IMPACT OF SOIL-STRUCTURE INTERACTION ON THE PROBABILISTIC FREQUENCY VARIATION OF CONCRETE STRUCTURES

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SUMMARY

Earthquake response of equipment in nuclear power plants is characterized by floor response spectra. Since these spectra peak at the natural frequencies of the structure, it is important, both from safety and cost standpoints, to determine the degree of the expected variability of the calculated structural frequencies. The present paper extends a previous work on the variability of the natural frequencies of structures due to the variations of concrete properties and presents a rigorous approach to evaluate frequency variations based on the probability distributions of both the structural and soil parameters and how they jointly determine the distributions of the natural frequencies. The impact of soil properties on the structural frequencies stems from the fact that soil-structure interaction is an important consideration for massive structures.

Although adequate data on the variability of concrete properties exists, very little is known concerning the exact probability distributions of those soil properties that determine the foundation impedance coefficients. To illustrate the methodology used, it is assumed that the soil-structure interaction coefficients are normally distributed. As data on soil property variations become available, the methodology developed here is capable of readily incorporating such data.

The methodology used and the results obtained are discussed. With the proper choice of coordinates, the simultaneous random variations of both the structural properties and the interaction coefficients can be incorporated in the eigenvalue problem. The key methodology problem is to obtain the probability distribution of eigenvalues of matrices with random variable elements. Since no analytic relation exists between the eigenvalues and the elements, a numerical procedure had to be designed. Since accuracy is most important at the extreme high and low fractile values, the Monte Carlo simulation technique was eliminated. It was found that the desired accuracy can be best achieved by splitting the joint variation into two parts: the marginal distribution of soil variations and the conditional distribution of structural variations at specific soil fractiles. Then after calculating the actual eigenvalues at judiciously selected paired values of soil and structure parameters, this information is recombined to obtain the desired cumulative distribution of natural frequencies.

Explicitly, $\omega_{n,p}$ (the pth fractile of $\tilde{\omega}_n$), can be found by an iterative procedure based on

$$p = P[\omega_n(\tilde{y}, \tilde{z}) \leqslant \omega_{n,p}] \approx \sum_{k=q}^{1-q} P[\omega_n(\tilde{y}, z_k) \leqslant \omega_{n,p}] \cdot P[z_{k-q} \leqslant \tilde{z} \leqslant z_{k+q}]$$

where $\tilde{y}$ and $\tilde{z}$ are random variables of actual-to-calculated structural and soil parameters, respectively.

The study indicates that, unlike fixed base structures, the frequency variation of structure-soil systems for any given exceedance level are not the same for all frequencies. Furthermore, the assumed variability of the impedance coefficients may increase or decrease the over-all frequency variation fractiles from those of the fixed base structure. As expected, the soil variability impacts only on the interacted frequencies. These frequencies are easily identified from the conditional cumulative distributions and other relationships presented in the paper.

1. Introduction

The variability of concrete properties away from their specified values is well known. The wide scatter of soil data is also commonly recognized. Thus, the variability of the frequencies of soil-structure systems away from their calculated values should be expected. These variations are usually accounted for by assigning limiting values to both the concrete and soil properties and conducting parametric studies using deterministic methods. The values resulting from this approach are often too extreme and lack a measure of how likely they are to occur. A more appropriate approach would be the use of probabilistic concepts. In an earlier paper [1] it was shown that the probability distributions of the natural frequencies of concrete structures can be derived from a) the recommendations of ACI 214-65 for the evaluation of compression test results of field concrete [2], b) data on variations of concrete modulus of elasticity about a least squares line [3] and c) data on variations of actual-to-calculated deflection ratios [4]. In that study only fixed-base continuous beams and lumped mass systems were adequately examined. The present paper extends that study to include soil-based structures and compares the results to those of the fixed-base structures. Lacking actual distribution data on soil properties a normal distribution of the impedance coefficients is assumed in the present study. However, the generality of the method developed is such that more appropriate distributions can be used when such data become available. Nevertheless, it is not expected that other distributions of soil data would alter the major findings of the present study.

2. Effects of Soil-Structure Interaction

One of the methods to account for the effects of soil deformations on the frequency and response of structures founded on soil is the use of impedance coefficients to represent the supporting medium [5]. The choice of the usual physical coordinate system for such a lumped parameter model would statically couple and dynamically uncouple the system. That is, the mass matrix would be diagonal. However, since different variabilities are associated with the structure stiffness and the soil-structure interaction parameters, it is desirable to have a coordinate system that statically uncouples the structure from the interaction parameters. Such a coordinate system is shown in Fig. 1, where x_j = horizontal displacement of the superstructure at the jth floor relative to the foundation excluding rotations; ν_g = free-field surface displacement; ν_o = translation of the foundation relative to the free-field motion; ϕ = rotation of the foundation; h_j = height of the jth story above the foundation; ν_j^t = total horizontal displacement of the jth mass with respect to a fixed vertical axis, i.e., $\nu_j^t = \nu_g + \nu_o + h_j\phi + \nu_j$; m_j = mass of the jth floor; m_o = mass of the foundation; and I_t = sum of the centroidal moments of inertia of the m+1 masses. The solution coordinates are given by the vector $(x_1, x_2, \ldots x_n, \nu_o, \phi)^T$.

The choice of the above coordinate system results in the following mass and stiffness matrices:

$$M = \left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right] \quad \text{and} \quad K = \left[\begin{array}{c|c} K_c & 0 \\ \hline 0 & K_s \end{array}\right]$$

where $[K_c]$ is the stiffness matrix of the fixed-base superstructure and $[K_s]$ is the matrix of impedance coefficients. Thus the structure is statically uncoupled from the foundation.

In the subsequent discussion it is assumed that the variability of the mass matrix is negligible and that the important variations in the system frequencies come from the stiffness properties of the superstructure and the foundation impedance coefficients. Two cases will be studied. In the first case it will be assumed that the impedance coefficients are known precisely. The second case will deal with the more useful problem where both the structure stiffness and impedance coefficients have different variabilities.

3. Effects of Soil-Structure Interaction - Soil Parameters Constant

To evaluate the interaction effects when the soil parameters are fixed, the submatrix $[K_s]$ associated with the interaction elements will be held constant. The variations in stiffness, $\tilde{y}$, are applied to the $[K_c]$ submatrix, and the resulting random submatrix is:

$$[\tilde{K}_c] = \tilde{y}\ [K_c]$$

Since the natural frequencies of the original structure depend on the characteristic roots of $[M]^{-1} [K]$, the distributions of probabilisitc frequencies will depend on the distribution of the random characteristic roots of $[M]^{-1} [\tilde{K}]$. This is a more complicated problem than the fixed base system, since only a submatrix of [K] is being made random and, correspondingly, only the left-most columns of $[M]^{-1} [K]$. Before delving into the solution of this problem it is useful to show that the characteristic roots of such a dynamical matrix, $D(y) = M^{-1} K(y)$, are monotone increasing functions of the scalar factor, y. To do this, let ω_n be the n^{th} natural frequency of a structure with dynamical matrix D. Further, let the K matrix be varied by multiplying its upper left submatrix, $[K_s]$, by y. Then K(y), and hence D(y), are matrix-valued functions of y, and $\omega_n(y)$ is a scalar-valued function of y corresponding to $\lambda_n(y)$ and $x_n(y)$, the n^{th} eigenvalue and eigenvector of D(y). By the definition of $\lambda(y)$ and $x(y)$,

$$D(y)\ x(y) = \lambda(y)\ x(y)$$

Taking derivatives with respect to y and premultiplying by x^T yields

$$x^T D' x + x^T D x' = x^T \lambda' x + x^T \lambda x'$$

Since D is symmetric, $x^T D = x^T \lambda$, and the terms involving x' may be cancelled to yield:

$$x^T D' x = x^T \lambda' x$$

Solving for the scalar quantity, $\lambda' = (x^T D' x)/(x^T x)$. By inspection, D' is the partitioned matrix $\left[M^{-1}.K_c \middle| 0\right]$, so that $x^T D' x$ is positive, as is $x^T x$. Hence λ' is always positive, or equivalently, $\lambda(y)$ is a strictly monotone increasing function of y.

Since each $\omega_n(y)$ is proportional to the square root of $\lambda_n(y)$, this assures the monotone increasing nature of $\omega(y)$. While the analytic form of $\omega_n(\cdot)$ is not known, a specific value of $\omega_n(y)$ can be numerically determined for any value of y. This leads directly to the method of finding the fractile values $\omega_{n,p}$. By the definition of y_p, $P\ [\tilde{y} \leq y_p] = p$. But since $\omega_n(\cdot)$ is strictly monotone increasing, $\tilde{y} \leq y_p$ if and only if $\omega_n(\tilde{y}) \leq \omega_n(y_p)$. Since $\omega_n(\tilde{y}) = \tilde{\omega}_n$, then $P\ [\tilde{\omega}_n \leq \omega_n(y_p)] = p$ and hence $\omega_n(y_p) = \omega_{n,p}$.

This property is not only useful for determining fractiles and confidence intervals when only $[K_c]$ is random, but is also the key to calculating fractiles of ω_n when $[K_s]$ also varies. For a numerical example of this procedure, consider the lumped mass representation of an auxiliary building in a nuclear power plant presented in Fig. 2.

The corresponding stiffness submatrices are:

$$[K_c] = \begin{bmatrix} 76.09 & -37.08 & 0.47 & 1.4 & 0.22 & 0.25 \\ -37.08 & 76.09 & -42.09 & 2.17 & 0.34 & 0.39 \\ 0.47 & -42.09 & 78.2 & -37.7 & 0.37 & 0.42 \\ 1.4 & 2.17 & -37.7 & 45.6 & -11.8 & -0.52 \\ 0.22 & 0.34 & 0.37 & -11.8 & 17.0 & -6.3 \\ 0.25 & 0.39 & 0.42 & -0.52 & -6.3 & 3.77 \end{bmatrix}$$

and

$$[K_s] = \begin{bmatrix} 6.75 \times 10^6 & 0 \\ 0 & 45.2.10^9 \end{bmatrix}$$

The units used are kip, ft, and radians.

The mass matrix in the adopted coordinates is:

$$[M] = \begin{bmatrix} 1421. & 0. & 0. & 0. & 0. & 0. & 1421. & 28420. \\ 0. & 1423. & 0. & 0. & 0. & 0. & 1423. & 56920. \\ 0. & 0. & 1363. & 0. & 0. & 0. & 1363. & 81780. \\ 0. & 0. & 0. & 1089. & 0. & 0. & 1089. & 87120. \\ 0. & 0. & 0. & 0. & 550. & 0. & 550. & 55000. \\ 0. & 0. & 0. & 0. & 0. & 733. & 733. & 87960. \\ 1421. & 1423. & 1363. & 1089. & 550. & 733. & 8107. & 397200. \\ 28420. & 56920. & 81780. & 87120. & 55000. & 87960. & 397200. & 38,737,470. \end{bmatrix}$$

The exact fractile values of the eight natural frequencies for selected values of p are given in Table I, followed by their percentage variations with respect to the nominal frequency values in Table II.

Considering each column separately, the most notable feature is that there is no longer the constant variation across all frequencies which is found in fixed-base systems. Further, for each frequency, the fractile limits are tightest for the lowest frequencies, and grow successively larger at the higher frequencies. However, at no time do they exceed the fractile limits established for the fixed-based system. In fact, at the lower frequencies, the limits are significantly less than were reported in [1] for fixed-base structures. This is primarily because only the $[K_c]$ portion of the stiffness matrix is subject to random variation, with $[K_s]$ being held constant.

4. Effects of Soil-Structure Interaction - Soil Parameters Variable

The previous section proceeded as though the soil stiffness parameters were known precisely. This is seldom the case. A more comprehensive methodology is required for simultaneous random variations in both the structure and the soil properties.

As before, let

$\tilde{y}$ = actual-to-calculated ratio of structural parameters

$\tilde{z}$ = actual-to-calculated ratio of soil parameters

$\tilde{\omega}_n = \omega_n(\tilde{y},\tilde{z})$ = nth natural frequency of the structure expressed as a function of $\tilde{y}$ and $\tilde{z}$.

While the $\omega_n(y,z)$ functions are not known in any analytic form, a numerical value of ω_n can be calculated for specific values of y and z. This, combined with the monotone nature of ω_n with respect to y, makes it possible to establish the expected value, fractiles, and confidence intervals of each $\tilde{\omega}_n$ directly from the probability distributions of $\tilde{y}$ and $\tilde{z}$.

Specifically, to find the pth fractile of $\tilde{\omega}_n$, we are interested in finding a value $\omega_{n,p}$ such that

$$P\;[\omega_n(\tilde{y},\tilde{z}) \leq \omega_{n,p}] = p.$$

This probability may be calculated by conditioning it on specific values of $\tilde{z}$ and integrating with respect to z:

$$P\;[\omega_n(\tilde{y},\tilde{z}) \leq \omega_{n,p}] = \int_{-\infty}^{+\infty} P\;[\omega_n(\tilde{y},z) \leq \omega_{n,p}] \cdot p(z)dz$$

where p(z) is the probability density function of z. While this integration cannot be accomplished analytically, an efficient numerical integration may be achieved by dividing the z-range into equally likely subintervals (the number of divisions being dictated by the accuracy desired) and evaluating the conditional probability at the fractile medians. For example, if N subintervals are chosen, and $\frac{1}{2N}$ is represented by q, then

$$\int_{-\infty}^{+\infty} P\;[\omega_n(\tilde{y},z) \leq \omega_{n,p}] \cdot p(z)dz$$

$$\approx \sum_{k=q}^{1-q} P\;[\omega_n(\tilde{y},z_k) \leq \omega_{n,p}] \cdot P\;[z_{k-q} \leq \tilde{z} \leq z_{k+q}$$

$$= \frac{1}{N} \sum_{k=q}^{1-q} P\;[\omega_n(\tilde{y},z_k) \leq \omega_{n,p}]$$

Typically values of N of at least 5 but not more than 10 produce sufficiently accurate results.

For the example problem Table III summarizes the results of such calculations. Note that the additional source of variability causes a widening of the probability distribution at all fractile values. For example, for ω_1, the 0.01 fractile is 13 percent below the nominal value (vs. -9.1 percent for structural variations only) and the 0.99 fractile is +10.8 percent above (vs. +5.2 percent previously). These numerical results are shown graphically in Fig. 3 where the fractile variations are plotted with respect to mode number.

Fig. 4 represents graphically the results for p = 0.99 and 0.01 of four separate cases:

1. Fixed based structure
2. Constant soil parameters σ = 0.0
3. Variable soil parameter σ = 0.15
4. Variable soil parameter σ = 0.50

The salient points from these curves are the following.

1. Soil structure interaction does not necessarily increase the variability in the system frequencies as compared to the fixed base frequencies. In fact, it may serve to reduce this variation for certain frequencies.

2. As the variability in the soil increases from $\sigma = 0.0$ to $\sigma = 0.5$, the frequency variation increases, but not by the same amount for all frequencies.

In order to test the generality of these observations, another structure of different characteristics was examined. This is discussed in the next section.

5. Application to a Containment Building

The auxiliary building studied provided a useful test case for developing and explaining the methodology. In this section, the full method is applied to a containment structure. The lumped-mass representation is shown in Fig. 5. The summary display of frequency fractiles as percentage deviations from their nominal values are shown graphically in Fig. 6.

The results shown in Fig. 6 differ from those of Fig. 3 in that the gradual and uniform widening from mode to mode does not hold in this case. To explain this phenomenon one could consider the conditional cumulative distributions. Nonetheless it is obvious from Figs. 3 and 6 that the interacted frequencies are the ones that show a significant sensitivity to soil parameter variations. Thus, for the auxiliary building the interacted frequencies are identified as the first and second modes with some minor impact of interaction in the third and fourth modes. For the containment building, the interacted frequencies are primarily the second, first, and fourth modes in that order. This effect is better visualized when the first three cases, similar to the auxiliary building, are plotted in Fig. 7.

6. Summary and Conclusions

Only the frequency aspect of the soil-structure interaction problem has been considered here; the problem of the amplitude of the peak response was not considered. Many assumptions were made either because pertinent data were not available or because the influence of the parameter in question was deemed negligible. The methods developed can, of course, be refined and extended as additional data become available or are considered to be important. Numerical results illustrated the use of the methodology in two specific instances.

Based on analytic results and numerical calculations, the principal findings of this and the previous study [1] are:

1. For continuous or lumped-mass fixed base structures, if it is assumed that the stiffness probability distribution function is the same throughout the structure, the variation of frequency is independent of the structural properties, for any probability of exceedance level. For this study, the appropriate probability distribution function was developed from data on the variability of deflections of beams.
2. The frequency variations for any given exceedance level are the same for all modes for fixed-based structures in contrast to systems that include soil-structure interaction. When soil-structure interaction is considered and the soil parameters are known, then the frequency variability is less than that in fixed base structures. Further, the amount of decrease is nonuniform, being greater for some modes than others.
3. When the soil parameters are also variable and soil-structure interaction is considered, this variability may either increase or decrease the overall frequency variation fractiles from those of the corresponding fixed base structure. As expected, the soil variability impacts only on the interacted frequencies. These

frequencies can be identified from conditional probability cumulative distributions, or from figures like Fig. 7.

In this study, actual data was used for concrete properties, but the soil stiffness properties (impedance coefficients) were assumed to be normally distributed. It is most desirable that actual soil property data be accumulated to increase the value of the numerical results.

The next major step in this development could be a study of the probabilistic responses of structures to earthquakes, including the inherent random nature of the input motion. Since the influence of damping at and close to resonance is important, data on damping of soils, structures, and equipment must also be accumulated and incorporated in any such effort.

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Table I

FREQUENCY FRACTILES WITH SOIL-STRUCTURE
INTERACTION EFFECTS: AUXILIARY BUILDING
(SOIL PARAMETERS CONSTANT)

p:	0.01	0.05	0.10	Nominal Frequency	0.90	0.95	0.99
ω_1	2.924	3.024	3.055	3.220 Hz	3.313	3.338	3.392
ω_2	7.136	7.393	7.506	7.988	8.249	8.322	8.461
ω_3	11.984	12.412	12.650	13.863	14.812	15.162	15.912
ω_4	16.177	17.066	17.446	19.748	21.496	22.153	23.625
ω_5	21.744	23.238	24.076	28.177	31.106	32.261	34.555
ω_6	26.759	28.623	29.682	34.772	38.440	39.869	42.712
ω_7	34.033	36.445	37.791	44.336	49.019	50.847	54.510
ω_8	39.183	41.993	43.573	51.191	56.651	58.797	63.038

Table II

PERCENTAGE VARIATIONS WITH SOIL-STRUCTURE
INTERACTION EFFECTS: AUXILIARY BUILDING
(SOIL PARAMETERS CONSTANT)

p:	0.01	0.05	0.10	0.90	0.95	0.99
ω_1	- 9.2%	- 6.1%	- 5.1%	+ 2.9%	+ 3.7%	+ 5.3%
ω_2	-10.7	- 7.4	- 6.0	+ 3.3	+ 4.2	+ 5.9%
ω_3	-13.6	-10.5	- 8.7	+ 6.8	+ 9.4	+14.8
ω_4	-18.1	-13.6	-11.7	+ 8.9	+12.2	+19.6
ω_5	-22.8	-17.5	-14.5	+10.4	+14.5	+22.6
ω_6	-23.0	-17.7	-14.6	+10.5	+14.7	+22.8
ω_7	-23.2	-17.8	-14.7	+10.7	+14.8	+22.9
ω_8	-23.4	-18.0	-14.8	+10.7	+14.9	+23.0

Table III

PERCENTAGE VARIATIONS WITH SOIL-STRUCTURE
INTERACTION EFFECTS: AUXILIARY BUILDING
(SOIL PARAMETERS VARYING: $\mu = 1.0$, $\sigma = .15$)

p:	0.01	0.05	0.10	0.90	0.95	0.99
ω_1	-13.0%	-10.3%	- 8.8%	+ 6.2%	+ 8.0%	+10.8%
ω_2	-12.8	-10.8	- 9.2	+ 6.3	+ 8.4	+11.0
ω_3	-15.4	-11.7	- 9.5	+ 8.2	+10.6	+15.0
ω_4	-18.5	-14.3	-12.1	+ 9.0	+12.5	+19.7
ω_5	-22.8	-17.4	-14.5	+10.7	+14.7	+22.7
ω_6	-23.1	-17.6	-14.6	+10.7	+14.8	+22.9
ω_7	-23.2	-14.7	-14.7	+10.7	+14.8	+22.9
ω_8	-23.4	-18.0	-14.8	+10.7	+14.9	+23.0

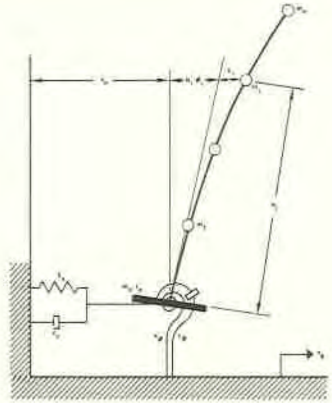


Figure 1

MODEL OF SOIL STRUCTURE SYSTEM

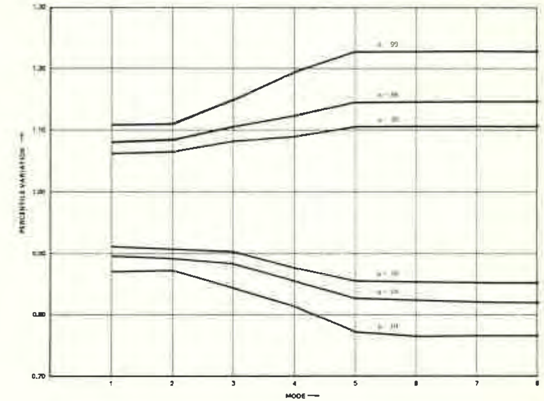


Figure 3

AUXILIARY BUILDING PERCENTILE VARIATIONS BY MODE FREQUENCY

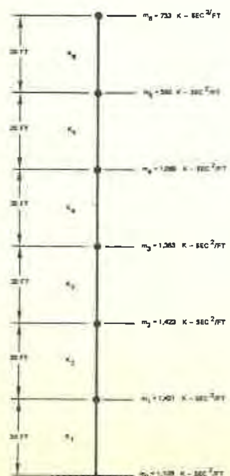


Figure 2

AUXILIARY BUILDING STICK MODEL

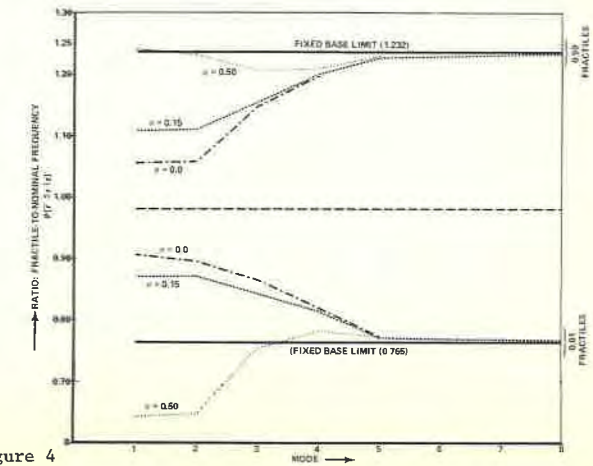


Figure 4

EFFECT OF SOIL VARIABILITY AND MODE LEVEL ON 0.01 AND 0.99 FREQUENCY FRACTILES: AUXILIARY BUILDING

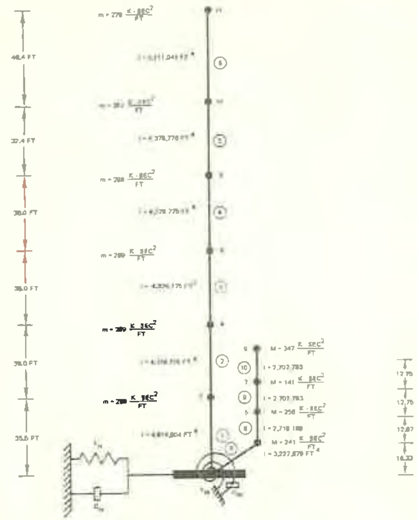
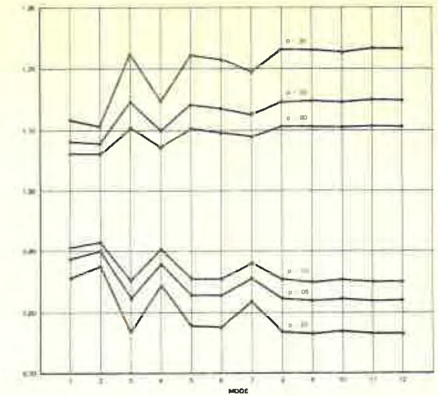


Figure 5 CONTAINMENT - STICK MODEL



Figure

CONTAINMENT STRUCTURE PERCENTILE VARIATIONS BY MODE FREQUENCY

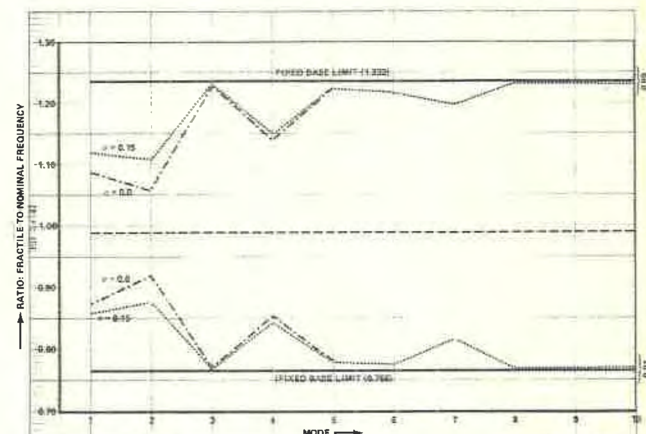


Figure 7 EFFECT OF SOIL VARIABILITY AND MODE LEVEL ON 0.01 AND 0.99 FREQUENCY FRACTILES: CONTAINMENT BUILDING