

Elastic-Plastic Analysis for the Ductile Growth of Surface Crack

Ning Jie, Gao Qing, Sun Xunfang
Southwestern Jiaotong University, Emei, Sichuan, PRC

ABSTRACT

This paper developed a rather direct engineering method to measure the J_r curve of surface crack. By means of simple tension experiment, we measured the J_r curve of surface cracked plate made of an aluminium alloy. Results indicated that the J_r curve of surface crack is geometry dependence and the material resistant force for the ductile growth of surface crack is much stronger than that of through crack.

INTRODUCTION

With the widespread and profound developing of Elastic-Plastic Fracture Mechanics (EPFM), the necessity of the EPFM analysis for structural flaw, especially the part-through surface crack in a plate or shell, is widely recognized. Because J_r curve method is able to predict the whole deformation process and crack growth features including of crack initiation load, steady crack growth amount, the failure load and the loading-displacement nature for the cracked body, it has been extensively concerned and used in the EPFM analysis for the through-crack with steady ductile crack growth. For surface crack problem, J_r curve method is almost not applied due to there is no an appropriate method to measure the J curve of surface crack.

The growth of surface crack is a three-dimensional crack growth problem. it is unquestionable that the three-dimensional finite element method is the most powerful tool to solve this problem. Unfortunately, because of high cost in calculation, complex numerical technique and dependence on larger-scale computer, it is inconvenient to make use of three-dimensional finite element method to measure J_r curve of surface crack. On the other hand, there are also numerous difficulties in measuring J_r curve of surface crack by means of experimental method. For example, the measure of local growth along the crack front is a very complicated matter. Therefore, it is necessary to develop an accurate and economical engineering approach to solve various problems at hand. In recent years, the Nonlinear Line-Spring Model (NLSM) [1-6] for the EPFM analysis of surface crack has been extensively developed owing to its simplicity and accuracy. By means of NLSM method, although the J curve of surface crack was primarily researched [7-8], there is a long distance to apply it into engineering structure analysis.

In this paper, we developed a rather direct engineering method to construct the J_r curve of surface crack by a simplified NLSM. In the following sections, we described the procedure of constructing J_r curve of surface crack. J_r curve of surface cracked specimens were briefly discussed, including of the effects

of thickness of the specimen and the geometry of the crack. Our results showed the geometry dependence of J curve of surface crack.

PROCEDURE OF CONSTRUCTING J CURVE OF SURFACE CRACK

For a surface cracked thin plate as shown in Fig.1, By means of line-spring model, this problem may be considered as a through-crack of total length $2c$ equal to the maximum length of the surface crack in the plate or shell mid-surface. For symmetric loading, the sides of this conceptual through-crack is connected by a series of line springs. The normal force and bending momentum acted upon the line spring are N and M respectively. These local forces have work-conjugate deformations (δ, θ) consisting of the relative separation and rotation of the sides of the model through-crack. The generalized nonlinear constitutive behavior of the line spring is simulated by the edge cracked strip. In this way, the three-dimensional problem is simplified as an imitative two-dimensional problem.

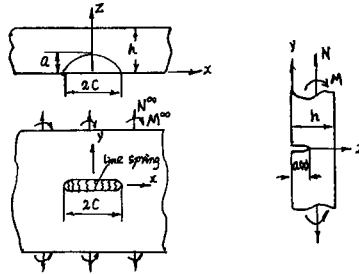


Fig.1 NLSM of surface of crack

By suitably simplifying the displacement fields (δ, θ) and interpolating the analysis of classical plate theory and Reissner plate theory a simplified NLSM of thin plate with a surface crack can be described by

$$\sigma_m - \sigma_m^{\infty} = -a_1 \bar{\delta} \quad (1a)$$

$$\sigma_b - \sigma_b^{\infty} = -a_4 \bar{\theta} \quad (1b)$$

where $a_1 = Eh/4c$, $a_4 = 3(3+\nu)Eh \zeta^p / 4(1+\nu)c + (1-\zeta^p)a_3$, $\sigma_m = N/h$,

$\sigma_b = 6M/h$, $a_3 = 3Eh/4c + 3E \int_0^{\infty} \psi(\alpha) J_1(\alpha c/h) d\alpha / \pi(1+\nu)$, $\zeta = a/h$,

$\psi(\alpha) = 0.5 - \alpha / \sqrt{\alpha^2 + 10} + \alpha$, $\sigma_m^{\infty} = N^{\infty}/h$, $\sigma_b^{\infty} = 6M^{\infty}/h$, $J_1(\cdot)$ is

the first Bessel function. α - integral variable, h - plate thickness, N^{∞} and M^{∞} are the tension and bending forces acted at far field, respectively. a - crack depth, c - half crack length, E - Young's modulus, ν - Poisson's ratio, $0 < p < 1$.

If the material constitutive behavior can be described by Ramberg-Osgood relation:

$$\epsilon / \epsilon_0 = \sigma / \sigma_0 + a (\sigma / \sigma_0)^n \quad (2)$$

where a and n are material hardening coefficients, ϵ_0 and σ_0 are the material flow strain and stress, respectively.

for an edge cracked strip shown in fig.1(c), its increments of generalized displacements can be described by

$$d\{\bar{q}\} = d\{\bar{q}\}^e + d\{\bar{q}\}^p \quad (3)$$

where $d\{\bar{q}\}^e$ is the elastic component and $d\{\bar{q}\}^p$ the plastic component.

Making use of generalized flow theory of plasticity, we obtained:

$$d\{\bar{q}\}^p = d\lambda \{\partial\phi / \partial\bar{\sigma}\} \quad (4)$$

where $\phi(\{\bar{\sigma}\}, \zeta)$ is the generalized yield surface [10]. The hardening

condition is given by

$$\phi(\{\bar{\sigma}\}, \xi) - F(\int |d\bar{q}^p|) = 0 \quad (5)$$

$$\text{where } |d\bar{q}^p| = \sqrt{(d\bar{\delta}^p)^2 + (d\bar{\theta}^p)^2} \quad (6)$$

At the uniaxial action of generalized normal force N , its solution was given by [11]

$$\bar{\delta}^p = \beta, \xi h_3(\xi, n) \bar{\alpha}_m^n \quad (7)$$

where $\beta, = \alpha \varepsilon \cdot \xi^n / [1.455(\sqrt{(1-\xi)^2 + \xi^2} - \xi)]^n$, $h_3(\xi, n)$ was given in ref. [11].

From eqn (3) - eqn (7), we can determine the function F and the parameter $d\lambda$. Through linearly simplifying the yield function ϕ , we can obtain the following nonlinear constitutive relation of the line spring:

$$\{\bar{q}\} = 2\varepsilon \cdot (1-\nu^2) [C] \{\bar{\sigma}\} + \beta, \xi h_3(\xi, n) [D_r^*] \bar{\sigma}_e^{n-1} \{\bar{\sigma}\} \quad (8)$$

where $\{\bar{q}\} = \{\bar{\delta}, \bar{\theta}\}$, $\{\bar{\sigma}\} = \{\sigma_m/\sigma_0, \sigma_b/\sigma_0\}$, $\bar{\sigma}_e$ was Irwin equivalent stress. $[C] = [\int_0^{\xi} g_r(\xi) g_s(\xi) d\xi]$ ($r, s = m, b$), ξ was Irwin equivalent crack size. $[D_r^*]$ was given in ref. [6]. $g_m(\xi)$ and $g_b(\xi)$ was given in ref. [12].

Eqn (1) and eqn (8) constituted the fundamental solution of the NLSM. Under the case of linear elastic analysis, numerical results showed that its calculation results agreed well with those of Newman's three-dimensional finite element method.

If we denoted V as the crack mouth opening displacement of the edge cracked strip as shown in Fig.1(c), we may assumed

$$V = V^e + V^p \quad (9)$$

where V^e is the elastic component and V^p the plastic component of the crack mouth opening displacement. $V^e(\xi_e)$ was given in ref. [12].

In the fully plastic state, by means of Cathetallory's theorem we obtained

$$V^p = h \xi \beta, h_3(\xi, n) \bar{\alpha}_e^n [1+k(\xi)] \quad (10)$$

where $k(\xi)$ is given in ref. [6].

Through experiment, we can measure P - V (loading-displacement) curve of surface cracked specimen. Subsequently, we may find the generalized stresses $\{\sigma\}$ and dimensionless crack growth amount $\Delta \xi$ by eqn (1)-eqn(10) and the P - V curve. Finally, we can calculate the J_r value by the following formula [6]:

$$J = (h \sigma_0^2 / E) [(1-\nu^2) \{\bar{\sigma}\}^T [C'] \{\bar{\sigma}\} + \beta_2 \bar{\sigma}_e^n B(\xi, n) / (n+1)] \quad (11)$$

where $\beta_2 = \alpha / 1.455^n$, $B(\xi, n) = A' \bar{\alpha}_e + (n+1) A k'(\xi) \bar{\sigma}_b$, $A' = dA/d\xi$

$$k'(\xi) = dk/d\xi, A = f(\xi) h_3(\xi, n), f(\xi) = \xi^n / (\sqrt{(1-\xi)^2 + \xi^2} - \xi)^n$$

EXPERIMENT

The surface crack specimens were made by thin plate whose material is LD10CS aluminum alloy. The material constants are $\sigma_s = 43(\text{Kg/mm}^2)$, $\sigma_b = 48(\text{Kg/mm}^2)$,

$E=7400(\text{Kg}/\text{mm}^2)$, $\nu=0.33$, respectively, where σ_s and σ_b are yield stress and ultimate strength of the material. To eliminate effects of the width of the specimen, the width of the specimen satisfied condition $W/2c > 5$, where W stands for the width of the specimen. Widths of the specimen are 80(mm) and 56(mm), and the thickness, 3(mm), 5(mm) and 7(mm), respectively. The details of experiment procedure was described in ref.[8].

Through the uniaxial material tension experimental results, the material hardening coefficients were determined by $\alpha = 0.45$, $n=20$, respectively. In order to investigate the effects of the crack length on the J_r curve, we also measured J_r curve of surface crack specimen made by electrosparking.

RESULTS AND DISCUSSIONS

Fig.2 described the J_r curves of surface cracked specimens which thickness was 7(mm). It showed that the slope of the J_r curve of surface crack is much steeper than that of through crack. This figure also indicated that the J_r curve of surface crack is sensitive to the crack depth. Fig.3 gave the same results. From Fig.2 and Fig.3, we may concluded that the J_r curve of surface is actually related to crack and structural geometries (crack depth, thickness of plate, etc.) the results given by white et al. [7] in bending experiment also showed the same conclusion.

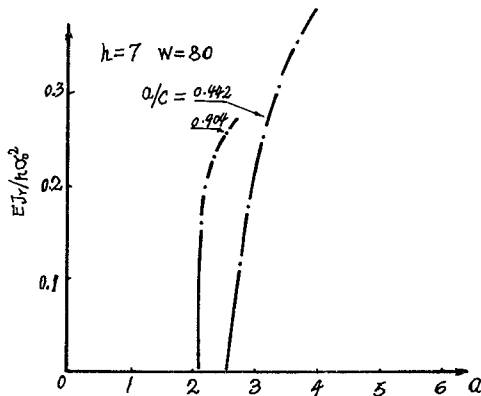


Fig.2 J_r curves of 7mm thickness specimens

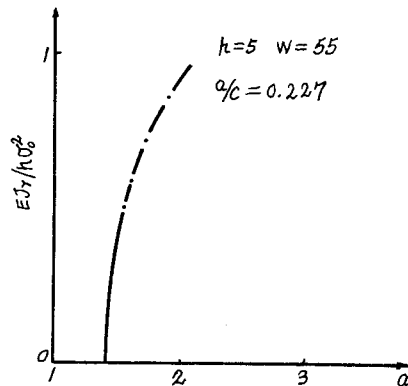


Fig.3 J_r curves of 5mm thickness specimens

In order to investigate the effects of the crack length on the J_r curve, we analysed experimental results of surface crack specimens made by electrosparking. The results were given in Fig.4 and Fig.5. In these figures, we may concluded that the effects of the crack length is weaker than those of crack depth. therefore, in our calculations, we assumed that the total length of surface crack was unvariation during the crack growth. In fact, the plastic deformation is larger at the adjacent location near the free surface, for a flat surface crack, so that the growth of $2c$ is small. If we assumed the growth of surface crack was similar growth, that is, $a/c = a_0/c_0$, where a_0 and c_0 are initial crack sizes, the corresponding J curve was somewhat lower than the J_r curve which was calculated under the condition $c=c_0$. (Fig.5[8]). The difference between them was small.

The validity of the J_r curve is whether it satisfies the J-dominance condition. In high constraint field, it is recommended that the ligament of crack L satisfy the condition $L \sigma_f / J > 25$, where σ_f is the fracture stress. According to our experimental results, $L \sigma_f / J = 192$, 104.26 and 71.7 for 7mm, 5mm and 3mm thickness specimen, respectively. Evidently, our results are all within the J-dominance range.

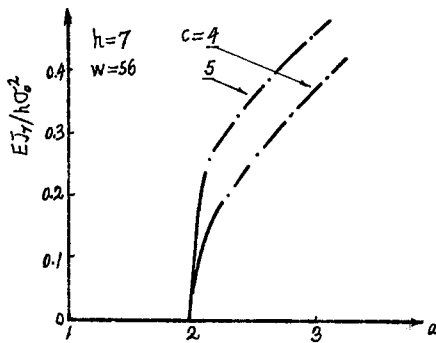


Fig.4 J_Y curves of 7mm thickness specimens made by electrosparking

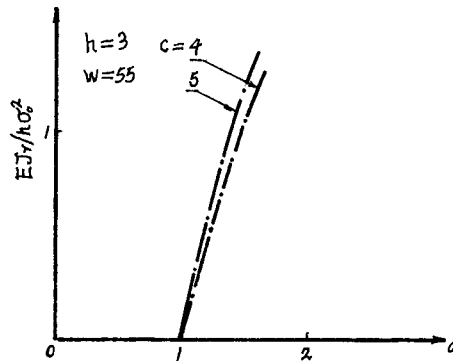


Fig.5 J_Y curves of 3mm thickness specimens made by electrosparking

CONCLUSIONS

In this paper, a rather direct method to construct J_Y curve of surface crack was developed. This method has the advantages of simplicity and economy. Our results showed that the slope of the J_Y curve of surface crack is much steeper, and the J_Y curve is geometry-dependence. The results also indicated the effects of the crack length is weaker than those of the crack depth.

ACKNOWLEDGEMENTS

The financial support of NSFC through Grant No. 1860386 is very appreciate.

REFERENCES

- [1] D. M. Parks, "The Inelastic Line-Spring Estimates of Elastic-Plastic Fracture Mechanics Parameters for Surface-Cracked Plates and Shells", ASME Journal of Pressure Vessel Technology, 103, 246-254 (1981).
- [2] D. M. Parks, and C.S. White, "Elastic-Plastic Line-Spring Finite Elements for Surface-Cracked Plates and Shells", ASME Journal of Pressure Vessel Technology, 104, 287-292 (1982).
- [3] F. Delale and F. Erdogan, "Line-Spring Model for Surface Cracks in a Reissner Plate", Int. J. Engin. Sci. 19, 1331-1340 (1981).
- [4] Ning Jie, "A Nonlinear Line-Spring Model for Surface Crack Analysis with Particular Reference to Reissner Effect", Acta Mechanica Sinica, 17, 435-444 (1985).
- [5] Ning Jie, "The Elastic-Plastic Fracture Mechanics Analysis for Surface Cracked Spherical Pressure Vessel", Proc. Int. Conf. on Computational Mechanics, Tokyo, v213-v218 (1986).
- [6] Lu, Y.C. and Ning Jie, "The Nonlinear Line-Spring Model for Surface Crack Analysis with Particular Reference to work Hardening Materials", Proc. ICF Int. Symposium on Fract. Mech., Beijing, 121-128 (1983).
- [7] C.S. White, R.O. Ritchie and D.M. Parks, "Ductile Growth of Part- Through Surface Cracks: Experiment and Analysis", Elastic-Plastic Fracture, ASTM STP 803, 384-409 (1983).
- [8] Ning Jie, "Jr Curves of Power Hardening Materials", Changsha Institute of Technology Tencnical Report, 83-1006 (1983).
- [9] Ning Jie and Gao Qing, "The Elastic-Plastic Failure Assessment Diagram of Surface Cracked Structure", Trans. of SMIRT-9, Vol.G, 183-188(1987).
- [10] J .R. Rice, "The Line Spring Model for Surface Flaws", The Surface Crack: Physical Problem and Computational solutions, (1972).
- [11] V. Kumar, M. D. German and C. F. shih, "An Engineering Approach for Elastic-Plastic Fracture Analysis", EPRI Topical Report NP-1931, Research Project 1237-1, (1981).
- [12] H. Tada, P. C. Paris and G. R. Irwin, The Stress Analysis of Cracks Handbook, Del Research Corporation, Hellertown (1973).

