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Estimating Probabilistic Distribution of Fatigue Strength for Practical Structure or Component

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ABSTRACT

In practical engineering, particularly in reliability–based design of fatigue, the probabilistic distribution of fatigue strength is very important. But this distribution could be impossibly obtained directly by the method of testing at an assiged fatigue lifetime, which can be calculated only by converting from P–S–N testing curve of fatigue life of structure or component. In this paper, author tries, only in need of small amount of testing of small specimens to obtain the related material constants, to estimate or predict the probabilistic distribution of fatigue strength for structure or component, in which the P–S–N testing curve of fatigue life of structure or component is not required. The paper gives the particulars of the proposed method and presents an example to show its application.

1 INTRODUCTION

In reliability–based design of fatigue, the stress–stength interference model is usually used, which requirs the precondition that the probabilistic distributions of both stress and strength are known. The former could be obtained directly by testing method, but for the latter, as stated in *American National Standard* (ANSI / ASTM E206–72): it is impossible to obtain the probabilistic distributon of fatigue strength at N cycles directly by testing method. Up to now the hypothese of *Normal* or *Weibull* distribution of fatigue strength is still used as the basis of reliability–based design of fatigue. In 1961, W. Weibull proposed an imagine that the failure probability of fatigue life is probably equal to that of fatigue strength. Now the equality has been mathematically verified (Fu and Gao, 1985). Therefore the probabilistic distribution of fatigue strength could be derived from that of fatigue life, which requirs the P–S–N testing curve of fatigue life of structure or component. Obviously this method will be limited in costs, periods and equipments for this kind of testing.

So far there have been some useful investigations and researchs on continuum damage mechanics (Kachanov, 1958; Rabotnov, 1969; Gurson, 1977. etc.), on micromechanism of fatigue process (Field and Behnaz, 1982), on stochastic and probablistic aspects of fatigue (Provan, 1980; Kozin *et al.* 1981). *Probablistic Fatigue Damage Mechanics* (PFDM) model (Zeng, 1991) is also a fatigue design theory comprehensively considering damage, probability and localed field effect of fatigue, which could be used to estimate the probablistic distribution of fatigue life. Based on PFDM model this paper proposes a method for predicting the probabilistic distribution of fatigue strength for structure or component aiming at its application only using material constants, not needing the P–S–N testing curve of structure. Main steps in-

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clude: obtaining the material constants of PFDM, analysing the damage field of structure by finite element method, predicting the probabilistic distribution of fatigue strength.

2 BASIC THEORY OF PFDM

2.1 PFDM model on *Log–normal* distribution

The density function of probabilistic distribution of fatigue life for a component could be estimated by

$$P_{ln}(t)=\frac{1}{\sqrt{2\pi\sigma_s^2}}exp\left\{-\frac{(t-\gamma_s)^2}{\sigma_s^2}\right\} \tag{1}$$

where the t denotes lg–life of component, γ_s, σ_s^2 are the mean value and variance of the distribution (1), which could be calculated by following formulae:

$$\gamma_s=-\frac{1}{\beta}\left[(1-RG_0)^{B_1}lgD_{eq}-(1-RG_0)^{B_2}lgC_F\right] \tag{2}$$

$$\sigma_s^2=\frac{f}{\int_{V0}dV}\left[(1-RG_0)^{A_1}b_1-(1-RG_0)^{A_2}b_2 lgD_{eq}\right] \tag{3}$$

in which

D_{eq}: equivalent damage value at the maximum damage point P_0 of the component

RG_0: equivalent damage gradient at P_0

f : factor for treating the mean stress caused by loading

V_0: damage space of the component

β, lgC_F, b_1, b_2: material constants of PFDM

A_1, A_2, B_1, B_2: coefficients of damage gradient in PFDM

D_{eq}, RG_0, f, V_0 could be calculated by FEM for the component, and the material constants and coefficients of damage gradient should be definitive to the material used in the component.

2.2 PFDM model on *weibull* distribution

Weibull distribution form of the density of probabilistic distribution of fatigue life for a component could be estimated by

$$P_w(N)=\frac{m_s}{\alpha_s}\left(\frac{N-\theta_s}{\alpha_s}\right)^{m_s-1}exp\left[-\left(\frac{N-\theta_s}{\alpha_s}\right)^{m_s}\right] \tag{4}$$

where m_s, α_s, θ_s could be calculated by γ_s and σ_s^2 of formulae (2) and (3) (Zeng *et al.* 1989). Here N indicates the cycle number of fatigue life.

3 DISTRIBUTION FUNCTION OF FATIGUE STRENGTH BASED ON PFDM

3.1 Basic formula

According to the equality of the failure probability between fatigue life and strength probabilistic distribution density of fatigue strength $P(S/N^*)$ at a given fatigue life N^* is derived from that of fatigue life $P(N/S)$ as follows:

$$P(S/N^*)=\frac{d}{dS}\int_{N_{min}}^{N^*}P(N/S)dN \tag{5}$$

in which S denotes the fatigue strength, N_{min} is minimum value of fatigue life. Formula (5) indicates a mathematical method that the $P(S/N^*)$, which is not obtained by testing, could be derived from

$P(N/S)$. Also, the $P(N/S)$ could be obtained by either testing or fatigue theory. Here the PFDM is applied to do this.

3.2 ON *Log–normal*–based fatigue life

Substitution of formula (1) into (5), noticing the relationship $t=lgN$, leads to the probabilistic distruibiution density at a given fatigue life N^* as following expression:

$$P(S/N^*)=\frac{\gamma_s(S)\sigma'_s(S)-\gamma'_s(S)\sigma_s(S)-\sigma'_s(S)lgN^*}{\sqrt{2\pi}\,\sigma_s^2(S)}exp\left\{-\frac{\left[lgN^*-\gamma_s(S)\right]^2}{\sigma_s^2(S)}\right\} \tag{6}$$

where $\gamma'_s(S)$ and $\sigma'_s(S)$ denote the first derivative of respectively $\gamma_s(S)$ and $\sigma_s(S)$, which could be solved from formulae (2) and (3).

3.3 On *Weibull*–based fatigue life

Similarly, substitution of formula (4) into (5) leads to the probabilistic distribution density of fatigue strength on *weibull*–based fatigue life as follows:

$$P(S/N^*)=\left[\frac{N^*-\theta_s(S)}{\alpha_s(S)}\right]^{m_s}\left[-m_s\left(\frac{\theta'_s(S)}{N^*-\theta_s(S)}+\frac{\alpha'_s(S)}{\alpha_s(S)}\right)\right]\cdot exp\left\{-\left[\frac{N^*-\theta_s(S)}{\alpha_s(S)}\right]^{m_s}\right\} \tag{7}$$

where $\theta'_s(S)$ and $\alpha'_s(S)$ denote the first derivative of respectively $\theta_s(S)$ and $\alpha_s(S)$, which could be solved by PFDM.

4 EXAMPLE

It is seen that the formulae (6) and (7) are very complex, so some simplifying treatment of (6) and (7) is needed in following example. The investigations of P–S–N testing curve of fatigue life have shown the fact that both the mean value and variance of fatigue life distribution are approximately the linear functions of Log–loading lgS. So one assumes the relationship:

$$\gamma_s(S)=k_1-k_2lgS \tag{8}$$

$$\sigma_s(S)=k_3-k_4lgS \tag{9}$$

where k_1, k_2, k_3, k_4 could be solved by the regression method based on PFDM. Hereof, the formula (6) could be expressed as follows:

$$P(lgS/N^*)=\frac{k_4lgN^*+k_2k_3-k_1k_4}{\sqrt{2\pi}\,(k_3-k_4lgS)^2}exp\left\{-\frac{\left(k_1-k_2lgS-lgN^*\right)^2}{(K_3-k_4lgS)^2}\right\} \tag{10}$$

If both the mean value and variance of fatigue life distribution are approximately the linear functions of loading S, i.e.

$$\gamma_s(S)=\bar{k}_1-\bar{k}_2S \tag{11}$$

$$\sigma_s(S)=\bar{k}_3-\bar{k}_4S \tag{12}$$

where $\bar{k}_1,\bar{k}_2,\bar{k}_3,\bar{k}_4$ could be also solved by the regression method based on PFDM, the formul (6) can be rewritten as

$$P(S/N^*)=\frac{\bar{k}_4lgN^*+\bar{k}_2\bar{k}_3-\bar{k}_1\bar{k}_4}{\sqrt{2\pi}\,(\bar{k}_3-\bar{k}_4S)^2}exp\left\{-\frac{\left(\bar{k}_1-\bar{k}_2S-lgN^*\right)^2}{(\bar{k}_3-\bar{k}_4S)^2}\right\} \tag{13}$$

Now the paper gives an example of the material 40Cr (Zeng, 1991) to show an application of the proposed method. The material constants based on PFDM are

$\beta = 0.07546$ $\qquad\qquad$ $lgC_F = 3.1254 \quad (lgMPa)$

$b_1 = -2.096\pi \times 10^{-7}\ (m^3)$ $\qquad\qquad$ $b_2 = 7.671\pi \times 10^{-8}\ (m^3 / lgMPa)$

$A_1 = -2.2942$ $\qquad\qquad$ $A_2 = -2.2917$

$B_1 = -0.1411$ $\qquad\qquad$ $B_2 = -0.1215$

Then applying the formula (10) the paper predicts the probabilistic density of fatigue strength lgS of specimen ASP−2 at $N^* = 2 \times 10^5$ cycles, mean stress $S_m = 0$, which is expressed as follows (the curve is shown in Fig. 1)

$$P(lgS / N^*) = \frac{0.3912}{(1.4041 - 0.4632 lgS)^2} exp\left\{ - \frac{(13.7260 - 5.2266)^2}{(1.4041 - 0.4632 lgS)^2} \right\} \qquad (14)$$

where the unit of S is MPa.

Similarly, applying formula (13) could predicts the distribution of fatigue strength S of the specimen, which is shown in Fig. 2.

It should be pointed out that which one of formulae (10) and (13) would be chosen depends on practical situation of structure or component.

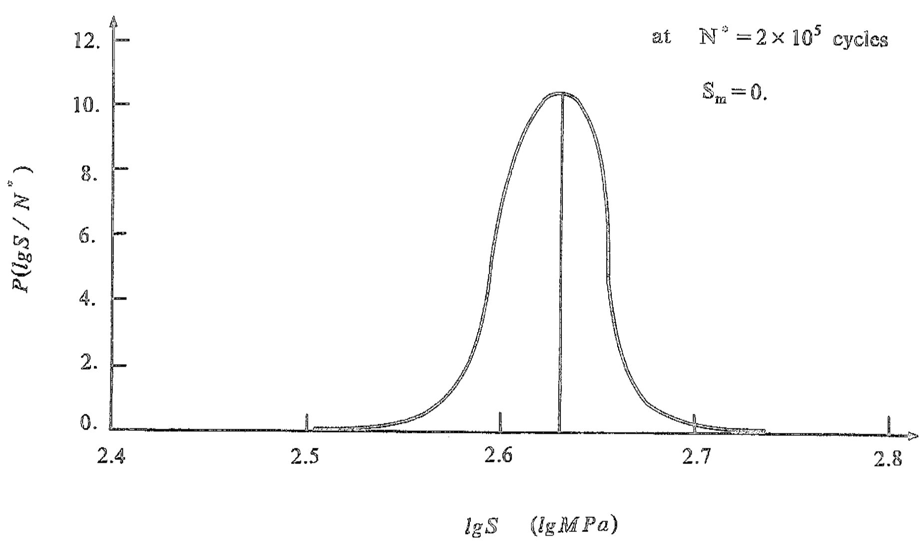

Fig. 1 The probabilistic density of fatigue strength lgS of ASP−2

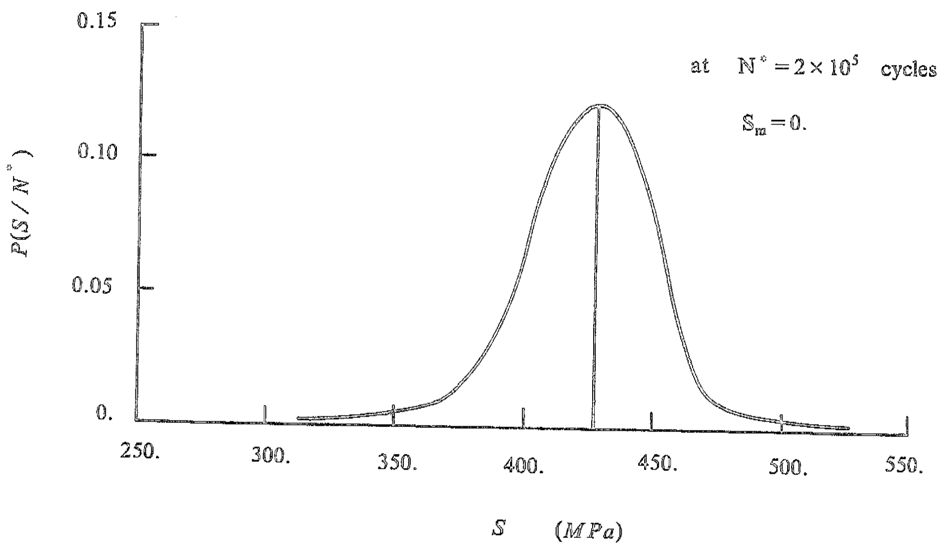

Fig. 2 The probabilistic density of fatigue strength S of ASP–2

5 SUMMARIES

Nowadays, the probabilistic aspect of fatigue and its application to engineering structure is a new tendency and advanced field in fatigue researchs. The paper has developed a method for predicting the probabilistic distribution of fatigue strength, of course, which needs further improvement in both theory and application. Some contents are summmaried as follows:

(1) The basic theory of PFDM is introduced as the basis for studying the probabilistic distribution of fatigue strength.

(2) It is presented that the mathematical formulae for predicting the probabilistic distribution of fatigue strength only on the condition of obtaining the related material constants.

(3) The proposed method does not need P–S–N testing curve of structure or component, except for small amount of testing of specimen.

(4) An example is given to show an application of the proposed method.

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