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RADIAL TENSION INDUCED BY PRESTRESSING FORCES AND MOMENTS

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ABSTRACT

Radial tension due to curved tendons has been known as one of the primary causes of concrete delamination occurred in prestressed concrete containments for nuclear power plants. The radial pressure due to curved tendons generates radial tension in an outer concrete section and radial compression in an inner section. The radial tension caused by curved tendons decreases proportionally as tendons are placed close to the outside surface of the section and becomes its maximum when tendons are located at the center of the section.

However, it is difficult to explain that most incidents of concrete delamination occurred during tendon tensioning or detensioning, or near a construction opening where the radial tension due to curved tendons does not produce the maximum radial tension. On the other hand, significant unbalanced moments may be introduced under above-mentioned circumstances. The mechanism how radial tensions are produced by unbalanced moments has not been clearly understood in nuclear industry.

This paper introduces a classical theory on a curved beam and provides the relationship between radial stresses and moments acting in a curved beam. Finite element analyses are further performed and the results are compared with the theoretic solutions. It is found that unbalanced moments can produce significant radial tension and should be considered for evaluating possible concrete delamination.

INTRODUCTION

Recent incident at Crystal River Unit 3 has brought attention to concrete delamination and its possible causes. The radial tension due to curved tendons is commonly referred as one of primary causes of concrete delamination (Moreadith and Pages, 1983; Stalnaker and Fugler, 1991; Ragunath et al., 2001; Acharya and Menon, 2003). The radial pressure due to curved tendons in prestressed concrete containments induces radial tension in the outer concrete section and radial compression in the inner section. Since the radial tension decreases proportionally as tendon locations move close to the outside surface of the section and becomes its maximum when tendons are located at the center of the section, the radial tension would be minimized with the placement of tendons at the outside surface of the section. The theory of the radial tension due to curved tendons provides a good explanation on the observed concrete crack pattern from delamination. However, it is difficult to explain that most incidents of concrete delamination occurred during tendon tensioning, de-detensioning or construction opening. The radial tension due to curved tendons reaches the maximum when the tensioning is complete but reduces during tensioning or detensioing. On the other hand, significant unbalanced moments may be introduced during tendon tensioning, detensioning or construction opening. Even though Gurfinkel (2000) recognized the importance of unbalanced moment on radial tension in post-tensioned silo structures, the mechanism of radial tension by unbalanced moments has not clearly understood in nuclear industry.

Classical theory of mechanics (Young and Budynas, 2002; Boresi and Schmidt, 2003) indicates that radial stresses can be generated by moments in a curved beam due to the radial components of the fiber stresses in a similar manner to the radial tension due to curved tendons. The radial components of the fiber stresses become tensile when the moment is applied to straighten the beam and become compressive under the moment in the opposite direction. It is important to note that a containment structure can be considered as curved beams in a cylindrical wall in the hoop direction and in a dome in both meridional and hoop directions.

This paper provides a brief background of the theories for radial stresses due to curved tendons and due to moments in a curved beam. The radial stresses and membrane stresses obtained from the theoretic formulas are compared with the results from finite element (FE) analyses. Moments generated by unbalanced moments and by the eccentricity of tendons are considered in this study.

THEORY FOR RADIAL STRESSES

A summary of mathematical formulas for radial stresses due to curved tendons and due to moments in a curved beam is provided. The radial stresses in a nuclear containment can be calculated by summing both radial stresses.

Radial Stresses due to Curved Tendons

Radial tension stresses are induced between tendons and the surrounding concrete when tendons deviate from a straight line. A tendon curving with a radius of R_{ps} pushes against the concrete with a force of P/R_{ps} per unit length. The stress distribution of radial tension in Fig. 1 can be calculated using the following expressions (Acharya and Menon, 2003):

$$p = \frac{P}{R_{ps} w} \qquad \text{Eq. (1)}$$

$$\sigma_{r,out} = \alpha p \qquad \text{Eq. (2)}$$

$$\sigma_{r,in} = (1-\alpha) p \qquad \text{Eq. (3)}$$

where p = radial pressure
P = prestressing force in tendon
R_{ps} = radius of prestressing tendon
w = width of cross section
$\alpha = \dfrac{b - R_{ps}}{b - a}$

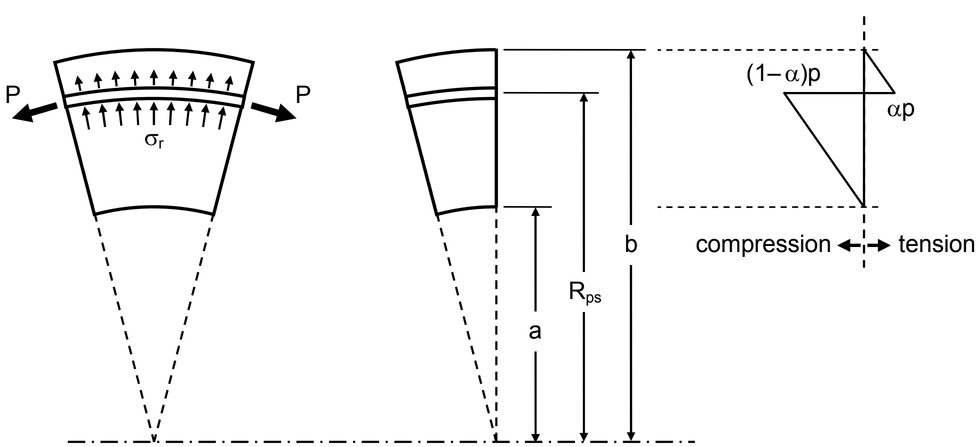


Figure 1. Radial stress due to curved tendon (Acharya and Menon, 2003)

Radial Stresses in a Curved Beam due to Moments

Radial stresses can be induced by moments in a curved beam due to the radial components of the fiber stresses (see Fig. 2). The radial stresses become tensile when the moment is applied to straighten the beam and become compressive under the moment in the opposite direction (Young and Budynas, 2002; Boresi and Schmidt, 2003). The radial stress due to moment, M, in a curved beam can be expressed as follows:

$$\sigma_r = \frac{R_c - e}{wAer} M \left(\int_a^r \frac{dA_1}{r_1} - \frac{A_r}{R_c - e} \right) \qquad \text{Eq. (4)}$$

where R_c = centerline radius

$e = \dfrac{I_c}{R_c A}$ for $R_c / t > 8$

t = depth of cross section = $b - a$

w = width of cross section

A = cross-sectional area

$A_r = \int_a^r dA_1$

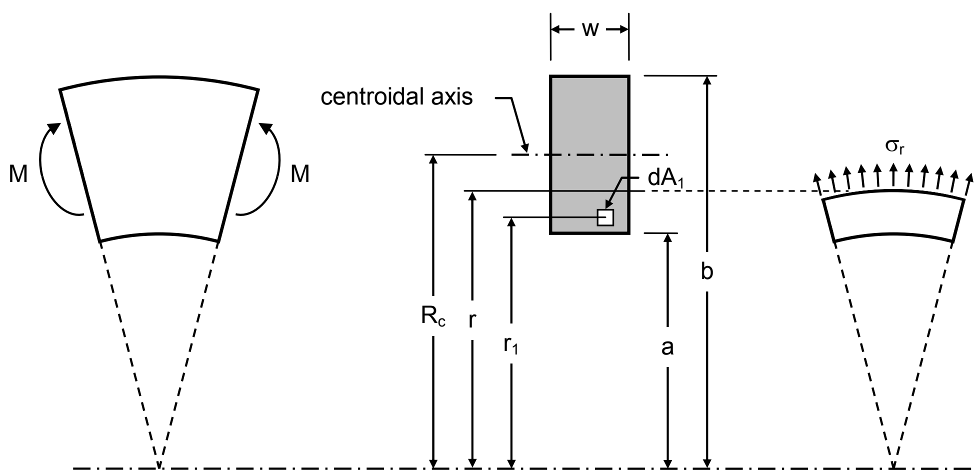


Figure 2. Radial stress in a curved beam due to moment (Young and Budynas, 2002)

EXAMINATION OF RADIAL STRESSES

A cylindrical wall and a dome of a prestressed nuclear containment can be considered as curved beams, which contain curved tendons. Therefore, radial stresses can be generated not only by curved tendons but also by moment. Moments studied in this study include unbalanced moments which may occur due to the tensioning/detensioning sequences or due to a construction opening in prestressed containments and moments due to eccentric tendon loading. Radial stresses are calculated by summing the stresses due to curved tendons and moments using Eq. (1) through Eq. (4). FE analyses are also conducted to get radial stresses and compared with theoretic solutions.

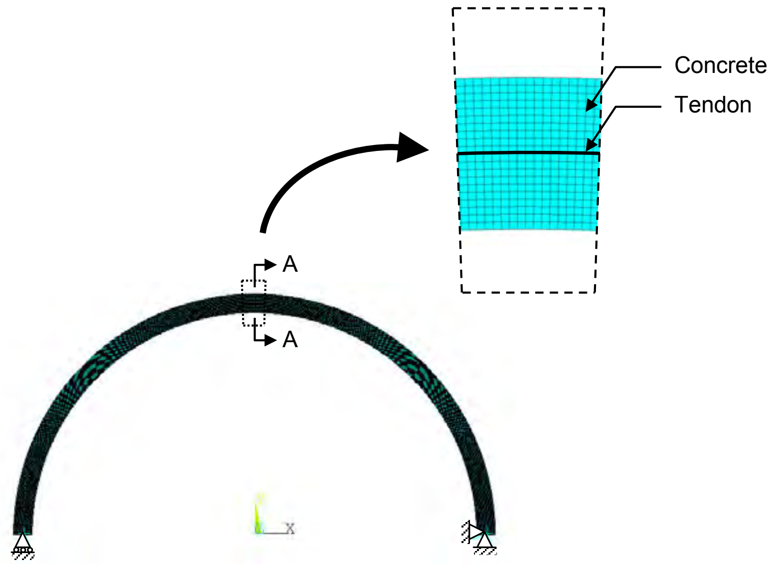


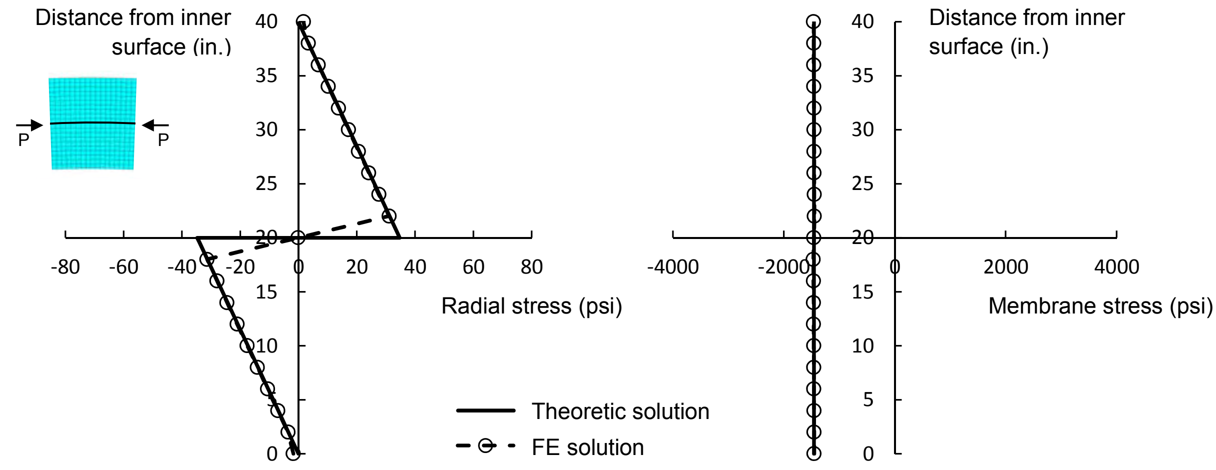
Figure 3. Finite element model

A FE model used in this study is illustrated in Fig. 3. A half of a cylindrical wall is modeled using a shell element. Tendons are modeled using a link (or truss) element. The model has a centerline radius of 70 ft with a section depth of 40 in. and a width of 12 in. The FE model contains a total of 20 elements across the section thickness to accurately capture the variation of stresses. ANSYS Professional FE analysis program (2010) is used. The results of analyses are obtained using PATH commands.

Radial and membrane stresses are calculated using theoretic equations and FE analyses. The stress distributions are examined under different loading conditions. The studied loading conditions include (1) prestressing force with zero eccentricity, (2) unbalanced moment only, (3) prestressing force with zero eccentricity and unbalanced moment, and (4) prestressing force with eccentricity of 0.25*t*. A prestressing force of 700 kips is applied. The magnitude of the unbalanced moment is 583.4 kip-ft, which is equivalent to the moment (700 kips×10 in.) due to the eccentricity of tendons.

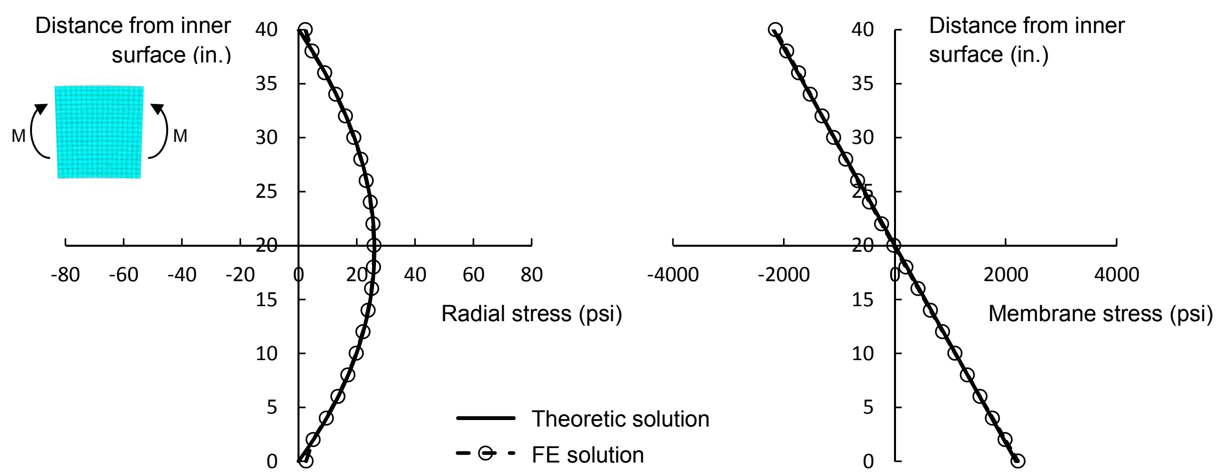
Fig. 4 shows the results for the case of the prestressing force with zero eccentricity. Positive sign indicates tension. Theoretic solutions show good agreement with the results from FE analysis. Interestingly, the radial stress from the FE results is zero at the center of the section where the radial stress becomes the maximum. This is due to the fact that a nodal force in a FE analysis is obtained by adding the contributions from all elements meeting at that node. Since the curved tendon generates equal but opposite radial forces at the center of the section, the radial force (or stress) from the FE analysis become zero at the center. If the number of elements across the section thickness is not sufficient, this limitation of FE analyses may result in a significant underestimation of radial stresses. The distribution of membrane stresses shows that uniform membrane stresses of 1750 psi are acting in the section.

The comparison of stress distributions for the case of the unbalanced moment only is presented in Fig. 5. The radial stresses are introduced due to the moment in a curved beam in this case. The figure shows that the theoretic and FE solutions agree well. The maximum radial tensile stress occurs at the center of the section. The observed maximum radial tensile stress is 26.1 psi. Considering that the maximum radial stress due to curved tendons is 34.7 psi, the amount of radial stress due to unbalanced moment is significant. The distribution of membrane stresses indicates that a pure bending force is acting to the section.



(a) Radial stress (b) Membrane stress

Figure 4. Stress distribution due to prestressing force with zero eccentricity



(a) Radial stress (b) Membrane stress

Figure 5. Stress distribution due to unbalanced moment only

The combined effect of curved tendons and unbalanced moment is shown in Fig. 6. The results can be viewed as the sum of stresses in Fig. 4 and Fig. 5. Note that the maximum radial tensile stress is significantly increased because maximum radial tensile stresses due to curved tendon and moment occur at the same location. The corresponding contour plot for radial stresses is shown in Fig. 7.

Fig. 8 illustrates the stress distribution due to prestressing force with eccentricity of 0.25*t*. The eccentrically-applied prestressing force introduces the same magnitude of the moment, which is 583.4 kip-ft. Hence, the moment generates the same stress distribution as shown in Fig. 5. However, the radial stress distribution due to curved tendons is affected by the eccentricity (see Fig. 1). As a result, the radial stress distribution in Fig. 8(a) becomes different from that in Fig. 6(a). On the other hand, the same membrane stress distribution is obtained.

Comparison of Fig. 4 and Fig.8 shows the effect of the tendon location on radial tensile stress. The magnitude of the maximum radial tensile stress is not reduced by the location of tendons (34.7 psi in Fig. 4 versus 35.3 psi in Fig. 8), which contradicts the conclusion based on the radial tension due to curved tendons only. This observation implies that moving tendon locations from the center to the outside surface of the section does not necessarily reduce the maximum radial tension. Rather, it may increase the rate of radial tension increase along with the depth of a section and reduce the outer thickness of the section at the same time. Therefore, the concrete section may become more vulnerable to the concrete delamination.

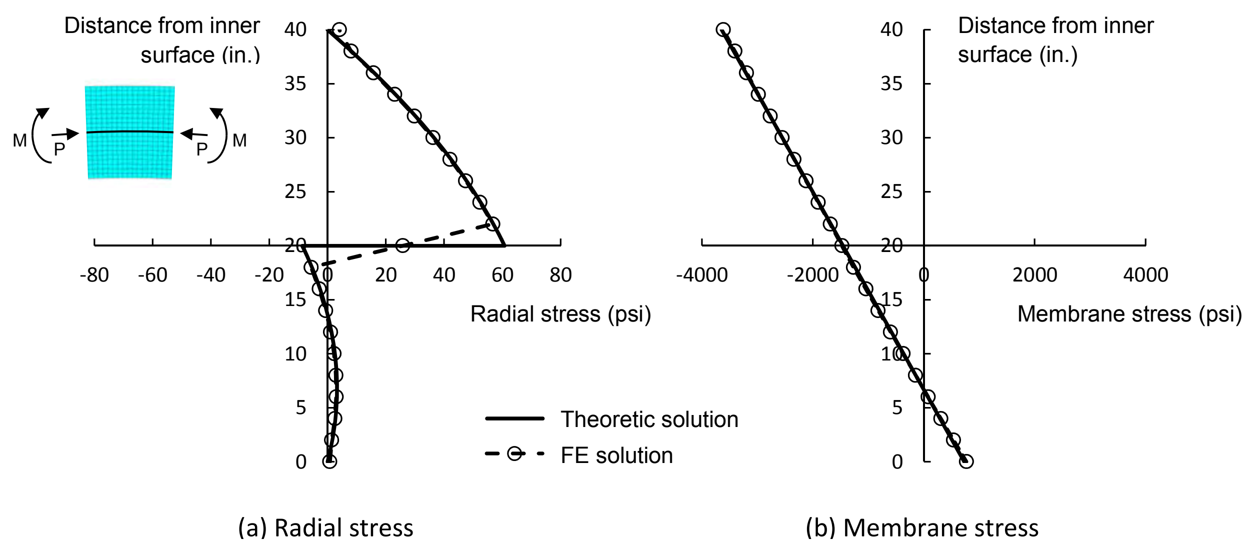


Figure 6. Stress distribution due to prestressing force with zero eccentricity and unbalanced moment

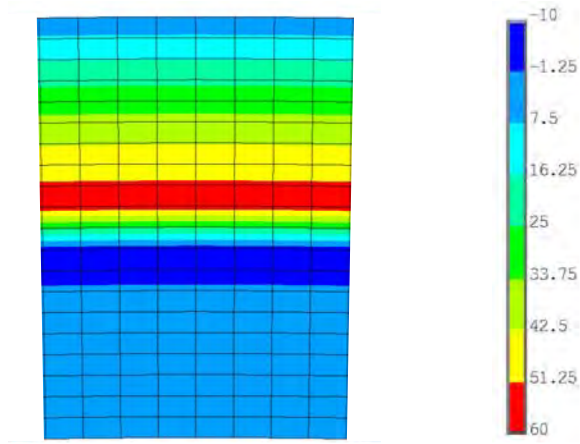


Figure 7. Contour plot of radial stress due to prestressing forces with zero eccentricity and unbalanced moment

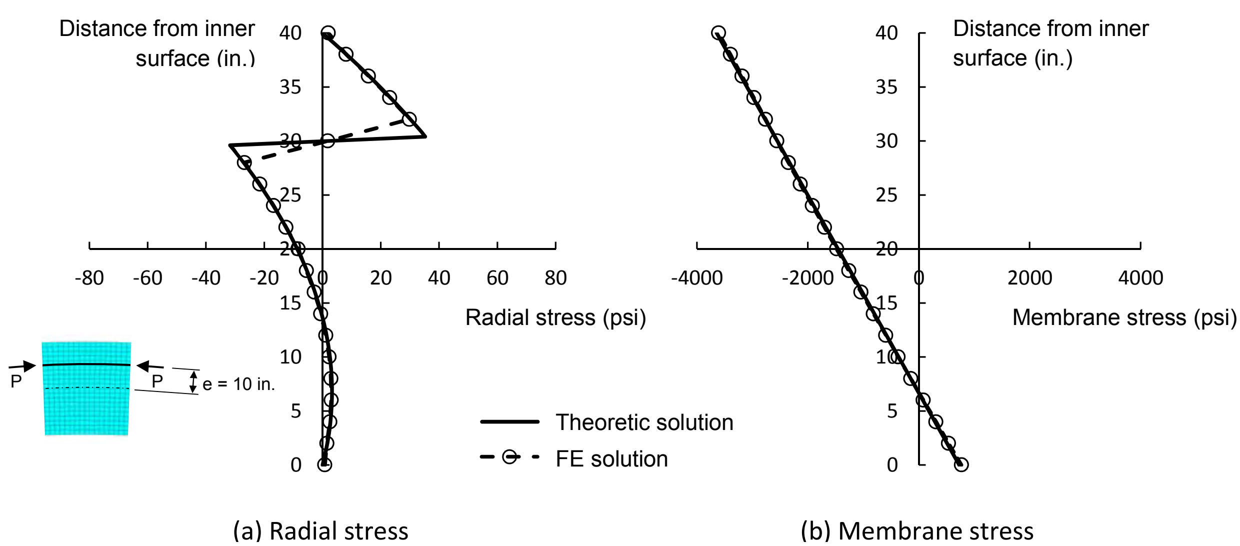


Figure 8. Stress distribution due to prestressing force with eccentricity of 0.25*t*

CONCLUSION

The effect of prestressing force and moment on radial stress in containment structures is studied in this paper. Radial stresses calculated by theoretic equations are compared with the results of FE analyses. Different loading conditions are examined to study their effects. The following conclusions can be reached based on the study reported herein:

1. Moments, which are out-of-plane moments, can produce significant radial stresses in containment structures. Sources of out-of-plane moments include not only unbalanced moments occurred at tendon tensioning/detensioning sequences or at construction opening, but also geometric irregularity, change of tendon curvature, tendon eccentricity and thermal gradient, and so on.
2. The maximum radial stress due to moment occurs at the center of a section. When tendons are placed at the center of the section, the maximum radial stresses due to curved tendons and due to moment occur at the same location, resulting in the most critical condition.
3. Placing tendons close to the outside surface of the section does not necessarily reduce the maximum radial tension. Rather, it may increase the rate of radial tension increase along with the depth of a section and reduce the outer thickness of the section at the same time. As such, the concrete section may become more vulnerable to the concrete delamination.
4. Predicting the maximum radial stress from the FE results is difficult because radial stresses due to curved tendons suddenly change its direction but a nodal force in a FE analysis is obtained by adding the contributions from all elements meeting at that node. If the number of elements across the section thickness is not sufficient, this limitation of FE analyses may result in a significant underestimation of radial stresses.
5. Radial stresses can be estimated using theoretic equations if prestressing force, tendon curvature, tendon eccentricity and moment acting in a section are known. Comparisons of radial stresses calculated by theoretic equations and FE analyses show that the maximum radial stresses can be better estimated by theoretic equations.

NOMENCLATURE

A	= cross-sectional area
p	= radial pressure
P	= prestressing force in tendon
R_c	= centerline radius
R_{ps}	= radius of prestressing tendon
t	= depth of cross section
w	= width of cross section
σ_r	= radial stress in concrete
$\sigma_{r,in}$	= radial stress in inside concrete fiber
$\sigma_{r,out}$	= radial stress in outside concrete fiber

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