

To study the skew supports effect' consider the global rotation matrix $[T]$:

$$\underset{\sim}{F}' = [T]\cdot\underset{\sim}{F} \qquad \text{eq. (3)}$$

where $\underset{\sim}{F}$ describes the unknowns based on the global axis system, and $\underset{\sim}{F}'$ is a "hybrid" vector containing unknowns referred to the global axis system and unknowns referred to the local reference system for skew supports. $[T]$ therefore is a banded matrix that can be partitioned in rotation submatrixes, each referred to a redundant support. The rotation submatrix is the identity matrix when the support is parallel to the global axis.

Pre-multiplying Equation 3 by $[T]^T$:

$$\underset{\sim}{F} = [T]^T\cdot\underset{\sim}{F}' \qquad \text{eq. (3a)}$$

Furthermore,

$$\underset{\sim}{\delta}' = [T]\cdot\underset{\sim}{\delta} \qquad \text{eq. (4)}$$

$$\underset{\sim}{\delta} = [T]^T\cdot\underset{\sim}{\delta}' \qquad \text{eq. (4a)}$$

where $\underset{\sim}{\delta}'$ is a displacement vector with elements referred to global coordinates, if the section or support is geometrically ortogonal, or having elements referred to locally-defined coordinates, if skew supports are involved. Supposing that $[C]$ was defined on the global coordinate system, from Equations 3, 4 and 2, we obtain the general matrix equation for the force method:

$$[T][C][T]^T\cdot\underset{\sim}{F}' = [T]\cdot\underset{\sim}{\delta} \qquad \text{eq. (5)}$$

2.3 A force-method based computer programme.

As discussed above, a computer programme based on the force method requires more programming work than a displacement-method based programme. The reason lies in the suppression of all redundant constraints, giving the "tree" a statically-determined structure. The number of unknowns depends on the number of suppressed constraints. Obviously, structures having a large number of constraints will be better suited for a displacement-method based analysis. Piping systems, however, usually have supports and hangers where almost all possible constraints are missing, thus making the force method an alternative worth considering for these particular structures.

Fig. 3 shows the computer programme flow chart we developed. Equation 5 is solved by the Gauss-Jordan technique [2] and requires little computation time when using computers such as the WANG VS80. Smaller computers restrict the complexity of the structure under analysis, but this can be overcome by using a Gauss-Seidel iterative solution [2] at the cost of more lengthy computation.

3. Dynamic Analysis

3.1 Dynamic models.

Matrix algorithms were developed for dynamic structural analysis. Dynamic equilibrium equations start with Equation 2: $[C]\cdot\underset{\sim}{F} = \underset{\sim}{\delta}$

generalized to dynamic studies as follows:

$$[C](\underset{\sim}{F_e} + \underset{\sim}{F_I}) = \underset{\sim}{\delta} \qquad \text{eq. (6)}$$

where $\underset{\sim}{F_e}$ represents the external force vector, $\underset{\sim}{F_I}$ is the inertia forces vector, and $\underset{\sim}{\delta}$ is the displacement vector. The inertia force vector $\underset{\sim}{F_I}$ arises from D'Alembert's theorem:

$$\underset{\sim}{F_I} = -[M]\cdot\ddot{\underset{\sim}{\delta}} \qquad \text{eq. (6a)}$$

therefore, Equation 6 may be written as follows:

$$[C]\cdot(\underset{\sim}{F_e} - [M]\ddot{\underset{\sim}{\delta}}) = \underset{\sim}{\delta} \qquad \text{eq. (7)}$$

We especially note that matrix [C] in Equation 6 is not the same as in Equation 2, for the static solution, as the degrees of freedom for dynamic study depend on the model adopted, no longer being redundant constraints. The accuracy of the dynamic solution depends on the definition of the dynamic degrees of freedom.

The mass matrix [M] depends on the dynamic model adopted, and this in turn depends on mass distribution criteria. We may assume that the uniformly distributed mass of the elements is concentrated in selected points of the structure, keeping the total mass constant.

Two mass discretization models are possible. Consider, for example, the frame in Fig. 4a, where each element has mass m per unit of length. If we set the geometric connections as dynamic nodal points (Fig. 4b), it is then possible to concentrate on each node, half of the total mass of each element connected to that node. Thus, node 1 will have a concentrated mass of mH/2 from half of the length of column 1; node 2 will have a concentrated mass of mH/2 from the column and mL/2 from the horizontal beam. As nodes 1 and 4 are fully constrained, the mass matrix for the Fig. 4b model is:

$$[M] = \begin{bmatrix} m\frac{H+L}{2} & & & & \\ & m\frac{H+L}{2} & & & \\ & & I_2 & & \\ & & & m\frac{H+L}{2} & \\ & & & & m\frac{H+L}{2} \\ & & & & & I_3 \end{bmatrix} \qquad (8)$$

where I_2 and I_3 are inertia moments of concentrated mass m_2 and m_3. Matrix 8 is diagonal as there is no inertial interaction between concentrated masses. The model could be simplified by supposing that lumped masses have zero-inertia moments, zero elements figuring in the leading diagonal of Equation 8. This would not be advisable for the solution of Equation 6, where "static condensation of mass" [3] should be performed in the first place.

The Fig. 4b dynamic model is preferable for displacement-method analysis, as mass-active nodes match on geometrically nodal points of the structure.

Another discreet mass model is presented in Fig. 4c, where mass is concentrated on the centroid of each element of the structure. Assuming three degrees of freedom, and not taking into account inertia moments:

$$[M] = \begin{bmatrix} mH & & & \\ & mH & & \\ & & mL & \\ & & & mL \\ & & & & mH \\ & & & & & mH \end{bmatrix} \qquad (9)$$

The partitions in matrices (8) and (9) correspond to influence areas of each mass-loaded node which is free to move and rotate in directions x, y and z. (x, y for 'plane' problems)

Mass matrix (9) can be used in Equation (7) for the force method as no static condensation of mass is required; in [M] and [C], lines and columns are deleted at the position of an inertia movement.

3.2 Eigenfrequencies and eigenmodes.

From Equation 7, with $\underset{\sim}{F_e}=0$:

$$-[C]\cdot[M]\,\underset{\sim}{\ddot{\delta}}_I = \underset{\sim}{\delta}_I \qquad \text{eq.} \quad (10)$$

Assuming that $\underset{\sim}{\delta}_I$ is a vector resulting from successive mode-superimposed vectors, each of the harmonic type, we get:

$$[C]\cdot\left(\omega^2[M]\,\underset{\sim}{\delta}_I\right) = \underset{\sim}{\delta}_I \qquad \text{eq.} \quad (11)$$

and the relevant solutions are obtained for:

$$\text{Det.}\left|[C][M] - \frac{1}{\omega^2}[I]\right| = 0 \qquad \text{eq.} \quad (12)$$

Solutions of (12) are eigenvalues of the problem, and each eigenvalue is associated to one eigenvector or vibration eigenmode.

3.3 Computer programme for eigenfrequency and eigenmode calculations.

In the eigenequation:

$$[A]\,\underset{\sim}{\phi} = \lambda\,\underset{\sim}{\phi} \qquad \text{eq.} \quad (13)$$

A may be identified with stiffness (or flexibility) matrixes when dealing with dynamic problems.

The main problem arising from Equations 11 or 12 is, which computer programme will define [C]? The procedure can be summarized as follows: Considering all centroids of the structural elements, a statically-equivalent problem is solved for each inertially unit-loaded centroid. This procedure gives the consequent redundant forces in all excessive constraints. Later, effects loading all eliminated redundancies are superimposed, together with the calculated substitution force vector, and the real displacements in all centroids are determined.

It is important to note that iterative routines for eigensolutions such as the Jacobi, or subspace iteration, [3,4] deal with symmetric matrixes, thus implying that Equation 12 must be modified as $[C]\cdot[M]$ is not necessarily symmetric. Equation 12, rewritten

$$[C][M]^{\frac{1}{2}}[M]^{\frac{1}{2}}\underset{\sim}{\delta} - \frac{1}{\omega^2}[I]\underset{\sim}{\delta} = 0 \qquad \text{eq. (14)}$$

gives us

$$([M]^{\frac{1}{2}}[C][M]^{\frac{1}{2}})([M]^{\frac{1}{2}}\underset{\sim}{\delta}) = \frac{1}{\omega^2}([M]^{\frac{1}{2}}\underset{\sim}{\delta}) \qquad \text{eq. (15)}$$

ready for a Jacobi or subspace iteration technique. Fig. 5 shows the flow chart which describes this programme.

4. Conclusions.

Accurate engineering results are obtained with the computer programme developed for the force-method dynamic linear analysis of piping structures.

5. References

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[2] DORN, W.S., McCRACKEN, D.D., Numerical Methods with FORTRAN IV Case Studies, John Wiley & Sons, 1972.

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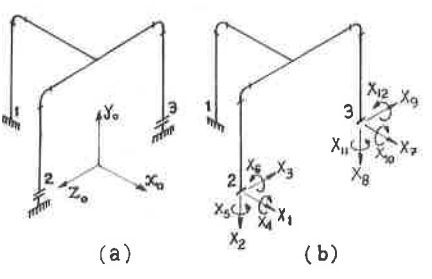

Fig. 1

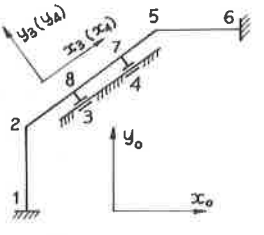

Fig. 2

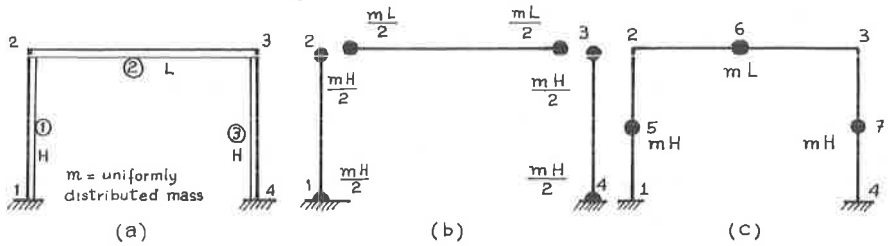

Fig. 4

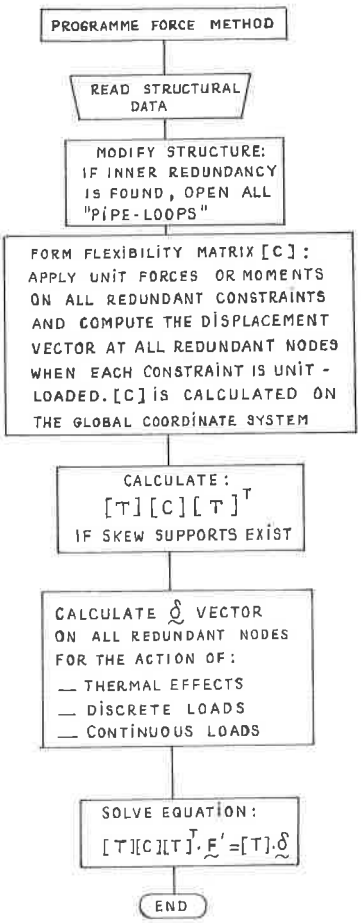

Fig. 3

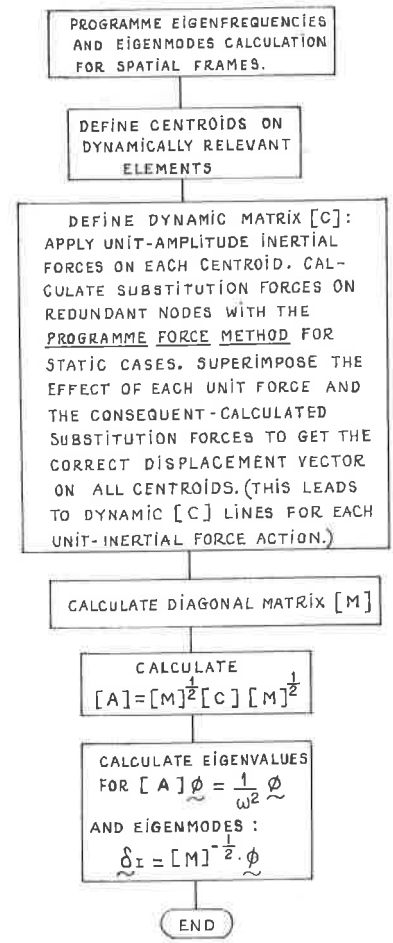

Fig. 5