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Orthotropic elastic beam-type formulation for elbows

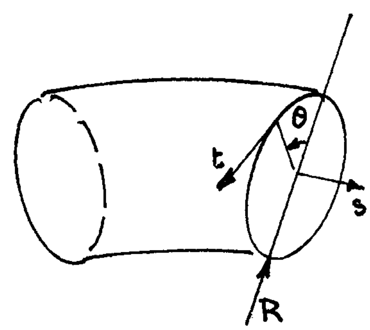
Berton, M.N., Lejeail, Y., Cabrillat, M.T.
CEA, DER/SERA, CEN Cadarache, St.-Paul-Lez-Durance Cedex, France

ABSTRACT : The construction of piping systems with composite materials raises the problem of beam-type calculations for non-isotropic materials. Indeed, whereas isotropic pipes are classically calculated like beams, with flexibility and stress intensification factors in elbows, there was no formulation for non-isotropic pipes, as far as we know. Furthermore, shell type calculations appear to be far too expensive for this kind of structure.

This paper describes an extension to orthotropic elbows used in classic calculations concerning elbow flexibility and stress intensification due to section deformation (these calculations are obtained with the asymptotic solution of Von Karman's method for "infinite elbows", i.e. elbows with no adjacent straight portion and with uniform bending). This method is applied to several configurations and then compared with shell-type calculations.

NOTATIONS :

e	thickness
r	mean radius
R	elbow curvature radius
$\lambda = eR/r^2$	characteristic parameter of elbows
v	Poisson's ratio
$I = \pi r^3 e$	moment of inertia of the section
$Z = \pi r^2 e$	
M	bending moment
O	ovalization: 2θ term of radial displacement with respect to mean radius r
t	index describing the circumferential direction
s	index describing the axial direction



1 INTRODUCTION

The section of elbows submitted to bending load is deformed, at first approximation it is ovalized. Ovalization results firstly in an additional bending of the elbow and secondly in local circumferential bending stresses.

The beam calculation theory is based on the undeformability of the straight sections. Amendments must therefore be made to this theory in order to perform elbow calculations :

- the additional bending due to ovalization is integrated using a flexibility coefficient which characterises the additional flexibility k of elbows compared to straight portions ;
- maximum circumferential stress resulting from ovalization is assessed using "beam" axial bending stress multiplied by the stress intensification coefficient C_2.

Coefficients k and C_2 are calculated using Von Karman's method (reference [1]) on elbows without straight sections, and which are submitted to uniform bending. Reference [2] describes calculations for k and C_2 which form the asymptotic solution of a Fourier series calculation applied to an elbow section.

The coefficients k and C_2 are used in all pipe analysis codes, as for example references [3] and [4].

This paper describes an extension of work conducted by Clark and Reissner (reference [2]) on orthotropic pipes.

2 CALCULATION OF AN ISOTROPIC ELBOW WITH UNIFORM BENDING FROM CLARK AND REISSNER (REFERENCE [2])

The authors of reference [2] calculate k and C_2 by the following method :

2.1 *The first stage consists in defining the orthotropic revolution shell equations, with :*

- writing of shell element equilibrium,
  - . for forces,
  - . for moments ;
- utilisation of a pseudo Hooke law (relation between generalized stresses (moments and forces) and generalized strain (bending and distortion curvatures) ;
- combination of the equations resulting in a system with two differential equations.

2.2 *These equations are simplified for the case of isotropic circular section elbows submitted to bending load :*

- considering the following assumption $r/R \simeq 0$. (This assumption was justified by previous works), established that the two differential equations are reduced and dependent on the λ parameter ;
- stresses calculations are based on moments and forces ;
- flexibility is defined as the ratio between the elbow curvature variation and the straight portion curvature variation.

2.3 *Asymptotic solution*

The two differential equations are reduced to a single complex differential equation. An accurate change of variables involving λ eliminates geometry from the equation.

A limited development in the vicinity of $\theta = \pi/2$ (elbow side) permits calculation of flexibility k and circumferential stress at $\theta = \pi/2$, (point of maximum circumferential stress).

Clark and Reissner obtained values for k and C_2, which, transposed with our notations give :

- for flexibility :

(1) $$k = \frac{\sqrt{12(1-\nu^2)}}{2}\frac{1}{\lambda}$$

i.e.: for $\nu = .3$, we obtain the classical $k = 1.65/\lambda$ of references [3] and [4].

- for stress loading intensification :

(2) $$C_2 = \frac{1.86176}{\left(1-\nu^2\right)^{1/6}}\frac{1}{\lambda^{2/3}}$$

i.e.: for $\nu = .3$ $C_2 = 1.89 / \lambda^{2/3}$ which differs little from the classical $1.95 / \lambda^{2/3}$ of references [3] and [4].

3 CALCULATION OF AN ORTHOTROPIC ELBOW

Starting from the end of step 2.1, we have used the previous calculation, remaining in the orthotropic formulation.

Using identical assumptions ($r/R \simeq o$), results in a differential system which depends on Young modulus Et and Es and on Poisson coefficients υ_{ts} and υ_{st}.

It appears that the differential system is the same when λ is replaced by $\lambda \sqrt{Et / Es}$.

Thanks to this change of variables, the calculation of the asymptotic solution is unchanged, and the following is obtained :

- for flexibility :

(3) $$k_{st} = \frac{\sqrt{12(1-\nu_{st}\,\nu_{ts})}}{2}\frac{1}{\lambda}\sqrt{\frac{E_s}{E_t}}$$

- and for stress loading intensification :

(4) $$C_{2st} = \frac{1.86176}{\left(1-\nu_{st}\,\nu_{ts}\right)^{1/6}}\frac{1}{\lambda^{2/3}}\left(\frac{E_t}{E_s}\right)^{1/6}$$

4 PRESENTATION OF THE VALIDATION "SHELL" CALCULATION

These formulations have been validated by comparison with shell-type finite element calculations performed with CASTEM 2000 (reference [5]).

Calculations were performed from a half-elbow section since elbows involved have no straight portions and are submitted to uniform bending load. This mesh, which comprises 96 shell-elements with 3 nodes, was optimised for bending stress calculations.

Boundary conditions are the following :

- symmetry in order to obtain the behaviour of a complete elbow section,
- symmetry on the 1st section,
- flatness relations with the 2nd section.

Displacements and stresses can be broken down into Fourier series via a specific procedure. This permits the calculation of ovalization.

Flexibility is calculated based on the rotation of the second section with respect to the first.

5 COMPARISON BETWEEN THE "SHELL" CALCULATION AND THE "BEAM" FORMULATION

5.1 *Flexibility (k)*

Figures 1 and 2 illustrate the comparison between flexibilities :

- resulting from the "shell" calculation with CASTEM 2000
- calculated using the "beam" formulation (3).

Figure 1 illustrates the comparison for different Et/Es values for $\lambda = .224$.

Figure 2 illustrates this comparison for different λ values and five Et/Es values.

5.2 *Ovalization (O)*

The classic relation between section ovalization and flexibility (see reference [6]) is expressed as follows :

$$(5)\quad O = \frac{4MR}{3E_sI}(k-1)$$

Figures 3 and 4 illustrates the comparison between ovalizations:

- resulting from "shell" calculations with CASTEM 2000,
- calculated using (5) and (3).

Figure 3 illustrates this comparison for different Et/Es values, for $\lambda = .224$.

Figure 4 illustrates this comparison for different λ values and five Et/Es values.

For these figures $MR/EI = 2.56\ 10^{-3}$.

5.3 *Maximum bending circumferential stress*

Figures 5 and 6 illustrate the comparison between σ_{tmax} stresses :

- resulting from the "shell" calculation with CASTEM 2000,
- calculating using C_2 from the "beam" formulation with $\sigma_{tmax} = C_2\, M/Z$, where C_2 is given by (4).

Figure 5 illustrates this comparison for different Et/Es values, for $\lambda = .224$.

Figure 6 illustrates this comparison for different λ values and five Et/Es values.

The moment applied results in an axial stress $M/Z = 177$ MPa.

6 CONCLUSIONS

Flexibility (k) and maximum bending circumferential stress (C_2) coefficient formulations for orthotropic elbows have been defined. These coefficients were validated by shell calculations for a wide elbow geometry and orthotropy variation range.

These formulations obviously have the same disadvantages as classical beam formulations since they result from calculations for elbows with no adjacent straight sections and with uniform bending. A finer approach would consist, for instance, in extending work presented in reference [6] to orthotropic configurations.

Complementary work on internal pressure loads is currently under way.

All this work improve composite material pipe calculations.

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Figure 1 : Orthotropic shell-beam comparison
Flexibility versus Young's modulus ratio
$\lambda = e R / r^2 = 0.224$

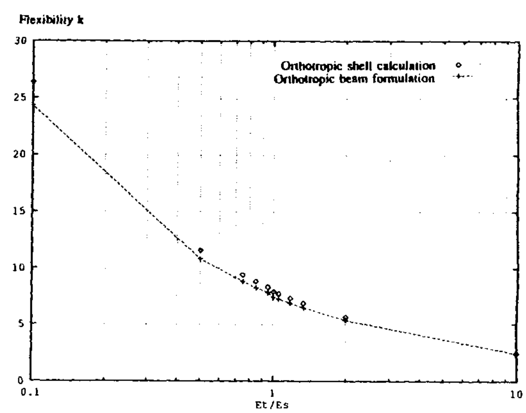
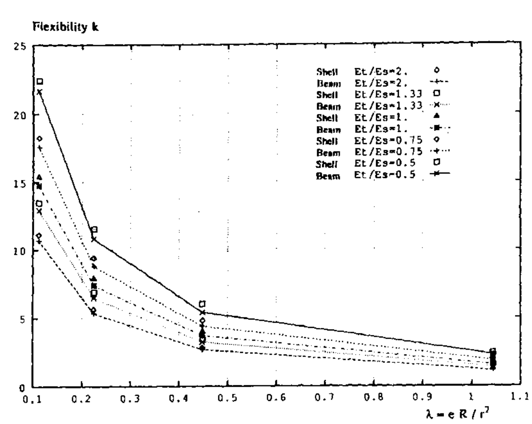


Figure 2 : Orthotropic shell-beam comparison
Flexibility versus $\lambda = e R / r^2$



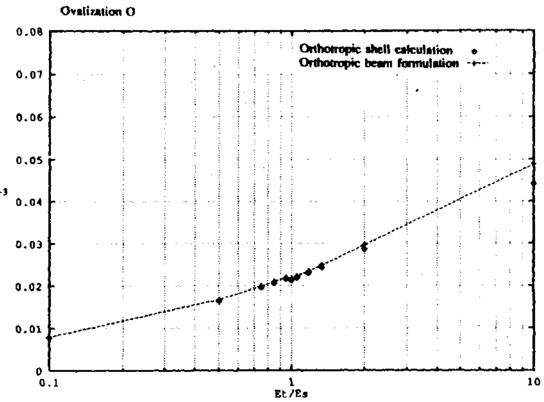


Figure 3 : Orthotropic shell-beam comparison
Ovalization versus Young's modulus ratio
$\lambda = e R / r^2 = 0.224$, $M R / E_s I = 2.56\ 10^{-3}$

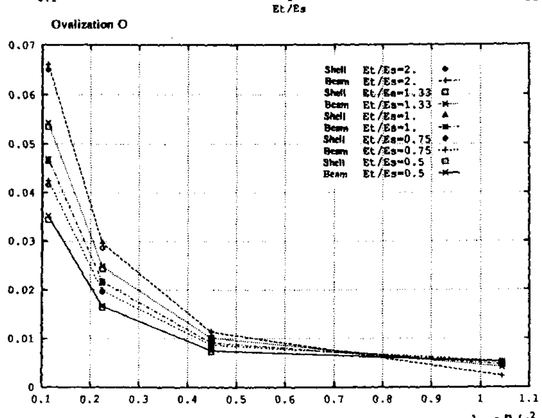


Figure 4 : Orthotropic shell-beam comparison
Ovalization versus $\lambda = e R / r^2$
$M R / E_s I = 2.56\ 10^{-3}$

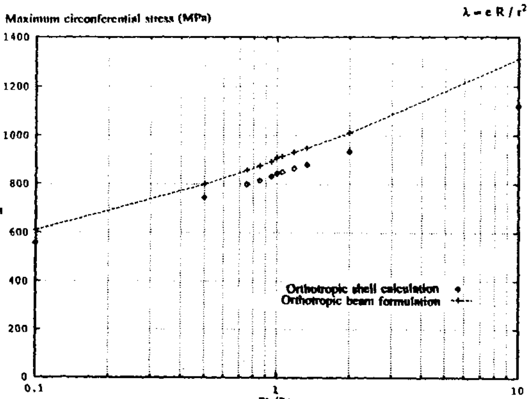


Figure 5 : Orthotropic shell-beam comparison
Maximum circonferential stress
versus Young's modulus ratio
$\lambda = e R / r^2 = 0.224$, $M / Z = 177$ MPa

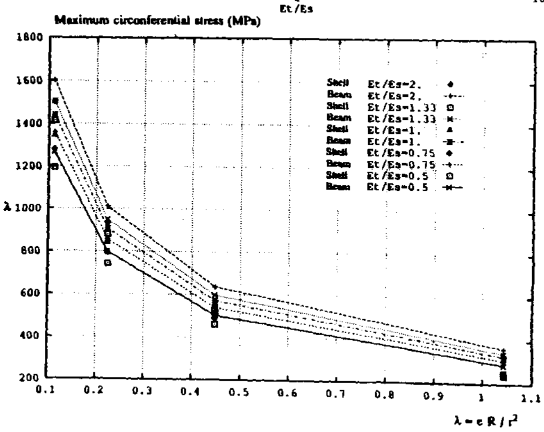


Figure 6 : Orthotropic shell-beam comparison
Maximum circonferential stress versus λ
$M / Z = 177$ MPa