Influence of the vertical motion component on the plastic collapse of earthquake excited tanks

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1 INTRODUCTION

Tanks damaged by earthquakes show different modes of buckling of the tank wall: diamond shaped buckles near the base, a bulge near the base or at a change of the wall thickness ("elephant footing") and, rather rarely, buckles near the top (Manos & Clough 1985). In addition to these modes of quasi-static buckling dynamic instabilities may appear (Chiba, Tani & Yamaki 1987) which are treatable by numerical methods as shown in (Rammerstorfer & Auli 1986).

The kind of damage appearing in an actual situation depends on the configuration of the tank, the liquid content, the foundation and on the characteristics of the earthquake. In this context the vertical component of the earthquake excitation in combination with the well studied effects caused by the horizontal component plays an important role. Furthermore, the nonlinearities due to plasticity and large deformations as well as the deformability of the foundation must not be ignored.

The dynamic analysis of a flexible tank filled with an incompressible, irrotational fluid and resting on an elastic foundation leads to a boundary value problem of linear potential theory with dynamic Neumann conditions in combination with the equations of motion of the elastic shell and the impedance functions of the ground (Fischer, Seeber & Rammerstorfer 1987).

The displacement potential which satisfies the potential equation and the dynamic boundary conditions equalizing the movements of the liquid and of the wetted boundary at right angles to the deformed spatial shape yields a dynamic liquid pressure which represents an additional term of damping and inertia in the equation of motion of the shell and of the foundation, respectively.

This leads to time and space dependent equations of vibration which are transformed into a discrete multi-degree-of-freedom system by a generalized coordinate approach calculating the generalized mass, stiffness and damping matrices as well as the load vector. The nonlinear eigenvalue problem obtained this way, where the impedance function of the ground is frequency dependent, is solved by the complex frequency response method giving natural frequencies and damping ratios as results.

The solution for vertically excited, axisymmetric vibrations of circular cylindrical tanks is shown in (Fischer & Seeber 1987). The results presented there demonstrate that the natural frequencies are not

very different from those calculated for a rigid foundation, whereas damping ratios are considerably increased by an elastic soil.

2 AN ENGINEERING APPROACH

The analytical model briefly discussed above requires a numerical solution procedure. In order to enable an engineer to obtain solutions very quickly by simply using a few diagrams and formulas an engineering approach is described below. The procedure is based on parametric studies performed for 14 typical steel tanks used to store light oil products and having different height vs. radius ratios (Holl 1987).

2.1 Horizontal excitation

In (Fischer & Rammerstorfer 1982) the fundamental concept of an iterative procedure is described for calculating the maximum dynamic pressure resulting from the common vibration of the elastic shell and the liquid. Based on this concept in (Holl 1987) a numerical algorithm has been developed using the "added mass concept".

Starting with an assumed vibration mode ${}^1f(\xi)$, circumferential wave number n = 1, the first guess (i = 1) of the dynamic pressure amplitude normalized to a horizontal excitation of 1g can be calculated from the solution of the potential equation

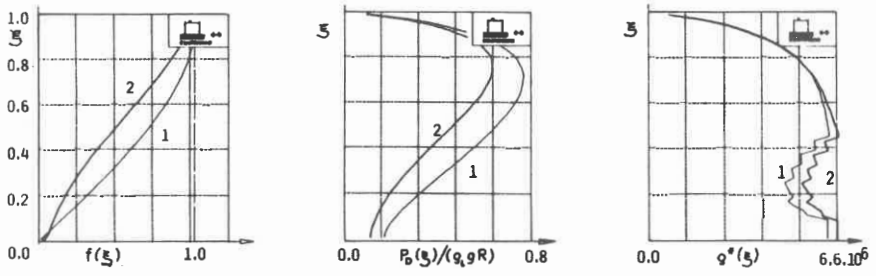
$$ {}^iP_D(\xi) = 2R\varrho_L g \sum_{j=1,3,..}^{\infty} (I_1(\lambda_j)/(\lambda_j I_1'(\lambda_j)) \left(\int_0^1 {}^if(\xi)\cos(j\tfrac{\pi}{2}\xi)\,d\xi\right)\cos(j\tfrac{\pi}{2}\xi)) \quad (1), $$

$$ \xi = x/H, \quad \lambda_j = j/(2\alpha), \quad \alpha = H/R. $$

Following the added mass concept an effective mass density of the tank wall at the axial coordinate ξ with the wall thickness $t(\xi)$ can be calculated:

$$ {}^i\varrho^*(\xi) = {}^iP_D(\xi)/(2gt(\xi)\,{}^if(\xi)) \quad (2). $$

For this modified shell the natural frequencies and mode shapes $f(\xi)$ can be computed by numerical methods such as used in the program KSHEL (Kalnins 1981). This mode shape ${}^{i+1}f$ can now be used in the (i+1)th iteration. In order to assure convergence of the iteration underrelaxation is necessary. The singularity of $\varrho^*(\xi)$ at $\xi = 0$ has been treated by a procedure based on the Rayleigh quotient. Initial as well as converged solutions are shown in fig.1 for a typical example. The discontinuities in $\varrho^*(\xi)$ results from the sudden changes of the thickness of the tank wall.



1 .. first guess, 2 .. converged solution, R=8m, H=32m, t=6-10.7mm

Fig.1: Mode shape, dynamic pressure amplitude and effective mass density

From the parametric study described above the diagrams shown in fig.2 are obtained by integration of the calculated pressure distributions which are used in the engineering approach. M_{SL}, M_B and M_D are the effective mass with respect to sloshing, to rigid tank motion and to interaction vibrations, respectively, which, multiplied by the corresponding maximum accelerations A_{SL}, A_B, A_D, lead to the maximum resultant horizontal earthquake loads. Multiplication with the related heights H_{SL}, H_B, H_D gives the individual contributions to the resultant overturning moment from which the maximum axial compressive membrane force used for buckling analysis is obtainable (Fischer & Rammerstorfer 1982).

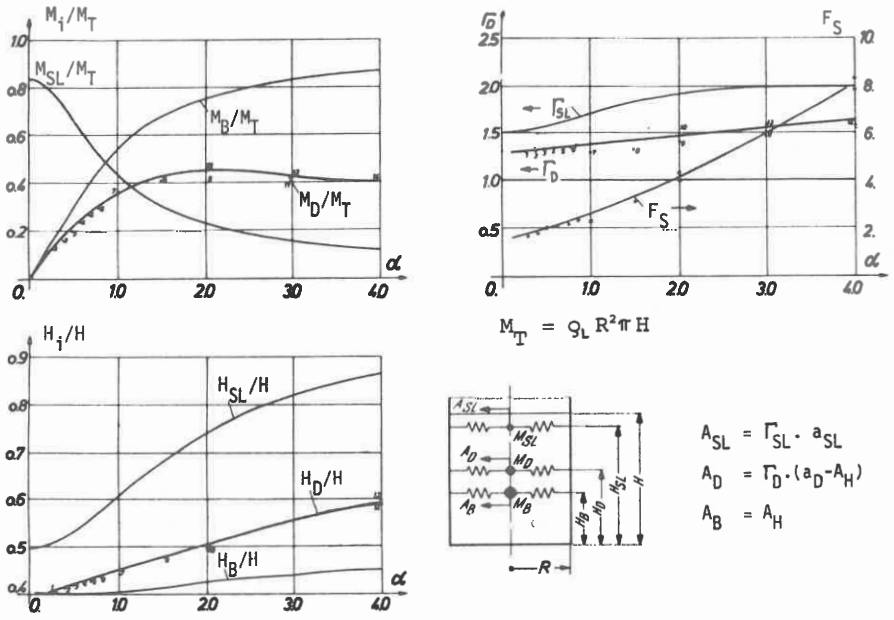


Fig.2: Effective masses, mode participation factors and frequency factor

In addition to the design response spectra the calculation of the relevant accelerations requires the knowledge of the natural frequencies for sloshing and interaction vibrations as well as the relevant mode participation factors.

The fundamental sloshing frequency is given by:

$$\omega_{SL} = \sqrt{1.84\ g\ \tanh(1.84\ \alpha)/R} \qquad [rad/s] \qquad (3),$$

and the fundamental natural frequency for the common vibration of the elastic tank wall and the liquid content can be approximated by

$$\omega_D = \sqrt{E t_{\frac{1}{3}}/(\varrho_L H)}\ \pi/(F_S(\alpha) R), \quad t_{\frac{1}{3}} = t(\xi = 1/3) \qquad (4),$$

with $F_S(\alpha)$ taken from fig. 2. The mode participation factors Γ_{SL}, Γ_D can be obtained from the same diagram.

A modified SRSS superposition (Fischer & Rammerstorfer 1982) should be applied in calculating the maximum resultant overturning moment.

2.2 Vertical excitation

Results of parametric studies based on the theory outlined in chapter 1 allow the conclusion that the horizontal earthquake excitation can, with the exception of the damping behaviour, be approximated by a mathematical model which does not take into account the deformability of the foundation. This means that the maximum pressure due to vertical excitation can be calculated by the following formula derived from the relations given for rigidly based tanks in (Luft 1984 and Fischer, Seeber & Rammerstorfer 1987):

$$p_V(\xi) = p_{BV} + p_{DV} = 0.815 H \varrho_L \cos(\tfrac{\pi}{2}\xi)(a_{DV} + A_V) + H\varrho_L(1 - \xi)A_V \quad (4).$$

p_{BV} results from the vertical excitation of the rigid tank and p_{DV} is the contribution due to the deformability of the elastic tank wall. A_V is the maximum vertical acceleration and a_{DV} is the spectral acceleration taken from the response spectrum for the vertical earthquake component at the fundamental natural frequency for axisymmetric interaction vibrations which can be estimated by

$$\omega_{DV} = \sqrt{2\pi E\, I_1(\lambda_1) t_{\frac{1}{3}} / (\varrho_L H(1-\nu^2) I_0(\lambda_1))}\ /(2R) \qquad (5)$$

and with a damping ratio as shown in fig.3 for different shear wave velocities of the soil and different radius to thickness ratios of the tank wall.

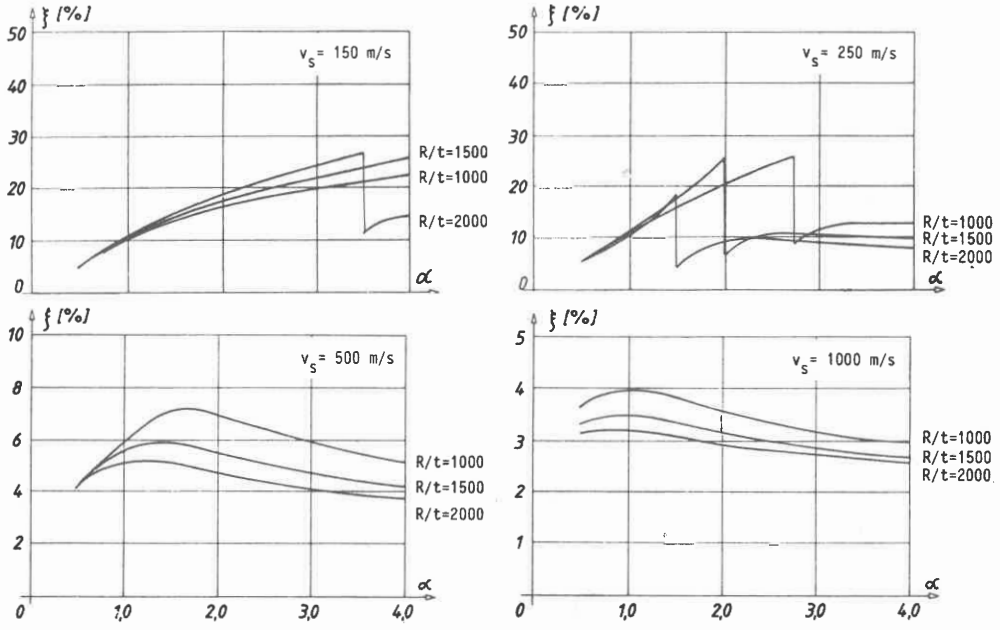
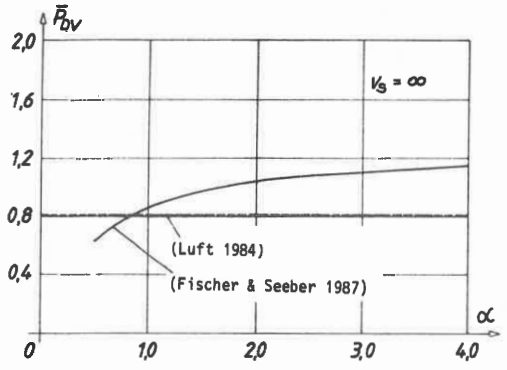


Fig.3: Damping ratio of the interaction vibration; vertical excitation

The contribution p_{DV} in eqn.(4) is estimated according to (Luft 1984) where the clamping condition of the tank wall at the bottom is neglected. Taking into account the correct boundary conditions the procedure described in chapter 1 leads to more accurate results as shown in fig. 4.



$$\bar{P}_{DV} = p_{DV}(\zeta=0)/(H\varrho_L g)$$

Fig.4: Maximum dynamically activated pressure $p_{DV}(\zeta=0)$ due to vertical acceleration of $(a_{DV} + A_V) = 1g$.

3 COMBINATION OF THE HORIZONTAL AND THE VERTICAL EARTHQUAKE COMPONENTS

Figure 5 shows three different possibilities of superposition of the contributions p_{stat}, p_H, p_V, the static pressure, the pressure due to the horizontal and the pressure due to the vertical earthquake component, respectively, which are most critical with respect to the individual buckling modes discussed in chapter 1.

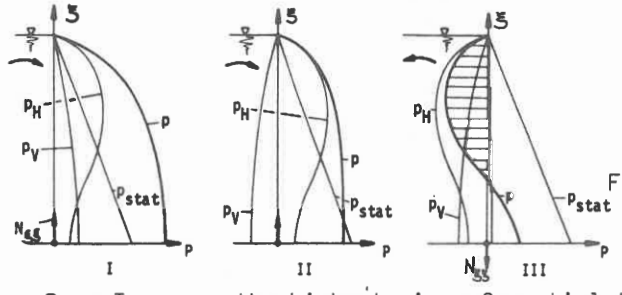


Fig. 5: Schematic presentation of different possible superpositions of the maximum pressure contributions

Case I causes the highest circumferential tensile stresses and leads, especially in the case of broad tanks, to plastic buckling as described in (Dorninger, Fischer, Rammerstorfer & Seeber 1986). This plastic collapse problem can be analyzed by the finite element method using an axisymmetric model. Fig. 6 shows the results obtained for a tank with a diameter of 40 m, $\alpha = 1$, assuming an earthquake characterized by the response spectra of the Friuli earthquake (Fritze & Grossmayer 1981). The bulge formation typical for "elephant footing" has been computed as the consequence of the plastification caused by high circumferential tensile stresses, mainly caused by the vertical earthquake component, in combination with the axial compression force due to the overturning moment, caused by the horizontal earthquake excitation.

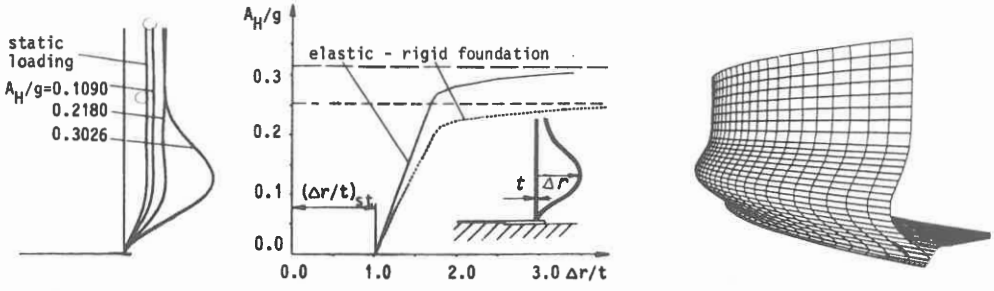


Fig. 6: "Elephant footing" treated as a plastic collapse problem

Case II is most critical with respect to "shell crippling", i.e. the elastic buckling due to the axial compression forces. The stabilizing effect of the internal pressure is reduced by subtracting the dynamic pressure caused by the vertical earthquake component. This mode of buckling can be treated in analogy to axially loaded cylinders as outlined in (Fischer & Rammerstorfer 1982).

Case III may lead to regions of low pressure mainly near the top of the tank where the wall is rather thin, and buckling due to external pressure as well as cavitation may occur. The treatment of this failure mode requires the knowledge of the exact pressure distribution over the tank's height as computable by the iterative procedure outlined in chapter 2.1.

It depends on the characteristics of the earthquake, the tank, the liquid content and the stiffness of the soil which failure mode is relevant. A number of stability analyses for tanks in operation show that broad tanks rather tend to plastic collapse and eventually to buckling caused by external pressure, whereas tall tanks preferably buckle in the elastic range showing diamond shaped buckles near the base.

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