Full Paper Transactions, SMiRT 28
Toronto, Canada, August 10-15, 2025

PHYSICS-ENCODED DEEP NEURAL NETWORKS FOR PREDICTION OF NONLINEAR SEISMIC RESPONSES UNDER NOISY SIGNALS

Dongjin Kim[1], Junho Song[2]

[1] PhD Candidate, Seoul National University, Seoul, Republic of Korea (rjsghksrhd16@snu.ac.kr)
[2] Professor, Seoul National University, Seoul, Republic of Korea (junhosong@snu.ac.kr)

ABSTRACT

Accurate prediction of structural responses to seismic loads is vital for assuring seismic performance. In this study, we propose a mode-ensemble Deep Neural Networks (DNN) model to address challenges in predicting seismic response time histories for structures, especially those exhibiting nonlinear behaviour. By utilizing a physics-encoded kernel initialization method based on structural dynamics theories, the model emulates the Duhamel convolution integral and incorporates mode shapes and modal participations. The model's architecture includes the kernel initialized with the mirror image of impulse response function, mode-shape encoding kernels, and modal participation kernel, to enhance its ability to generalize across diverse structural behaviours, including bilinear material nonlinearity model with different stiffness reduction ratio under varying levels of input noise. The model's robustness is evaluated across low, medium, and high levels of Gaussian noises introduced to the input seismic motions. Numerical investigations demonstrate that the proposed DNN achieves high prediction accuracy and robustness, highlighting its potential for applications in structural health monitoring and seismic risk assessment.

INTRODUCTION

Predicting seismic response is essential for ensuring safety and structural performance, but traditional approaches such as finite element (FE) analysis may face challenges when it comes to rapid or real-time predictions of nonlinear responses. This presents additional challenges within the performance-based earthquake engineering framework, which requires numerous nonlinear time history analyses, as discussed by Günay and Mosalam (2013). To address this by an efficient surrogate model, recent studies including Raissi et al. (2019) and Zhang et al. (2020) explored physics-encoded deep neural networks (DNN) models that incorporate physical principles to improve prediction accuracy. However, limitations still persist in predicting responses from noisy input signals, which can lead to misinterpretation of structural behavior.

To overcome such limitations, this study proposes a physics-encoded DNN model for predicting nonlinear seismic responses under varying levels of input signal noises, providing a practical tool for rapid seismic response predictions. The proposed DNN model includes two main components: one that removes noises from input seismic motions and the other that predicts the response time history of the target structure based on the denoised seismic motions. The noise reduction component is inspired by the architectures of well-established deep learning-based denoising methods that have been actively studied in recent years. Moreover, the proposed DNN model integrates physics into the architecture to guide the learning process, incorporating theories of structural dynamics to capture nonlinear structural behaviour. Once trained, the DNN model reduces noises from the input seismic motions, and predicts the response time history of the target structure from the denoised seismic motions. The model predicts the entire time history of nonlinear structural responses and can also process multiple input seismic motions in batches, offering an efficient alternative to traditional FE-based models, which handle each motion individually.

The first section reviews relevant research on DNN models developed for response prediction and denoising methods. The second section introduces the proposed physics-encoded DNN model, which incorporates a denoising autoencoder architecture. Section three details the numerical investigations, followed by the discussion of the results in section four. Finally, conclusions are drawn in the last section.

BACKGROUNDS

Response Prediction Through DNN Model

Predicting time series or sequential data, such as structural responses under dynamic loading, has garnered significant attention in recent years, particularly due to the emergence of advanced deep learning techniques. Traditional methods for analyzing these responses, such as FE analysis, often require high computational costs and struggle to effectively support real-time prediction needs, as highlighted by Sherstinsky (2020). Deep learning models, such as recurrent neural networks and long short-term memory networks, have shown promising capabilities in modelling sequential dependencies by capturing temporal patterns and maintaining memory of past states as studied by Dudek et al. (2022). One-dimensional convolutional neural networks (CNN) have been effectively applied to time series data, demonstrating robust performance in tasks like fault diagnosis and classification, particularly in noisy environments. Shenfield and Howarth (2020) highlighted the advantages of hybrid architectures that integrate CNN for feature extraction and recurrent layers for learning temporal patterns, achieving improved prediction accuracy and noise resilience. Additionally, Karniadakis et al. (2021) emphasized the potential of physics-informed neural networks to integrate physical laws directly into the learning framework, enhancing the consistency and reliability of predictions. These advancements illustrate how DNN models, through the incorporation of domain knowledge and innovative architectures, can provide robust and efficient tools for addressing the complexities of response time history prediction.

Denoising Methods

Noises in seismic motions typically arise as aleatoric uncertainty during sensor measurements and are often modeled as Gaussian noises. Its intensity is quantified using the signal-to-noise ratio (SNR), expressed in decibels (dB), which measures the ratio of signal power to noise power, that is,

$$SNR = 10 \cdot \log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \tag{1}$$

where P_{signal} and P_{noise} are the power of signal and noise time series, respectively. A high SNR indicates cleaner signals, while low SNR representing greater noise interference.

Various digital signal processing techniques such as moving average filtering, wavelet transforms, and Fourier transforms have been widely applied for noise reduction. While effective in specific cases, these methods often struggle to balance signal preservation and noise suppression under complex noise conditions. For instance, according to Nason et al. (1996), wavelet shrinkage can reduce noise in specific frequency bands but requires precise parameter tuning and may cause boundary effects.

Recent advancements in deep learning have revolutionized denoising strategies. Neural network-based approaches, particularly denoising autoencoder (DAE), can leverage large datasets to effectively isolate and remove noises. Chiang et al. (2019) demonstrated the ability of DAE to reconstruct clean biomedical signals from noisy inputs, showcasing their robust performance in diverse applications. Fully convolutional DAE architectures further enhance computational efficiency and simplify model complexity, making them suitable for denoising purpose.

PROPOSED METHODS

This study aims to develop a DNN model that can predict the structural response time histories of a target structure when input noisy seismic motions are obtained. The proposed DNN model is composed of two main components: (1) neural network (NN) model that reduces noise from the input seismic motions to

produce denoised input signals and (2) NN model that predicts the structural response of the target structure based on the denoised input signals.

The first component leverages the architecture of a DAE to reduce noise effectively from the noisy seismic motions. The second component integrates theories in structural dynamics into the deep learning framework using a physics-encoding approach, enabling this part of DNN model to simulate the process of structural dynamic analysis. Together, these components offer an effective approach for accurately predicting structural responses under noisy input conditions. The following subsections provide detailed descriptions of each component.

Denoising Neural Networks Based on Architecture of Denoising Autoencoder

Denoising NN is developed to effectively remove noises from seismic motion signals. The DAE architecture employed in this study leverages the inherent structure of time-series data, making it well-suited for seismic motion signal denoising by effectively isolating noise while preserving key signal characteristics, ensuring robust performance across various noise levels. Details of the architecture of the networks are summarized in Table 1.

Table 1: Detailed specifications of denoising NN.

Layer	Type	Input channels	Output Channels	Kernel Size	Stride	Padding	Output Padding
Encoder1	Conv1d	1	32	11	4	4	-
MaxPool	MaxPool1d	-	-	5	1	2	-
Encoder2	Conv1d	32	32	9	4	3	-
MaxPool	MaxPool1d	-	-	5	1	2	-
Decoder2	ConvTranspose1d	32	32	9	4	3	1
MaxPool	MaxPool1d	-	-	5	1	2	-
Decoder1	ConvTranspose1d	32	1	11	4	4	1

The encoding layers are designed to progressively reduce the input signal's length by half with each layer while simultaneously decreasing the convolutional kernel size, effectively compressing the signal information for mapping into the embedding space. Conversely, the decoding layers are designed to progressively double the signal length with each layer, while also symmetrically increasing the kernel size in a progressively larger manner.

In this study, two encoding layers and two decoding layers are implemented, forming a total of four layers. This configuration achieves a balance between denoising performance and signal retention, as fewer layers caused the loss of critical signal information such as amplitude of the signal, while adding more layers caused overfitting, reducing overall effectiveness.

Mode-ensemble Neural Networks

The mode-ensemble NN utilizes denoised seismic acceleration data to predict the response time history of a target structure subjected to seismic loads. Acting as a surrogate model for high-cost FE model, this NN encodes physics in structural dynamic to simulate the structural dynamic algorithms. The architecture is designed with three main components, each tailored to imitate or capture critical aspects of structural dynamics: the mirror-IRF kernel, the mode-shape encoding kernel, and the modal participation kernel.

The mirror-IRF kernel is constructed to replicate the impulse response functions (IRFs) that form the foundation of dynamic analysis. For single-degree-of-freedom linear structures, IRFs represent the system's response to an impulse input, derived through the Duhamel convolution integral. The IRF is described as

$$h(t) = \frac{1}{\omega_D} e^{-\xi\omega_N t} \sin(\omega_D t) \tag{2}$$

where ζ is the damping ratio, ω_N denotes the natural angular frequency, $\omega_D = \omega_N \sqrt{1-\zeta^2}$ is the damped angular frequency, and t is the time. For multi-degree-of-freedom structures, mode-specific IRFs are used to calculate modal responses using Duhamel convolution integral, i.e.,

$$q_i(t) = \int_0^t p(\tau) \cdot h_i(t-\tau) d\tau \tag{3}$$

where $q_i(t)$ and $h_i(\cdot)$ respectively denote the response time history and the IRF of the ith mode, and $p(\cdot)$ is the input excitation. Using the mode shape matrix $\boldsymbol{\Phi}$ and $\boldsymbol{q}(t) = \{q_q(t), \ldots, q_n(t)\}$, the overall structural responses can be obtained by superimposing these modal contributions, that is,

$$\boldsymbol{u}(t) = \boldsymbol{\Phi}\boldsymbol{q}(t). \tag{4}$$

Based on these dynamics, a 1D CNN architecture can be introduced to simulate the Duhamel convolution integral in Equation 3. In this approach, the future time-step components of the 1D CNN kernel weights are set to zero, while the weights for past time-step components are initialized as the mirror image of the IRF. This configuration allows the 1D CNN layer to numerically simulate the Duhamel convolution integral. The proposed 1D CNN kernel is referred to as the mirror-IRF kernel, and the 1D CNN layer utilizing this kernel is referred to as the Duhamel Layer. The mirror-IRF kernel ensures that the model captures the causal nature of structural dynamics, reflecting only the influences of the past on the current response. A schematic representation of the mirror-IRF kernel's operation within the Duhamel Layer is shown in Figure 1.

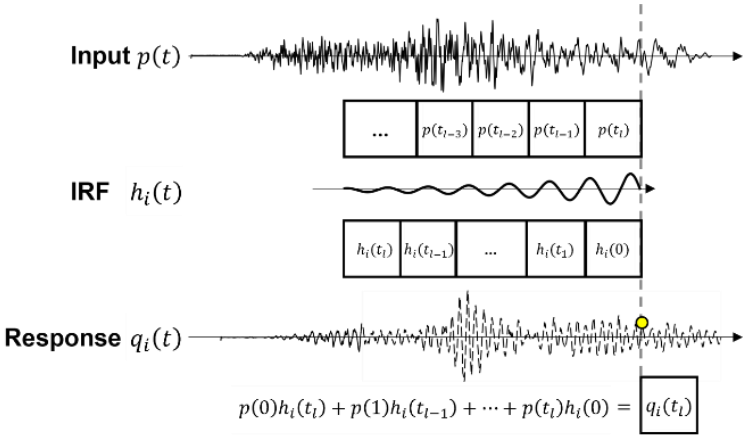

Figure 1. Operation of the mirror-IRF kernel within the Duhamel Layer.

If computing resources were unlimited, the kernel size of the mirror-IRF kernel could be set to infinity, with weights initialized as the mirror image of the ith mode IRF. However, due to computational constraints, the kernel size must be predefined within the Duhamel Layer. To achieve this, we incorporate the physics of decay of motion, which refers to the gradual reduction in oscillation amplitude over time, typically due to damping. This decay limits the influence of earlier excitations on the present responses, enabling kernel truncation with minimal loss of predictive accuracy. The relationship between the jth

positive peak and the first positive peak value of the ith mode IRF of the target structure is given as

$$u_1/u_j = \exp[(j-1)\cdot 2\pi\zeta] \tag{5}$$

where u_j is the jth positive peak. Then the kernel size k of the ith mode mirror-IRF kernel can be expressed as

$$k = T_{i_D} j/dt = \frac{2\pi}{\omega_{iN}\sqrt{1-\zeta^2}} \frac{1}{2\pi\zeta} \ln\frac{u_1}{u_{j+1}}/dt \tag{6}$$

where T_{i_D} is the ith damped natural period, and dt is the time interval between input excitation time history data. As shown in Equation 6, a smaller ratio u_{j+1}/u_1 results in a longer kernel size. However, an excessively long kernel size may lead to the loss of detailed characteristics in the frequency domain. In this study, we set the ratio u_{j+1}/u_1 to 10%, ensuring that the kernel size provides a sufficient time window for the DNN model to learn the frequency related properties of the target structure accurately.

The mode-shape encoding kernel is introduced after the Duhamel Layer to encode the nodal information into the nonlinear modal response. This matrix kernel learns mode-like shapes of nonlinear system by simulating Equation 4 from linear dynamic analysis process. Although eigenvalue analysis is not applicable to nonlinear systems, nonlinear structural systems possess certain nonlinear basis vectors, which can potentially be combined to approximate the behaviour of the target structure. As a matrix kernel, each row learns the basis vector corresponding to the target nonlinear system during training phase. Since modal analysis is valid only for linear systems, the term basis vector is used instead of mode shape or mode vector.

After passing through the mode-shape encoding kernel, a nonlinearity-embedding deep CNN layer is introduced to further embed the nonlinearities. This layer uses conventional CNN operations with random kernel weight initialization to maximize the effectiveness of the training dataset. By embedding nonlinearities and capturing spatial correlations among nodes, this layer enhances the accuracy of the predicted dynamic responses. The tensor, after passing through the nonlinearity-embedding deep CNN layer, encodes influence of modal contributions and guides the accurate reconstruction of node-specific response time histories. This physics-encoded kernel helps ensure that the predicted response time history reflects not only the external seismic loads but also the intrinsic dynamic characteristics of the structure.

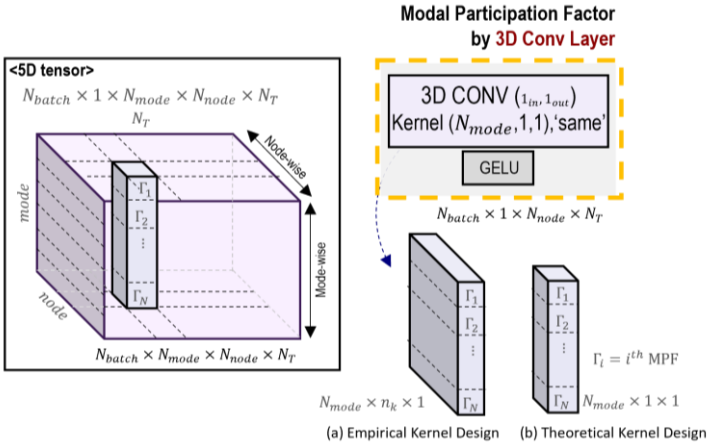

Figure 2. Modal participation kernel design in 3D CNN layer: (a) empirical kernel, and (b) theoretical kernel.

The modal participation kernel encodes the participations of each basis vector of a target nonlinear structure. Designed as a 3D CNN layer, this kernel learns the contributions of each basis vector to the structural response. In theory, as shown in Figure 2(b), if a CNN kernel is designed with dimensions of $N_{mode} \times 1 \times 1$, the corresponding CNN layer with this kernel can learn the participation ratio of each nonlinear basis vector. However, since nonlinear systems exhibit correlations between nodes and modes, a kernel with dimensions of $N_{mode} \times n_k \times 1$, as depicted in Figure 2(a), could be utilized to account for these correlations, where n_k is a kernel size for correlated nodes. In this study, we apply the modal participation kernel with the design shown in Figure 2(b) and initialize it randomly. The modal participation kernel is particularly important for nonlinear systems where the participation of higher modes significantly affects the overall structural responses in some seismic scenarios. The main role of this kernel is to encode the interaction between modes in both spatial and temporal dimensions. This approach enables the DNN model to capture the theoretical modal contributions accurately and adapt to structural systems with varying degrees of nonlinearity.

Together, these components enable the mode-ensemble DNN to predict nonlinear structural responses even when inputting noisy seismic motion data. The physics-encoded architecture in Figure 3 not only provides a computationally efficient alternative to FE models but also captures the underlying dynamic behaviours, including nonlinear basis vectors and their participations. This capability ensures the model's robustness and applicability across a wide range of structural configurations and seismic motions.

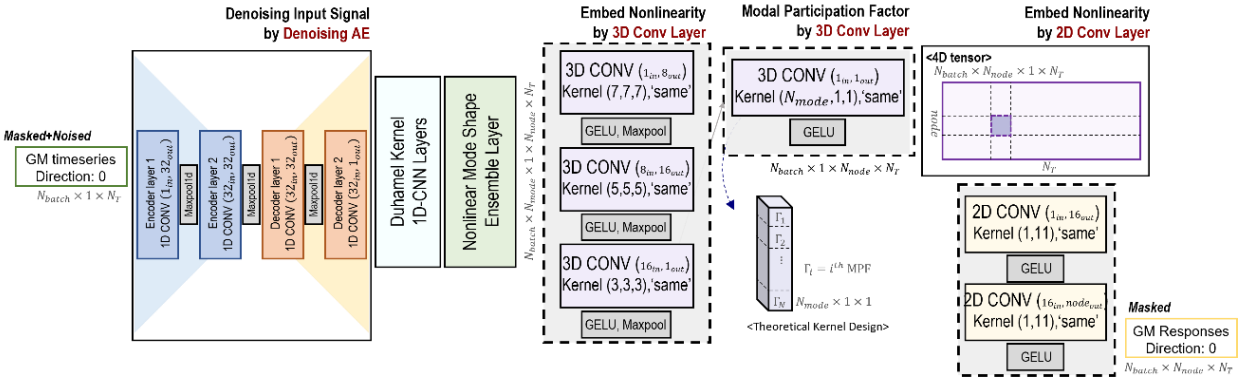

Figure 3. Illustrative diagram of the Mode-ensemble DNN architecture.

NUMERICAL INVESTIGATION

To demonstrate the applicability of the proposed method, a 3-story reinforced concrete shear building is considered as the target structure. The numerical model of the building was constructed using OpenSeesPy, and its properties are summarized as follows: the building features a rectangular cross-section with a width of 0.5m and a depth of 0.6m, with story heights of 3m. The elastic modulus of reinforcement steel is set at 410 MPa, which reflects typical initial stiffness of steel. A 5% Rayleigh damping ratio is used to account for dynamic behaviour under seismic motions.

To evaluate the performance of the proposed DNN model in predicting nonlinear structural responses, a material nonlinearity utilizing bilinear model is employed. The robustness of the model is assessed under varying degrees of nonlinearity by considering three stiffness reduction ratios: 5%, 20%, and 45%.

The input seismic motions are obtained from the NGA WEST2 database curated by the PEER center, comprising a total of 10,566 earthquake records. These seismic motion datasets are treated as clean input data, and three levels of Gaussian noises, represented by power-based SNR of 40 dB, 20 dB, and 10 dB, are artificially added to simulate observed accelerometer data under noisy conditions. The performance of the DNN models is evaluated using the Mean Squared Error (MSE) as the loss metric.

RESULTS

Denoising Neural Networks

The training process utilized 1,000 seismic motion acceleration data, starting with the low-noise SNR 40 dB dataset. Once the denoising NN stabilized, it was further trained on the medium-noise SNR 20 dB dataset, and finally, the high-noise SNR 10 dB dataset. This sequential approach was adopted to ensure the model to adapt effectively to increasing noise levels without overfitting to any specific noise intensity. The loss trends for the MSE metric, depicted in Figure 4, show a temporary increase in loss when transitioning between dataset with different noise levels, followed by stabilization as the model converged for each noise intensity. The transitions between datasets across epochs are represented in Figure 4 by shaded blue for SNR 40 dB, dark purple for SNR 20 dB, and light purple for SNR 10 dB.

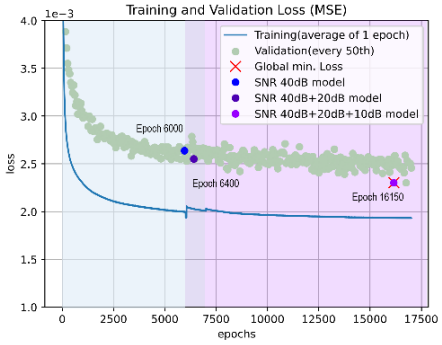

Figure 4. MSE loss trends during sequential training for different SNR levels.

The denoising performance was evaluated on test datasets with random SNR levels between 10 dB and 40 dB. Figure 5 compares an example of the denoised output from the denoising NN with the corresponding clean seismic motion signals. The results indicate that the denoising NN effectively reconstructs the original signals while preserving key features such as signal power, maximum and minimum values, and temporal patterns. For instance, as shown in right column of Figure 5 with SNR 10 dB, the denoised signal retained 91.84% of the clean signal's maximum value (0.8727 / 0.8015) $\mathrm{m/s^2}$, 96.44% of the minimum value $(-0.8101 / -0.7812)\ \mathrm{m/s^2}$, and 98.45% of the power $(0.0114 / 0.0113)\ (\mathrm{m/s^2})^2$, with a mean squared error (MSE) of 3.68% $(0.0004)\ (\mathrm{m/s^2})^2$. Even under high noise conditions, the model demonstrated robust performance, effectively reducing noise levels while maintaining the integrity of signal characteristics.

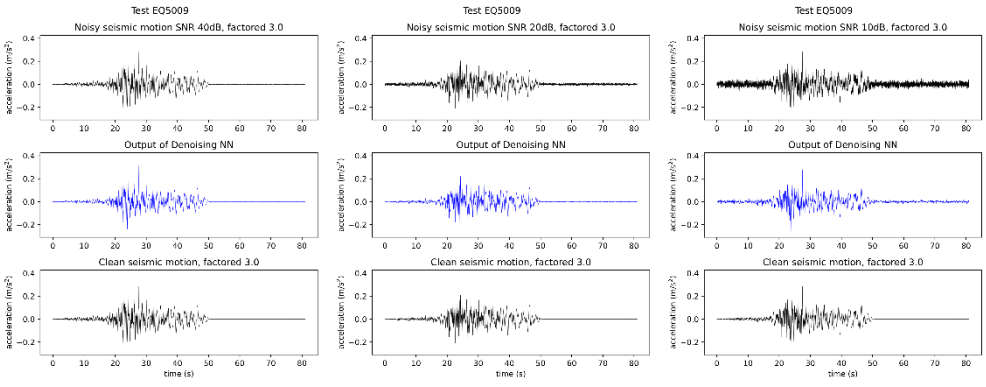

Figure 5. Comparison of noisy and denoised seismic motion signals.

Mode-ensemble Neural Networks

The mode-ensemble NN were designed to predict the response time history of structures with bilinear material nonlinearity. This component of the DNN model uses clean or denoised seismic motion inputs to predict time histories of floor accelerations for each story. The training database was constructed using a numerical model of a 3-story shear building with OpenSeesPy. The database consisted of acceleration response time histories generated by inputting 4,000 seismic motions, amplified with a factor of 3.0 to ensure sufficient nonlinear behaviour. While training the mode-ensemble NN, training data are clean seismic motion acceleration time series and target data are clean story response time histories.

The role of the Duhamel Layer is investigated through comparative experiments. Three variations in the Duhamel Layer are considered in this study. Model A0 utilized a 1D CNN layer without physics-encoded kernel initialization. Model A1 incorporated a single layer of physics-encoded mirror-IRF kernel, while Model A2 employed three stacked layers of physics-encoded mirror-IRF kernel. Validation error trends, as shown in Figure 6, reveal that models with physics-encoded initialization exhibit improved resistance to overfitting compared to Model A0. Therefore, Model A2 architecture is adopted for further model training.

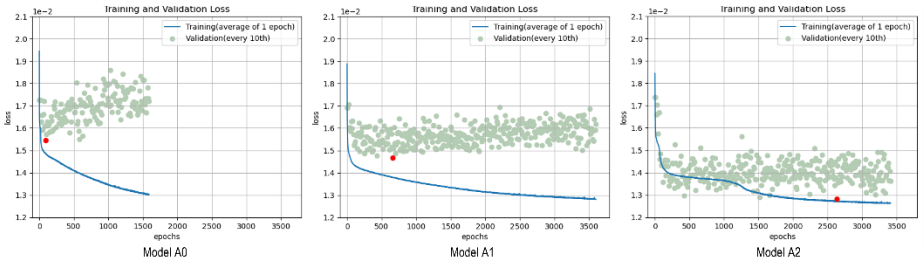

Figure 6. Training and validation loss trends of model A0, A1, and A2.

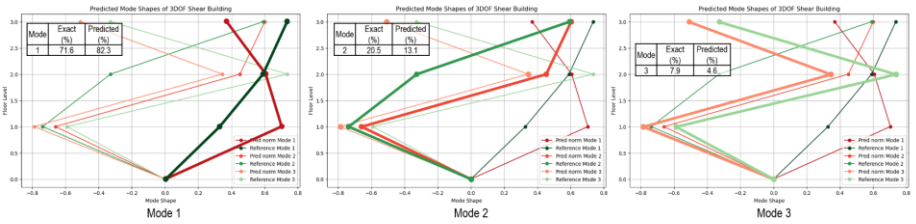

Figure 7. Comparison of nonlinear basis vectors learned by the mode-shape encoding kernel with mode shapes from eigenvalue analysis.

The trained mode-ensemble NN not only predicts response time histories but also encodes theories in structural dynamics. The mode-shape matrix kernel learns the nonlinear basis vectors of the target structure, while the modal participation kernel captures the contribution of each nonlinear basis vector to the structural response. A comparative analysis between the nonlinear basis vectors learned by the mode-shape matrix kernel and the mode shapes obtained from eigenvalue analysis of a linearized structure is presented in Figure 7, demonstrating the kernel's capability to adapt to basis vectors effectively. In the figure, the red-shaded graphs represent the nonlinear basis vectors, also referred to as mode-like shapes, learned by the mode-shape matrix kernel. In contrast, the green-shaded graphs correspond to the linear mode shapes derived from eigenvalue analysis of a linear structure with identical geometric properties but assuming an initial stiffness-based linear material model. Despite the random initialization of the kernel weights in mode-shape matrix kernel, the results confirm that the kernel successfully learns the nonlinear basis vectors of the target structure through the training process. This highlights the effectiveness of the physics-encoding approach in capturing the inherent nonlinear characteristics of the structural system. The participations for

each basis vector predicted by the modal participation kernel are also shown at the bottom of each graph in Figure 7, compared with modal participations from eigenvalue analysis of the linear structure.

To evaluate robustness, the mode-ensemble NN were trained on structures with varying bilinear material stiffness reduction ratios of 5%, 20%, and 45%. Figure 8 presents the results of arbitrary test seismic motions applied to structures following the material bilinear model with stiffness reduction ratios of 5%, 20%, and 45% in the left, center, and right columns, respectively. The first row of Figure 8 compares the predicted (in red) and target (in blue) acceleration response time histories for the first and third floors of the target structure in both the time and frequency domains. The title of each subplot indicates the predicted-to-target ratio, which represents the ratio of the maximum value of the predicted seismic response time history to that of the target response. The second row of Figure 8 illustrates the hysteresis loops for the test seismic motions, allowing an assessment of whether the structure experienced sufficient nonlinearity. Since the input seismic motions were scaled by an amplification factor of 3.0, it was confirmed that all test cases exhibited sufficient bilinear behaviour. The trained deep learning model effectively reconstructs the target seismic response time histories, regardless of the stiffness reduction ratio of the bilinear material model, as shown by both time-domain and frequency-domain analyses.

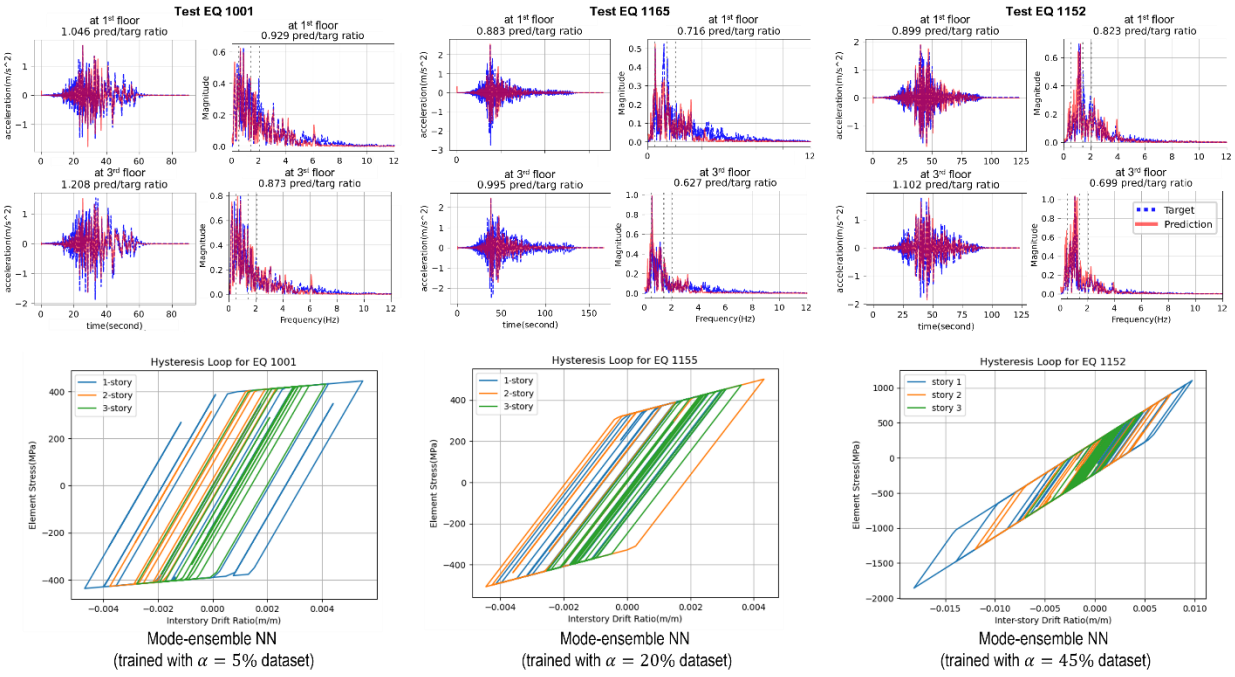

Figure 8. Comparison of predicted and target time histories for various stiffness reduction ratios.

Integrated Deep Neural Networks Model

The integrated DNN model consists of two main components, namely the denoising NN and the mode-ensemble NN. This modular and unified architecture processes noisy seismic motions as input and predicts the nonlinear response time history of the target structure. Each component is trained separately, and the physics-encoding approach is employed to embed theories in structural dynamics into the NN, ensuring an accurate representation of the structural response. The integrated model's ability to process noisy seismic signals and predict nonlinear structural responses highlights its robustness and applicability across diverse input conditions. This end-to-end framework offers a practical and efficient solution for structural response prediction under real-world noisy environments.

CONCLUSIONS

This study presented a physics-encoded deep neural network model to predict the nonlinear seismic responses of structures under noisy input conditions. The proposed model effectively integrates a denoising autoencoder to remove noises from seismic motion signals and a mode-ensemble neural network to predict the structural response, utilizing physics-encoded kernel initialization inspired by theories of structural dynamics.

The numerical investigations demonstrated that the proposed model achieves high accuracy and robustness across varying levels of input noises and structural nonlinearity. The denoising component successfully reconstructed clean seismic signals while preserving critical signal characteristics, and the response prediction component accurately estimated the structural response time history under nonlinear behaviour. Validation results confirmed that the model can provide consistent performance across different noise intensities and stiffness reduction ratios.

The proposed DNN model offers a computationally efficient alternative to traditional finite element analysis methods, which are often resource-intensive and time-consuming. Its ability to process noisy seismic inputs and deliver accurate response predictions in real-time makes it a promising tool for applications such as structural health monitoring and seismic risk assessment.

AKNOWLEDGEMENT

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (Ministry of Science and ICT, No. RS-2022-00144434).

REFERENCES

Chiang, H. T., Hsieh, Y. Y., Fu, S. W., Hung, K. H., Tsao, Y., & Chien, S. Y. (2019). “Noise reduction in ECG signals using fully convolutional denoising autoencoders,” *Ieee Access*, 7, 60806-60813.

Dudek, G., Smyl, S., & Pełka, P. (2022). “Recurrent neural networks for forecasting time series with multiple seasonality: a comparative study,” *International Conference on Time Series and Forecasting (ITISE 2022)*, Spain, 171-185.

Günay, S. and Mosalam, K. M. (2013). “PEER performance-based earthquake engineering methodology, revisited,” *Journal of Earthquake Engineering*, 17(6), 829-858.

Karniadakis, G. E., Kevrekidis, I. G., Lu, L., Perdikaris, P., Wang, S., & Yang, L. (2021). “Physics-informed machine learning,” *Nature Reviews Physics*, 3(6), 422-440.

Nason, G. P. (1996). “Wavelet shrinkage using cross-validation,” *Journal of the Royal Statistical Society: Series B (Methodological)*, 58(2), 463-479.

Raissi, M., Perdikaris, P., and Karniadakis, G. E. (2019). “Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving PDE,” *Journal of Computational Physics*, 378, 686-707.

Shenfield, A., and Howarth, M. (2020). “A novel deep learning model for the detection and identification of rolling element-bearing faults,” *Sensors*, 20(18), 5112.

Sherstinsky, A. (2020). “Fundamentals of recurrent neural network (RNN) and long short-term memory (LSTM) network,” *Physica D: Nonlinear Phenomena*, 404, 132306.

Zhang, R., Liu, Y., and Sun, H. (2020). “Physics-informed multi-LSTM networks for metamodeling of nonlinear structures,” *Computer Methods in Applied Mechanics and Engineering*, 369, 113226.