Application of probabilistic methodology to the assessment of pressurised components containing defects

G.W.Gould, P.J.Browne & Z.A.Gralewski
Electrowatt Engineering Services (UK) Ltd, Horsham, Sussex
N.F.Haines & R.Wilson
CEGB, Berkeley Nuclear Laboratories, UK

1. INTRODUCTION

The use of fracture mechanics is now well established in the design and assessment of pressure vessels and piping for nuclear power plants and other industrial facilities. Over the last few years a number of design procedures for assessing defects in such components have been developed specifically for use by the designer rather than the fracture mechanics specialist. One such method is the CEGB R6 procedure (Milne I, et al, 1986) which has been in use for several years and is well validated by a considerable amount of experimental and analytical work (Milne I. et al, 1987).

The deterministic methods have been developed to give conservative assessments of defective structures which in turn leads to a high level of confidence that failure will be avoided. Although the deterministic methods have these in-built safety factors there may still be a finite probability of failure, albeit a very small one, due to uncertainties in material properties, NDE data and loading conditions. The aim of the probabilistic approach is to provide an estimate of the failure probability of a structure. In addition, a clearer indication of the residual life of a component can be obtained using probabilistic analysis. Thus, application of probabilistic methodology should be seen as an extension of the deterministic "safe/unsafe" concept rather than an alternative.

In particular, if the probabilistic concept is combined with a well established deterministic procedure it is hoped a useful assessment tool will result. It was with this objective that a computer code has been developed in which probabilistic methodology has been linked to the CEGB R6 procedure.

2. THE COMPUTER MODEL

The computer code developed by BNL and EWE comprises two main parts. There is the deterministic aspect which considers failure assessment given all the input parameters and there is the probabilistic aspect which handles the sampling and final estimation of the failure probabilities. Such a division has the important result that it is possible to verify separately the deterministic and probabilistic portions of the program. A simplified flow chart of the computer code is shown in Figure 1.

2.1 Deterministic Analysis

The program uses the CEGB R6 procedure (Milne, Ainsworth, Dowling and Stewart 1986) for the criterion for failure by either fracture or plastic collapse. This approach is well established and it is simply necessary to select the appropriate form of the diagram for the materials under consideration. A typical failure assessment diagram is shown in Figure 2. The failure criterion for creep rupture is taken from the appropriate diagram of the ISO Data 1 Report (1978).

The relevant assessment parameters are calculated by the program using the available fracture mechanics and creep rupture models as selected by the user.

2.2 Probabilistic Analysis

Monte Carlo simulation is used to estimate the distributions of the various failure parameters given the probability distributions of the stochastic input variables such as fracture toughness, yield strength and initial defect size. In effect a large number of assessments are carried out for each case using different values for the stochastic variables in each assessment. These values are selected from the appropriate distributions using a random number generator. A count is kept of both the total number of assessments performed and those for which failure is predicted (or at least where the assessment does not fall within the 'failure avoidance' region on the diagram). The failure probability is simply the ratio of the number of predicted failures to the total number of assessments. A major difficulty with the small expected failure probabilities is that a very large number of samples must be taken for the uncertainty in the probability to be suitably small. This may require excessive computer time and therefore the computing costs may be prohibitive. Fortunately techniques are available which (under suitable conditions) greatly reduce the time to obtain a result of the required accuracy. Two such techniques, stratified and importance sampling are employed in the program and further details are given below.

3. SAMPLING TECHNIQUES

3.1 Stratified Sampling

The purpose of a so-called stratified sampling scheme is to ensure that specified intervals in the distribution of sample values contain the appropriate number of samples. This will ensure that the 'tails' of the distribution (i.e. the regions which often have the greatest effect on the output parameters) are not empty or severely under-filled due to chance. Instead each interval contains the correct number of samples (rounded to the nearest integer). Sampling is then carried out using the standard method first of generating a pseudo-random number in the range 0 - 1 and taking this to be the cumulative probability for the sample variate. The value of the sample is then calculated according to the distribution assumed.

After each set of sample variates have been generated (one value for each of the variables) the deterministic assessment is carried out using these values. The whole process is repeated until the required number of cases has been run (for example 1000 runs). The distributions included in the program at present are normal, log-normal, uniform, log-uniform, Weibull and Gumbel. Others could be added if necessary.

3.2 Importance Sampling

Importance sampling can reduce the variance of an output variable for a given number of samples by concentrating the sampling on those regions of the distribution which have the greatest effect on the relevant output parameter (e.g. in the tails of the distributions).

The biasing of certain parts of the distribution is achieved by applying suitable weighting factors (>1.0) to the relevant intervals in the stratified sampling scheme. The numbers in each interval and the cumulative probabilities corresponding to each interval boundary are modified accordingly. Sampling followed by a deterministic calculation is then carried out as before for the required number of samples. It is of course necessary to process the results to remove the effect of the biasing before the failure probability can be determined and this is done within the program. The values of the weighting factors to be used are determined from an initial calculation without weights. This analysis is facilitated by selecting an option in the program which will calculate the necessary functions which can then be plotted and analysed to give the weights.

The effectiveness of importance sampling is illustrated in Figure 3. This shows, for a hypothetical load case, the cumulative probability of failure plotted against the number of runs using a log scale. It is clear that IS shows an oscillation that is very much smaller than that shown by RS. This is supported by Figure 4 which shows, for the same load case, the standard deviation of the cumulative failure probability plotted against the number of runs using a log scale. The increased efficiency of importance sampling over random sampling may be seen directly.

4. APPLICATION OF COMPUTER CODE

Consider a longitudinal weld in a large diameter pipe. The component has been in service for some time and results from a large number of non-destructive examinations of the weld are available. A finite element analysis has been carried out to determine the stresses in the weld under safety valve pressure conditions. The owner wishes to operate the plant for at least a further five years so the probability of failure of the pipe at that time needs to be determined.

4.1 Loading Data and Material Thickness

Loading data required by the computer code are, applied stresses (divided into membrane and bending), operating temperature and pressure, and the safety-valve lift-off pressure. Other loading conditions can be introduced so that fault conditions and transients may be examined. Probability distributions may be associated with these parameters but, for this example, constant values have been used as follows:

Applied membrane stress = 100 MPa
Operating Pressure = 2.1 MPa
Safety Valve Pressure = 2.5 MPa
Nominal Operating Temperature = 370°C.

4.2 Material Properties and Defect Depth Distribution

The material properties required are yield stress, ultimate tensile strength and fracture toughness. For this example, normal

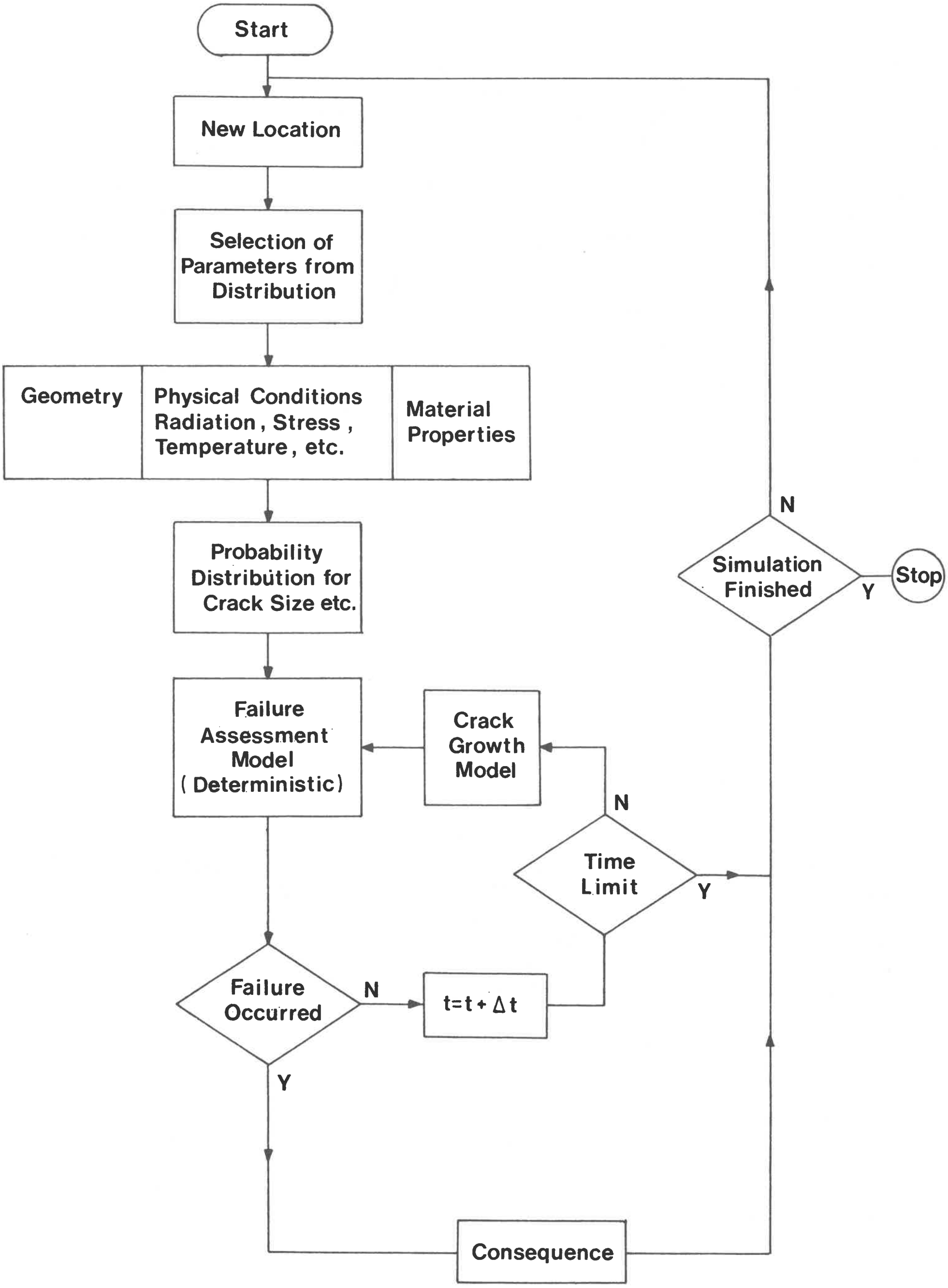

Figure 1: Flowchart for Combined Deterministic and Statistical Model

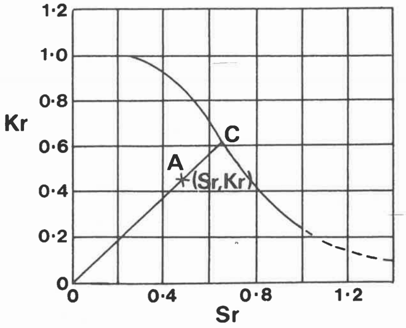

Figure 2: Failure Assessment Diagram

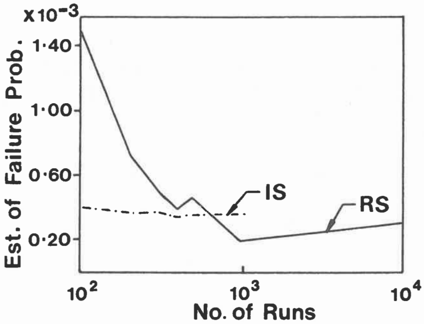

Figure 3: IS vs RS Estimation of Cumulative Probability

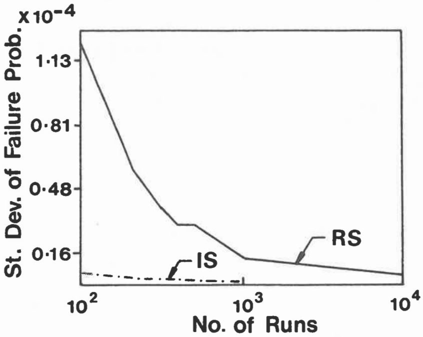

Figure 4: IS vs RS Standard Deviations

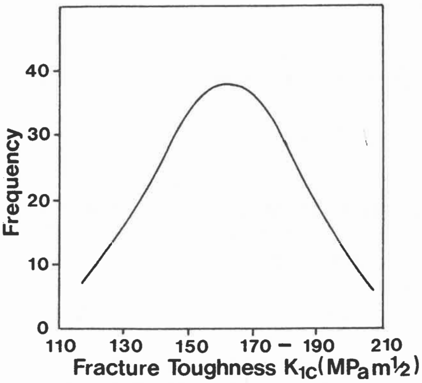

Figure 5: Material Fracture Toughness Distribution

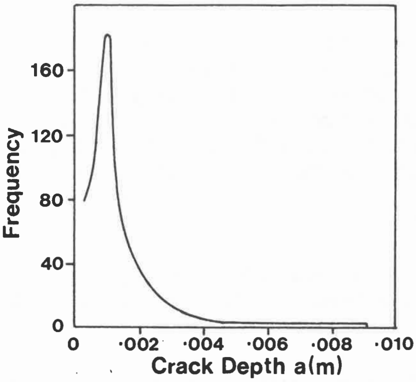

Figure 6: Input Crack Distribution

distributions are used to reflect the variability of these properties, defined as follows:

Yield Stress = 175 MPa with a standard deviation of 16.53 MPa
UTS = 433 MPa with a standard deviation of 15.81 MPa
Fracture
Toughness = 164 MPa $m^{1/2}$ with a standard deviation of 26.51 MPa $m^{1/2}$

These distributions are curtailed at $\pm 2\sigma$. The input distribution for fracture toughness, K_{1c}, is shown in Figure 5.

For this example the Gumbel extreme value function has been chosen for the defect depth distribution as shown in Figure 6. This can be defined as follows:

$$\mathrm{PDF}(X) = BA^{B} \exp\left[- \frac{X}{A}^{-B} \right] X^{-(B+1)} \quad ; \quad A = 0.04,\ B = 3.00$$

where, X = normalised defect depth = $\frac{\text{defect depth}}{\text{Component thickness}}$

4.3 Importance Weighting

We are interested in the probability that the R6 safety factor is less than 1.0, that is, the probability of failure. This occurs for crack depth values in the upper tail of the Gumbel distribution and therefore importance weighting has been applied to this region.

4.4 Results

In the following results it should be noted that for this simple example it was assumed the defect exists along the full length of the weld. If the length of the defect had been included the predicted failure probability would be considerably smaller.

A failure probability of 6.8×10^{-4} is predicted using a sample size of 1000 with importance sampling of the defect depth distribution. With no importance weighting, that is stratified sampling only, no failures at all were predicted for this number of runs. When such small probabilities are involved the number of samples would have to be $>10^{4}$ if no importance sampling was employed.

5. CONCLUSIONS

A computer code is under development which applies probabilistic methodology to a well established fracture mechanics assessment procedure. Preliminary results show that it could become a valid and useful tool for the safety assessment of pressurised components containing defects. Prediction of residual life of components in terms of their failure probabilities is possible with this code.

REFERENCES

Milne I, Ainsworth R.A., Dowling A.R., and Stewart A.T., 1986. Assessment of the Integrity of Structures Containing Defects. CEGB Report R/H/R6 - Rev 3.

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