A New Stochastic FEM-Based Reliability Assessment of BWR Mark-II Type Reactor Building

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Abstract

The successive perturbation method is newly developed both for static and eigenvalue finite element analyses. The method copes with any magnitude of change of parameters and approximates the structural behavior in good accuracy based on the sole solution of governing equation. When combined with an iterative algorithm, the successive perturbation method enables us to evaluate the reliability index of Advanced First-Order Second-Moment method efficiently in the framework of finite element method. Reliability assessment of Mark-II type reactor building with fourteen random variables is exemplified to show the utility of the proposed method.

1. Introduction

Analysis and assessment are two legs needed for structural safety and reliability to stand up. Finite element method has a forte for numerical analysis of structure with various parameters and many degrees of freedom included in the modeling. Reliability indices have been groped hitherto in cases that uncertainties involved in structural system are of small number and can be conjoined theoretically with behavior of structure.

To make full use of the forte of numerical analysis and theoretical assessment, a bridge seems to be needed between the finite element method and reliability index analysis. Any structure is designed so that failure is not likely to occur, that is to say, the failure surface lies far away from the expected point. This calls for the finite element method which is able to cope with variation of parameters over wide range.

Suppose a static SDOF system of Eq. (1) for the convenience of discussion.

$$k \cdot u = k^0(1+\alpha) \cdot u = f \tag{1}$$

where k^0 is the expected stiffness and α is a non-dimensional uncertain parameter with zero expectation. The mean-centered perturbation solutions for this problem are inaccurate for the wide range of α as depicted by broken lines in Fig.1. In order to improve the accuracy, we now introduce the "successive perturbation method" as described below. This notion was suggested by Der Kiureghian [1] and emphasis must be put on the fact that the advantage of stochastic FEM is still preserved, that is to say, the governing equation need not be solved repeatedly. Displacement u at $\alpha+\Delta\alpha$ may be written as

$$u_{\alpha+\Delta\alpha} = k^{-1}_{\alpha+\Delta\alpha} \cdot f \tag{2}$$

where the inverse of stiffness at $\alpha+\Delta\alpha$ is approximated by

$$k^{-1}_{\alpha+\Delta\alpha} = k^{-1}_{\alpha} + \left.\frac{dk^{-1}}{d\alpha}\right|_{\alpha} \cdot \Delta\alpha = k^{-1}_{\alpha} - k^{-1}_{\alpha} \left.\frac{dk}{d\alpha}\right|_{\alpha} k^{-1}_{\alpha} \cdot \Delta\alpha \tag{3}$$

The relation $dk^{-1}/d\alpha = -k^{-1} \cdot dk/d\alpha \cdot k^{-1}$ is obtained by the differentiation of $kk^{-1}=1$ with α. Once k^{-1} is computed for an arbitrary starting point α_0(e.g.$\alpha_0=0$),k^{-1} for any α and therefore u_α can be approximated by repeating the above sequence with suitable step width $\Delta\alpha$. The successive perturbation solutions for the present problem are also depicted by solid lines in Fig. 1, α_0 being set zero. As seen in the figure, the successive perturbation solution with $\Delta\alpha=0.1$ approximates the exact solution fairly well and almost exactly with $\Delta\alpha=0.02$.

In Section 2 of the present paper, the general formulae of successive perturbation method are constructed based on this notion both for static and eigenvalue problems. The formula for eigenvalue problem is then applied to the evaluation of reliability index β in Section 3. Reliability assessment of Mark-II type reactor building is exemplified to show the utility of the proposed method.

2. GENERAL FORMULAE OF SUCCESSIVE PERTURBATION METHOD

Static Problem

By extending the above case of SDOF system, the successive first order perturbation formula is easily given as follows.

$$U_{\alpha+\Delta\alpha}=K^{-1}_{\alpha+\Delta\alpha}F \tag{4}$$

$$K^{-1}_{\alpha+\Delta\alpha}=K^{-1}_{\alpha}+\sum_k \frac{\partial K^{-1}}{\partial \alpha_k}\bigg|_{\alpha}\cdot\Delta\alpha_k=K^{-1}_{\alpha}-\sum_k K^{-1}_{\alpha}\frac{\partial K}{\partial \alpha_k}\bigg|_{\alpha}K^{-1}_{\alpha}\cdot\Delta\alpha_k \tag{5}$$

where α represents the random variables $\{\alpha_1,\alpha_2,\cdots,\alpha_m\}$ involved in the system. The successive second order perturbation formula is constructed similarly by adding

$$\frac{1}{2}\sum_k\sum_l \frac{\partial^2 K^{-1}}{\partial\alpha_k\partial\alpha_l}\bigg|_{\alpha}\cdot\Delta\alpha_k\Delta\alpha_l=-\frac{1}{2}\sum_k\sum_l K^{-1}_{\alpha}\left(\frac{\partial^2 K}{\partial\alpha_k\partial\alpha_l}\bigg|_{\alpha}K^{-1}_{\alpha}+\frac{\partial K^{-1}}{\partial\alpha_k}\bigg|_{\alpha}\frac{\partial K}{\partial\alpha_l}\bigg|_{\alpha}+\frac{\partial K}{\partial\alpha_k}\bigg|_{\alpha}\frac{\partial K^{-1}}{\partial\alpha_l}\bigg|_{\alpha}\right)\cdot\Delta\alpha_k\Delta\alpha_l \tag{6}$$

to the right side of Eq. (5).

Eigenvalue problem

Suppose the eigenvalue λ and eigenvector ϕ of N-DOF eigenvalue problem

$$(K-\lambda M)\phi=0 \tag{7}$$

are known at a certain α value. Then λ and ϕ at $\alpha+\Delta\alpha$ are approximated as follows.

$$\lambda_{\alpha+\Delta\alpha}=\lambda_{\alpha}+\sum_k\frac{\partial\lambda}{\partial\alpha_k}\bigg|_{\alpha}\cdot\Delta\alpha_k\equiv\lambda_{\alpha}+\sum_k\lambda^{I}_{k\alpha}\cdot\Delta\alpha_k \tag{8}$$

$$\phi_{\alpha+\Delta\alpha}=\phi_{\alpha}+\sum_k\frac{\partial\phi}{\partial\alpha_k}\bigg|_{\alpha}\cdot\Delta\alpha_k\equiv\phi_{\alpha}+\sum_k\phi^{I}_{k\alpha}\cdot\Delta\alpha_k \tag{9}$$

As is well known[2], $\lambda^{I}_{k\alpha}$ in Eq. (8) is evaluated by

$$\lambda^{I}_{k\alpha}=\{\phi^{T}_{\alpha}(K^{I}_{k\alpha}-\lambda_{\alpha}M^{I}_{k\alpha})\phi_{\alpha}\}/\phi^{T}_{\alpha}M\phi_{\alpha} \tag{10}$$

where $K^{I}_{k\alpha}=\partial K/\partial\alpha_k|_{\alpha}$ and $M^{I}_{k\alpha}=\partial M/\partial\alpha_k|_{\alpha}$. On the other hand, some methods are known [2,3,4] as regards the evaluation of $\phi^{I}_{k\alpha}$. The simplest way [4] is employed here, that is to say, a suitable component of $\phi^{I}_{k\alpha}$ is set zero and the other components denoted by $\tilde{\phi}^{I}_{k\alpha}$ are solved as follows.

$$\tilde{\phi}^{I}_{k\alpha}=-(\tilde{K}_{\alpha}-\lambda_{\alpha}\tilde{M}_{\alpha})^{-1}(\hat{K}^{I}_{k\alpha}-\lambda^{I}_{k\alpha}\hat{M}_{\alpha}-\lambda_{\alpha}\hat{M}^{I}_{k\alpha})\phi_{\alpha} \tag{11}$$

In the right side of the above equation, $\sim$ and $\wedge$ mean that the sizes of matrix are $(N-1)\times(N-1)$ and $(N-1)\times N$ respectively. Denoting $\tilde{K}_{\alpha}-\lambda_{\alpha}\tilde{M}_{\alpha}$ by X_{α}, we can approximate X^{-1}_{α} based on $X^{-1}_{\alpha-\Delta\alpha}$ following the static case as

$$X^{-1}_{\alpha}=X^{-1}_{\alpha-\Delta\alpha}+\sum_k\frac{\partial X^{-1}}{\partial\alpha_k}\bigg|_{\alpha-\Delta\alpha}\cdot\Delta\alpha_k \tag{12}$$

where $\partial X^{-1}/\partial\alpha_k|_{\alpha-\Delta\alpha}$ is derived as

$$\frac{\partial X^{-1}}{\partial\alpha_k}\bigg|_{\alpha-\Delta\alpha}=-X^{-1}_{\alpha-\Delta\alpha}(\tilde{K}^{I}_{k\alpha-\Delta\alpha}-\lambda^{I}_{k\alpha-\Delta\alpha}\tilde{M}_{\alpha-\Delta\alpha}-\lambda_{\alpha-\Delta\alpha}\tilde{M}^{I}_{k\alpha-\Delta\alpha})X^{-1}_{\alpha-\Delta\alpha}\quad. \tag{13}$$

Because $\lambda_{\alpha-\Delta\alpha}$ and $\lambda^{I}_{k\alpha-\Delta\alpha}$ are already known at the preceding step, X^{-1}_{α} and therefore $\tilde{\phi}^{I}_{k\alpha}$ are approximated based on $X^{-1}_{\alpha-\Delta\alpha}$ without the literal execution of Eq.(11). The successive second order perturbation formula for the eigenvalue is given by adding

$$\frac{1}{2}\sum_k\sum_l\frac{\partial^2\lambda}{\partial\alpha_k\partial\alpha_l}\bigg|_{\alpha}\cdot\Delta\alpha_k\Delta\alpha_l\equiv\frac{1}{2}\sum_k\sum_l\lambda^{II}_{kl\alpha}\cdot\Delta\alpha_k\Delta\alpha_l \tag{14}$$

to the right side of Eqs. (8), $\lambda^{II}_{kl\alpha}$ being evaluated as follows.

$$\lambda^{II}_{kl\alpha}=\phi^{T}_{\alpha}\{(K^{I}_{k\alpha}-\lambda^{I}_{k\alpha}M_{\alpha}-\lambda_{\alpha}M^{I}_{k\alpha})\phi^{I}_{l\alpha}+(K^{I}_{l\alpha}-\lambda^{I}_{l\alpha}M_{\alpha}-\lambda_{\alpha}M^{I}_{l\alpha})\phi^{I}_{k\alpha}+(K^{II}_{kl\alpha}-\lambda^{I}_{k\alpha}M^{I}_{l\alpha}-\lambda^{I}_{l\alpha}M^{I}_{k\alpha}-\lambda_{\alpha}M^{II}_{kl\alpha})\phi_{\alpha}\}/\phi^{T}_{\alpha}M_{\alpha}\phi_{\alpha} \tag{15}$$

In the right side of the above equation, $K^{II}_{kl\alpha}$ and $M^{II}_{kl\alpha}$ stand for $\partial^2K/\partial\alpha_k\partial\alpha_l|_{\alpha}$ and $\partial^2M/\partial\alpha_k\partial\alpha_l|_{\alpha}$, and the other components are already known.

3. Successive Perturbation Method Applied to Evaluation of Reliability Index of Mark-II type Reactor Building

It is shown in the preceding section that K^{-1} and $\partial K^{-1}/\partial\alpha_k$, or λ and $\partial\lambda/\partial\alpha_k$ are evaluated for arbitrary values of $\boldsymbol{\alpha}$ by solving the governing equation only once. This feature can be utilized when FEM-based structural reliability analysis is carried out on the line of Advanced First-Order Second-Moment (AFOSM) method. In other words, when the successive perturbation technique is combined effectively with the iterative algorithm to evaluate the reliability index β, then CPU time is saved much.

According to the AFOSM method, the reliability index β is defined as the minimum distance from the origin to the failure (limit state) surface where the performance function $G(Y)$ equals zero in the standardized space[5]. Y denotes reduced (standardized) random variables. When finite element method is concerned in the analysis, the performance function consists of K^{-1} in the static problem, and λ in the eigenvalue problem. It is therefore necessary in the search algorithm to repeat the computation of K^{-1} and $\partial K^{-1}/\partial\alpha_k$, or λ and $\partial\lambda/\partial\alpha_k$ so that we reach the design point on the failure surface. A simple search algorithm in conjunction with the successive first order perturbation method is summarized as follows.

[1] Expand the performance function $G(Y)$ at $Y^{(0)}=0$ to make the tangential hyperplane. Assuming this hyperplane as $G(Y)$, evaluate the corresponding design point $Y^{(1)}$ according to the following equation.

$$Y_i^{(n+1)}=\frac{-G(Y^{(n)})+\sum_i \partial G(Y)/\partial Y_i|_{Y=Y^{(n)}}\cdot Y_i^{(n)}}{\sum_i\{\partial G(Y)/\partial Y_i|_{Y=Y^{(n)}}\}^2}\cdot\frac{\partial G(Y)}{\partial Y_i}\Big|_{Y=Y^{(n)}} \tag{16}$$

The governing equation and the derivatives must be computed, and K^{-1} and $\partial K^{-1}/\partial Y_i$, or λ and $\partial\lambda/\partial Y_i$ are stored at this stage.

[2] Set $n=1$.

[3] Divide the vectorical distance from $Y^{(0)}=0$ to $Y^{(n)}$ into M pieces. Apply the successive perturbation technique with the step width $(Y^{(n)}-Y^{(0)})/M$ to approximate K^{-1} and $\partial K^{-1}/\partial Y_i$, or λ and $\partial\lambda/\partial Y_i$ at $Y^{(n)}$.

[4] Calculate $G(Y^{(n)})$ and $\partial G(Y)/\partial Y_i|_{Y=Y^{(n)}}$ based on the above result. Expand $G(Y)$ at $Y^{(n)}$ to make the new hyperplane and calculate the corresponding design point $Y^{(n+1)}$ using Eq. (16).

[5] Set $n=n+1$.

[6] Repeat step [3] to [5] until $Y^{(n)}$ and $\beta^{(n)}=\sqrt{Y^{(n)T}Y^{(n)}}$ converge, and confirm $G(Y^{(n)})\fallingdotseq 0$. In this sequence the governing equation is not solved. $\beta^{(n)}$ and $Y^{(n)}$ may be taken as target reliability index β and the design point Y^*.

In order to demonstrate the validity of the proposed successive perturbation method, the reliability assessment is carried out of eigenvalue problem of Mark-II type reactor building with the algorithm described above.

Figure 2 schematically shows finite element idealization of Mark-II type reactor building and foundation system, in which various uncertainties are involved inevitably. For instance, the rigidity of building storeys is often idealized by a beam element with the stiffness matrix including shear deformation correction as follows.

$$k_j^0=\frac{EI}{l^3(1+\phi)}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & (4+\phi)l^2 & -6l & (2-\phi)l^2 \\ & & 12 & -6l \\ \text{SYM.} & & & (4+\phi)l^2 \end{bmatrix} \tag{17}$$

$$\phi=12EI/GAl^2 \tag{17-a}$$

It is not easy, however, to determine equivalent sectional area A, second moment of inertia I, moduli E and G in the above stiffness matrix. The uncertainty thus brought about is associated with engineer's judgement rather than inherent variabilities of nature. The difficulty also arises from the determination of foundation stiffness K_θ, K_{SS} and K_v. It turns out that the response against earthquake of the building and the equipments settled in it is uncertain.

In the following numerical example, the uncertainties of stiffness as mentioned above are assumed in form of

$$\left.\begin{aligned} &k_j=k_j^0(1+\alpha_j):j=1\sim 11\\ &K_\theta=K_\theta^0(1+\alpha_{12})\\ &K_{SS}=K_{SS}^0(1+\alpha_{13})\\ &K_v=K_v^0(1+\alpha_{14})\end{aligned}\right\} \tag{18}$$

where $\{\alpha_1,\alpha_2,\cdots,\alpha_{14}\}\equiv\boldsymbol{\alpha}$ is random vector with zero-mean components. All the other parameters are given deterministically in this study. The equipments are usually designed so that its natural frequency exceeds that of building of first and/or second mode.

In this sense the performance function for the equipments may be given as

$$Z(\alpha)=(\omega_c^i)^2-(\omega^i(\alpha))^2 \tag{19}$$

where $\omega^i(\alpha)$ is the natural circular frequency of i-th mode of the building, and ω_c^i the limit state which is assumed as $\omega_c^i=1.15\omega^i(0)$ in the following computation. The above performance function is set for the equipments, not for the building.

Reliability analysis is then carried out based on the technique as stated above. The specifications of the model including the assumed statistics of α are summarized in Table 1. Figure 3 shows the convergence of the reliability index β versus the number of iterations n. The number of division M in the above algorithm is set eight. The convergence of β evaluated exactly (the eigenvalue problem is solved at every iteration step) is also shown in the figure. Table 2 summarizes the converged design point α^* in the basic-variable space together with Y^* in the standardized space and the reliability index β. Those as evaluated in the exact manner are also listed in parentheses. The accuracy of the successive perturbation method seems to be satisfactory from the practical point of view. It goes without saying that we have more accurate solution by setting larger M.

In the remainder of this section, discussion is made for the probability of failure P_f with the aid of some approximation techniques for failure surface. Namely, P_f can be estimated as follows when α is assumed as Gaussian random vector.

The simplest way to estimate P_f is the use of the tangential linearization of the failure surface at the design point. Under this hyperplane-approximation, P_f is easily given as follows with the standard normal distribution function Φ.

$$P_f=\Phi(-\beta)=\begin{cases}\Phi(-2.817)=2.43\times10^{-3}\ :\ i=1\\ \Phi(-3.175)=7.58\times10^{-4}\ :\ i=2\end{cases} \tag{20}$$

In an extreme case the failure surface is substituted by the hypersphere whose radius is β. This constitutes the upper bound for P_f which is conveniently evaluated as follows with the aid of chi-squared distribution [6].

$$P_f<1-\chi_m^2(\beta^2)=\begin{cases}1-\chi_{14}^2(7.935)=0.89\ :\ i=1\\ 1-\chi_{14}^2(10.08)=0.75\ :\ i=2\end{cases} \tag{21}$$

Since the conservativeness depends on the number of degree of freedom m, the formula yields unpractically high values in the present case of $m=14$.

The more sophisticated method has been developed by Fiessler et al., in which quadratic expansion of the failure surface is made at the design point [7]. The method meets the formulae developed in the preceding section, because second-order perturbation terms are considered there. The quadratic form of the failure surface is, therefore, derived as follows

$$10.173(\tilde{Y}_1-2.915)^2+5.374(\tilde{Y}_2-0.121)^2+3.515(\tilde{Y}_3-0.011)^2+2.545(\tilde{Y}_4-1.268)^2+2.093(\tilde{Y}_5+5.808)^2+1.381(\tilde{Y}_6+2.844)^2+1.019(\tilde{Y}_7+5.796)^2+0.992(\tilde{Y}_8+6.941)^2+0.651(\tilde{Y}_9+0.497)^2+0.524(\tilde{Y}_{10}-8.740)^2+0.128(\tilde{Y}_{11}-218.306)^2+0.014(\tilde{Y}_{12}+23.355)^2+0.002(\tilde{Y}_{13}-133.263)^2-0.006(\tilde{Y}_{14}+2.447)^2=6225.525 \quad :\ i=1$$

$$42.362(\tilde{Y}_1-2.749)^2+18.339(\tilde{Y}_2-1.068)^2+14.812(\tilde{Y}_3+0.705)^2+10.166(\tilde{Y}_4-1.787)^2+9.075(\tilde{Y}_5-7.477)^2+6.731(\tilde{Y}_6-1.859)^2+4.723(\tilde{Y}_7-2.788)^2+3.949(\tilde{Y}_8-24.710)^2+2.717(\tilde{Y}_9-0.736)^2+1.572(\tilde{Y}_{10}-2.544)^2+0.219(\tilde{Y}_{11}-63.562)^2+0.100(\tilde{Y}_{12}-333.876)^2+0.005(\tilde{Y}_{13}-13.167)^2-0.042(\tilde{Y}_{14}+0.465)^2=14635.065 \quad :\ i=2 \tag{22}$$

where the standardized random vector Y is again transformed to the new standardized vector $\tilde{Y}$ so that second-order cross terms vanish as shown above. Applying the non-central chi-squared distribution formula [8] to the above equations, we have the probabilities of failure as given below.

$$P_f=\begin{cases}1.24\times10^{-2}\ :\ i=1\\ 9.01\times10^{-3}\ :\ i=2\end{cases} \tag{23}$$

4. Conclusion

It seems to be impracticable to compute P_f for such problem with many random variables as stated in the preceding section. For instance, it will be hard to carry out (1) rigorous evaluation of the true failure surface in fourteen-dimensional space, and (2) fourteen-fold integral of the joint probability density function of the basic variables over the unsafe domain. Although Eqs. (20) and (23) yield different values of P_f, the magnitude of P_f might be considered small enough in spite of the severe limit state (ω_c^i is set only 15% larger than the design value $\omega^i(0)$) and rather large standard deviations assumed for α.

The successive perturbation method can be engaged well with FEM-based structural reliabilty analysis as shown in the present paper. Further case study should be done with various problems to cultivate the potential of the proposed method in practice.

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Table 1

(a) (Expected)Specifications of Finite Element Model

Length ℓ (m)	Mass number	Mass (10^3 Kgf)	Moment of inertia of rotation (10^9 Kgf·m²/rad.)	Sectional area A(m²)	Moment of inertia of cross-section I (10^4 m⁴)
	1	3500	1.0		
12.0				55.0	3.0
	2	4000	1.0		
8.0				70.0	4.0
	3	15000	3.0		
10.0				185.0	5.0
	4	30000	6.0		
8.0				210.0	8.0
	5	20000	4.0		
8.0				250.0	10.0
	6	25000	13.0		
6.0				560.0	40.0
	7	45000	25.0		
6.0				600.0	45.0
	8	35000	18.5		
6.0				630.0	50.0
	9	30000	16.5		
6.0				630.0	50.0
	10	0	0.0		
3.0				630.0	330.0
	11	125000	66.0		
3.0				630.0	330.0
	12	0	0.0		

Young's modulus $E=2.1\times10^9$ Kgf/m² Poisson's ratio $\nu=0.167$

Constants of Foundation	Natural Period (sec)
$K_\theta=5.0\times10^{13}$ Kgf·m/rad.	Mode
$K_{ss}=2.0\times10^{10}$ Kgf/m	1st 0.3048
$K_g=3.5\times10^{10}$ Kgf/m	2nd 0.1455
	3rd 0.0991

(b) Assumed Statistics for Random Variables α

Standard Deviations

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\sqrt{Var[\alpha_j]}$	0.05	0.05	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3

Correlation Coefficient Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	0.8	0	0	0	0	0	0	0	0	0	0	0	0
2		1	0.4	0	0	0	0	0	0	0	0	0	0	0
3			1	0.4	0	0	0	0	0	0	0	0	0	0
4				1	0.4	0	0	0	0	0	0	0	0	0
5					1	0.4	0	0	0	0	0	0	0	0
6						1	0.4	0	0	0	0	0	0	0
7							1	0.4	0	0	0	0	0	0
8								1	0.4	0	0	0	0	0
9									1	0.4	0	0	0	0
10										1	0.4	0	0	0
11											1	0	0	0
12												1	0.8	0.8
13													1	0.8
14	SYM.													1

Table 2 Design Point and Reliability Index Obtained by Successive First-Order Perturbation Method (Values in Parentheses: Exact Solution)

	j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
$i=1$	Y_j^*	0.014 (0.014)	0.031 (0.032)	0.299 (0.299)	0.650 (0.651)	0.818 (0.820)	0.552 (0.552)	0.630 (0.631)	0.801 (0.802)	0.861 (0.862)	0.449 (0.449)	0.197 (0.197)	1.482 (1.504)	1.071 (1.086)	1.042 (1.057)	β = 2.817 (2.842)
	α_j^*	0.001 (0.001)	0.005 (0.005)	0.083 (0.083)	0.169 (0.170)	0.204 (0.204)	0.161 (0.162)	0.171 (0.171)	0.213 (0 213)	0.211 (0.211)	0.128 (0.128)	0.054 (0.054)	0.618 (0.627)	0.563 (0.571)	0.559 (0.567)	
$i=2$	Y_j^*	0.152 (0.155)	0.230 (0.233)	0.991 (0.995)	1.090 (1.094)	0.958 (0.962)	0.418 (0.419)	0.441 (0.441)	0.686 (0.688)	0.820 (0.820)	0.530 (0.530)	0.395 (0.394)	0.951 (0.976)	1.400 (1.430)	1.449 (1.482)	β = 3.175 (3.215)
	α_j^*	0.011 (0.011)	0.028 (0.028)	0.238 (0.239)	0.288 (0.290)	0.241 (0.242)	0.133 (0.133)	0.124 (0.124)	0.182 (0.182)	0.205 (0.205)	0.150 (0.150)	0.096 (0.096)	0.570 (0.584)	0.631 (0.645)	0.637 (0.652)	

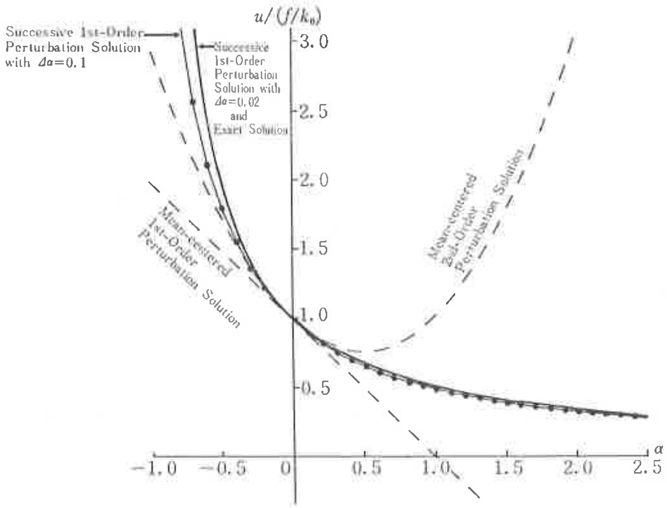

Fig.1 Successive Perturbation Solutions Compared with Mean-Centered Perturbation Solutions

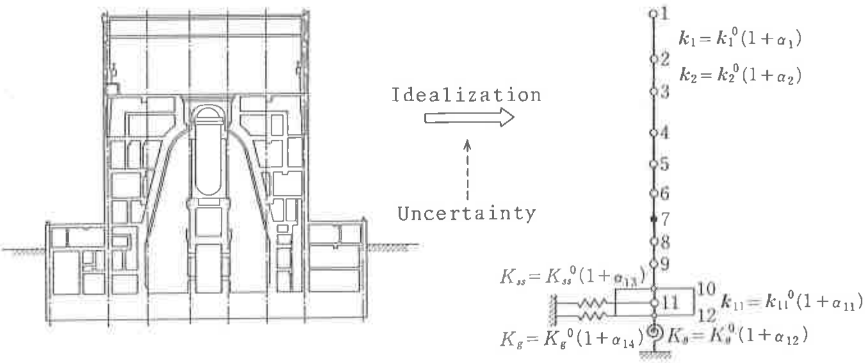

Fig.2 Finite Element Idealization of Mark-II Type Reactor Building and Foundation System

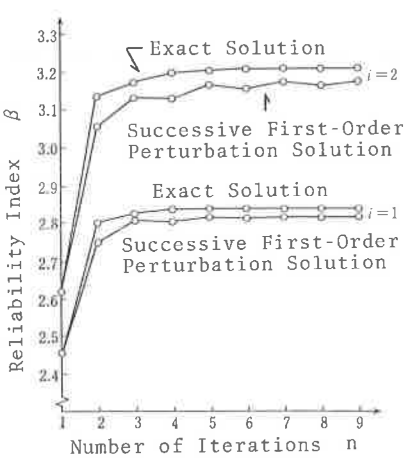

Fig.3 Convergence of Reliability Index β versus Number of Iterations n