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ANALYSIS OF DUCTILE CRACK GROWTH BY A SIMPLE DAMAGE MODEL

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1. INTRODUCTION

A strip damage–zone model of the Dugdale–Barenblatt–model type [1] [2] is presented in this paper for analyzing crack growth in ductile materials with damage evolution. In particular, a semi-infinite Mode–I crack in plane stress or plane strain is considered. The damage is assumed to be present in form of dispersed microvoids, which are localized into a narrow strip direct ahead of a crack–tip. This configuration approximates the real situation naturally arising due to the high stress and strain concentrations in the proximity of the crack–tip. A simple damage model of the Gurson–model type [3] [4] is developed for uniaxial tension to describe the macroscopic propeties of the strip damage–zone. Under small–scale yielding and small–scale damage conditions, a system of nonlinear integral equations for the plastic strain and the length of the damage–zone is derived. Numerical results are presented and discussed for the crack opening displacement, the stress and damage distribution within the plastic/damage zone, and the crack resistance curve. Special attention is devoted to reveal the effect of damage evolution on the ductile crack growth.

2. CONSTITUTIVE EQUATIONS

In our previous work [5], a simplified Gurson–model is developed for uniaxial tension, which is very suitable for implementation in a strip damage–zone model. Without going into details, the simplified Gurson–model can be described by the following constitutive equations:

$$\text{strain equivalence:} \quad \epsilon^p = \epsilon^p_M , \tag{1}$$

$$\text{undamaged:} \quad \frac{\sigma_M}{\sigma_0} = \begin{cases} \epsilon_M/\epsilon_0 , & \sigma_M \le \sigma_0 , \\ (\epsilon_M/\epsilon_0)^{\frac{1}{n}} , & \sigma_M > \sigma_0 , \end{cases} \tag{2}$$

$$\text{damaged:} \quad \sigma = (1 - q_1 f^*)\sigma_M , \tag{3}$$

$$\text{damage evolution:} \quad \frac{df}{d\epsilon^p} = \frac{3q_1q_2f^*(1-f)}{2(1-q_1f^*)} \sinh[\frac{q_2}{2}(1-q_1f^*)] + D . \tag{4}$$

Here, σ is the macroscopic stress, ϵ is the macroscopic strain, and ϵ^p is the macroscopic plastic strain. The corresponding quantities of the matrix material are designated by a subscript "M". Also, σ_0 and $\epsilon_0 = \sigma_0/E$ are the equivalent flow stress and flow strain, while f^* and D are given by

$$f^* = \begin{cases} f\,, & \text{if} \quad f \le f_c\,, \\ f_c + \dfrac{f_u^* - f_c}{f_F - f_c}(f - f_c)\,, & \text{if} \quad f > f_c\,. \end{cases} \tag{5}$$

$$D = \frac{f_N}{s_N\sqrt{2\pi}} \exp\left\{ -\frac{1}{2}\left(\frac{\epsilon_M^p - \epsilon_N}{s_N}\right)^2 \right\}, \tag{6}$$

where q_1, q_2, f_c, f_F, $f_u^* = 1/q_1$, f_N, s_N and ϵ_N are material parameters arising in the modified Gurson–model. In this paper, the following material parameters are used:

$$f_N = 0.04\,, \quad s_N = 0.1\,, \quad \epsilon_N = 0.3\,, \quad f_c = 0.15\,, \quad f_F = 0.25\,,$$
$$q_1 = 1.25\,, \quad q_2 = 1.0\,, \quad \nu = 0.3\,, \quad \sigma_0/E = 1/300\,.$$

In the original Gurson–model, $q_1 = q_2 = 1$ and $f^* = f$. The parameters q_1, q_2 and f^* have been introduced by Tvergaard [4] in order to acquire a better agrement between the results from the Gurson–model and his numerical investigation on a periodic array of cylindrical voids. As shown in [5], the simplified Gurson–model (1)–(4) agrees very well with the modified Gurson–model, at least for uniaxial tension.

3. A STRIP DAMAGE–ZONE MODEL

Let us consider a semi–infinite Mode–I crack in plane stress or plane strain. The plasticity and the damage are assumed to be localized into a narrow strip direct ahead of the crack–tip as shown in Fig. 1. The finite but small thickness t of the plastic/damage zone is artificially introduced in order to make the simplified Gurson–model of Section 2 directly applicable.

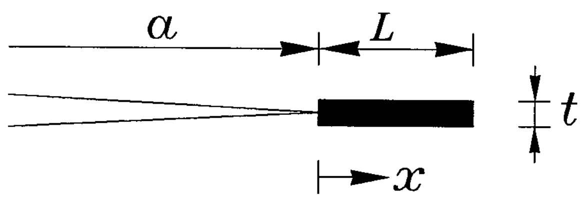

Figure 1: Strip damage–zone model for a Mode–I crack

The relative opening displacement $\delta(x)$ within the plastic/damage zone can be written as

$$\delta(x) = \frac{8K_I^A}{E^*\sqrt{2\pi}}\sqrt{L-x} - \frac{4}{E^*\pi}\int_0^L \sigma(x') \log \frac{\sqrt{L-x} + \sqrt{L-x'}}{\left|\sqrt{L-x} - \sqrt{L-x'}\right|} dx'\,, \tag{7}$$

where K_I^A represents the applied stress intensity factor, while $E^* = E$ for plane stress and $E^* = E/(1-\nu^2)$ for plane strain. The first term of Eq. (7) represents the relative opening displacement induced by the applied stress intensity factor K_I^A, while the second term denotes the contribution of the macroscopic stress $\sigma(x')$ acting within the plastic/damage zone. The condition of finite stress at the fictive crack–tip $x = L$ results in

$$K_I^A - \sqrt{\frac{2}{\pi}} \int_0^L \frac{\sigma(x')}{\sqrt{L-x'}} dx' = 0\,. \tag{8}$$

Neglecting the elastic strain ϵ^e in the plastic/damage zone, introducing the new variables $\xi = x/L$ and $\eta = x'/L$, appealing to the well-known relation between the K_I^A–factor and the J–integral

$$J = (K_I^A)^2/E^*\,, \tag{9}$$

and using the following kinematic relations [5]

$$\left.\begin{array}{l}\epsilon^p \simeq 2\delta/t \\ \epsilon^p_c \simeq 2\delta_c/t\end{array}\right\} \quad\Longrightarrow\quad \epsilon^p \simeq \frac{\epsilon^p_c}{\delta_c}\delta\,, \tag{10}$$

equations (7) and (8) can be rewritten as

$$\epsilon^p(\xi) = \frac{4\epsilon^p_c}{\pi}\frac{L}{\delta_c}\epsilon_0 \int_0^1 \frac{\sigma(\eta)}{\sigma_0}\left(2\frac{\sqrt{1-\xi}}{\sqrt{1-\eta}} - \log\frac{\sqrt{1-\xi}+\sqrt{1-\eta}}{|\sqrt{1-\xi}-\sqrt{1-\eta}\,|}\right)d\eta\,, \tag{11}$$

$$\frac{J}{\sigma_0\delta_c} = \frac{2}{\pi}\frac{L}{\delta_c}\epsilon_0\left(\int_0^1 \frac{\sigma(\eta)}{\sigma_0}\frac{1}{\sqrt{1-\eta}}d\eta\right)^2\,, \tag{12}$$

where ϵ^p_c and δ_c are the critical plastic strain and critical crack–tip opening displacement. For a given applied loading J, equations (11) and (12) in conjunction with the constitutive equations (1)–(4) form a system of nonlinear integral equations for the unknown plastic strain ϵ^p and the length L of the plastic/damage zone. A collocation method in conjunction with Newton–Raphson method has been applied for solving the nonlinear integral equations [5].

One important advantage of the present model is that no special crack growth criterion has to be postulated. Here, crack growth is controlled by the damage parameter f or f^*. Once the condition $f = f_F$ or $f^* = f^*_u$ at a material point is meet, the macroscopic material stress carrying capacity is lost at this point, and local failure takes place. Due to numerical difficulties, the constitutive equations (1)–(4) will be used once a damage level $f = 0.975 f_F$ is attained. Thereafter, $f = 0.975 f_F$ is kept as constant, and subsequent damage evolution is frozen. Henceforth, the plastic/damage zone behaves as nearly ideally plastic with a small flow stress.

4. RESULTS

Numerical calculations have been carried out for a material hardening exponent $n = 15$ and for plane stress. Figure 2 shows the normalized relative opening displacement δ/δ_c versus the dimensionless distance x/L from the crack–tip. With increasing x/L, δ/δ_c decreases from its maximum at the crack–tip to zero at the end of the plastic/damage zone. On the other hand, δ/δ_c increases with increasing J, and crack growth initiation takes place at $J = J_c$ or $\delta_t = \delta(0) = \delta_c$.

The stress and the damage distribution within the plastic/damage zone is shown in Figs. 3 and 4. The damage parameter f increases with increasing applied loading J, while it decreases with rising distance x/L from its maximum to zero. As long as the applied loading J is small, the normalized stress σ/σ_0 rises with increasing J. In this case, σ/σ_0 is largest at the crack–tip $x = 0$ and smallest at the end of the plastic/damage zone $x = L$. A somewhat

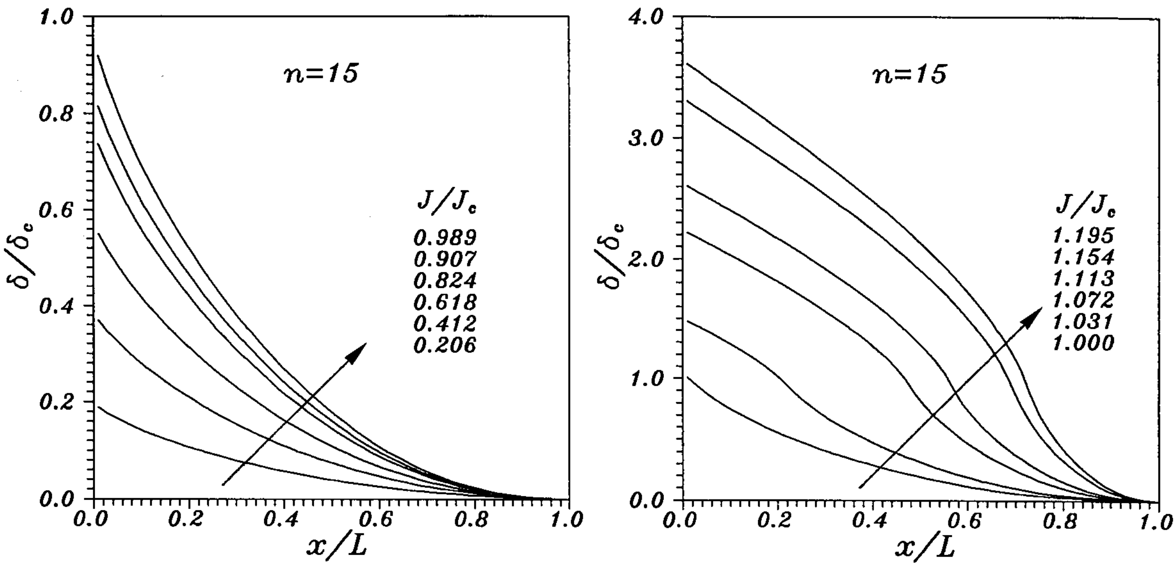

Figure 2: δ/δ_c versus x/L

different behavior of σ/σ_0 is, however, noted for large J. Here, the normalized stress σ/σ_0 in the immediate proximity of the crack–tip decreases, which is caused by the intense damage evolution in that region. Note here that similarly to the behavior before crack initiation, the damage parameter f during the crack growth is largest at the crack–tip and smallest at the end of the plastic/damage zone.

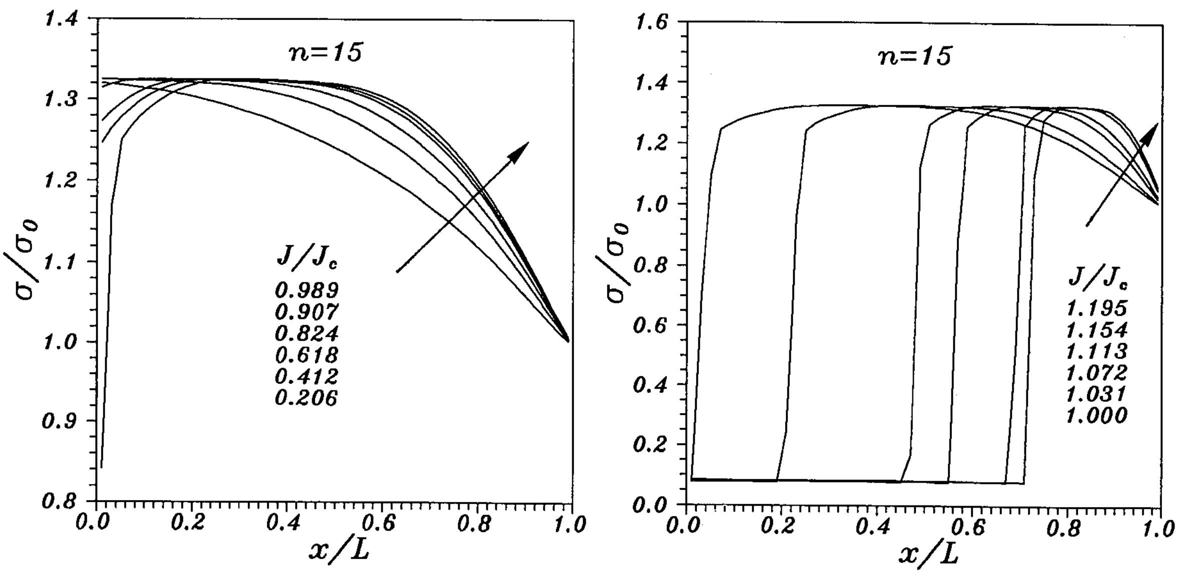

Figure 3: σ/σ_0 versus x/L

Figure 5 shows the dependence of the normalized crack–tip opening displacement δ_t/δ_c and the normalized length of the plastic/damage zone $L\epsilon_0/v_c$ on the applied loading $J/(\sigma_0\delta_c)$ and $J/(\sigma_0 v_c)$, where $v_c = \delta_c/2$. Both δ_t/δ_c and $L\epsilon_0/v_c$ before crack growth initiation increase nearly linearly with increasing $J/(\sigma_0\delta_c)$ and $J/(\sigma_0 v_c)$. This behavior can be approximated by the following relations

$$\delta_t = d_n \frac{J}{\sigma_0}, \qquad L = r_n \frac{J}{\sigma_0 \epsilon_0}, \tag{13}$$

where $d_n = 0.80$ and $r_n = 0.28$. After crack growth initiation, a complicated feature is noted. While δ_t/δ_c oscillates about unity, $L\epsilon_0/v_c$ shows a slightly rising tendency. The oscillations are induced by the applied numerical procedure.

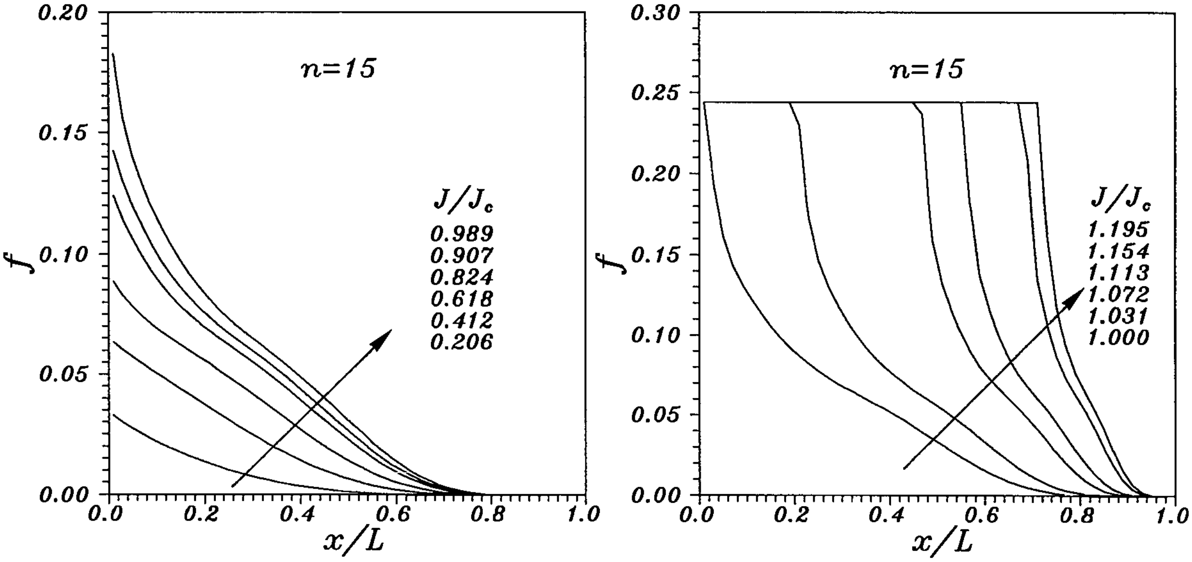

Figure 4: f versus x/L

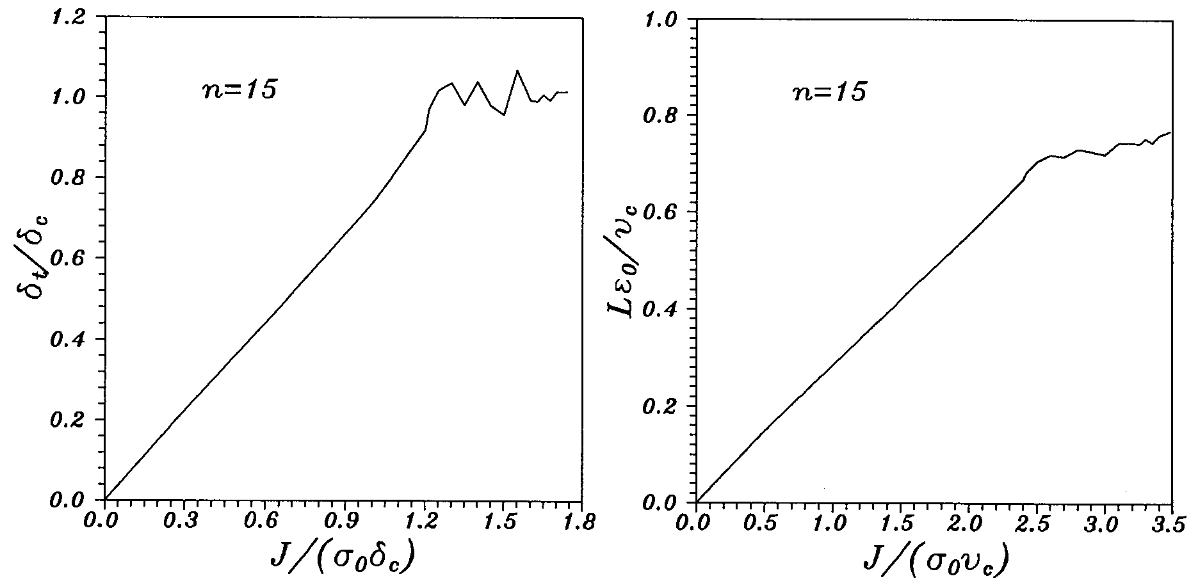

Figure 5: δ_t/δ_c versus $J/(\sigma_0\delta_c)$ and $L\epsilon_0/v_c$ versus $J/(\sigma_0 v_c)$

The normalized crack–tip opening displacement δ_t/δ_c versus the normalized crack extension $\Delta a/[J_c/(\sigma_0\epsilon_0)]$ is presented in Fig. 6. Here, circles designate the discrete numerical results, while the solid line represents their interpolation. Figure 6 reveals that the normalized crack–tip opening displacement δ_t/δ_c varies only slightly about unity, with a maximum deviation somewhat about 7%.

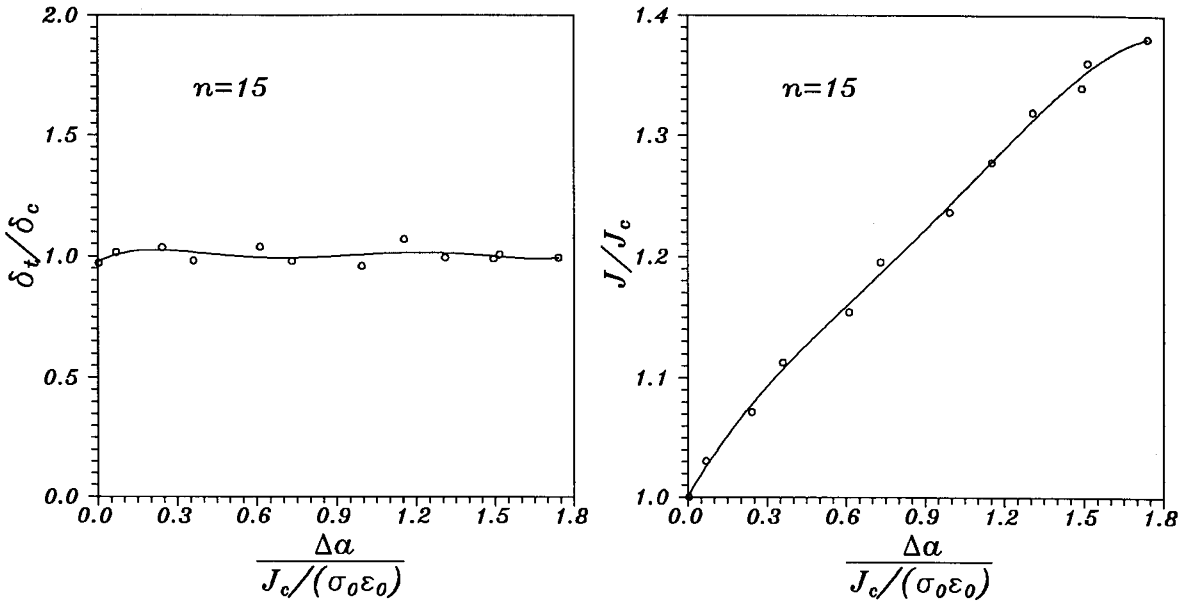

Figure 6: δ_t/δ_c and J/J_c versus $\Delta a/[J_c/(\sigma_0\epsilon_0)]$

The normalized crack resistance J/J_c is also shown in Fig. 6. Qualitatively, this figure can render the essential features of a typical crack resistance curve $J_R(\Delta a)$. It shows for instance, that the applied loading J has to be increased to achieve a continuuing crack growth after its initiation. This rising phase describes the resistance of a crack against the crack growth, and it is of practical importance to exploit this property in engineering applications. The plateau of the $J_R(\Delta a)$–curve characterizing the steady state crack growth $(dJ/da = 0)$ can yet not be simulated by the present model due to numerical difficulties.

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