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Correlation of responses with the energy dissipated by a structure during earthquake motions

Kubo, T.
Nagoya Institute of Tecnology, Dept. of Architecture, Nagoya, Japan

ABSTRACT : Correlation between the energies dissipated by a structure and the inelastic responses generated during strong earthquake ground excitation is evaluated for reinforced concrete structures. Emphasis is placed on the energies dissipated by the inelastic load-deflection characteristics. The dissipated energies reveal a good correlation with the inelastic responses expressed in terms of the ductility factor, provided that the energies are normalized as well by the so-named yielding potential energy introduced herein. It is concluded that the seismic performance of a structure can be examined from a point of view on the energies that are dissipated by and applied to a structure.

1 INTRODUCTION

In the seismic response analysis on a structure, it is of much concern to evaluate inelastic responses of the structure obtained when subjected to strong earthquake motions. To examine structural performance against an earthquake action, the structure is concluded safe provided that the structure can dissipate the energy that is produced during a seismic excitation without developing excessively large inelastic deflection. The structure can be judged safe provided that it produces less significant seismic response dissipating the energy applied during an earthquake excitation by its inelastic characteristics with stable load-deflection hysteresis loops. Dissipating a larger amount of the applied energy, structural responses during an excitation develop less and remain within a response smaller than the critical response not producing structural damages.

2 ANALYTICAL MODEL AND ANALYTICAL CONDITION

2.1 Analytical model

Within the study herein, a structure is represented by a single-degree-of-freedom oscillating system with the condition that the dominant deflection of a structure, even multi-storied, is well described by the fundamental mode.

The equation of motion for the structure subjected to a ground motion is given by:

$$m\ddot{x}(t) + c\dot{x}(t) + F(t) = -m\ddot{y}_0(t) \qquad (1)$$

in which mass m and damping coefficient c are taken unity and determined so as the fraction to critical damping equal to 0.05. The restoring force F(t) will be established by the inelastic load-deflection characteristics specified in the analysis.

Emphasis is placed on reinforced concrete structures, whose inelastic load-deflection characteristics are represented by the Degrading Trilinear Takeda model for flexural failure structures and by the Origin-oriented model for shear failure structures.

2.2 Analytical conditions

The structural model employed in this study can be uniquely established by the initial stiffness, which corresponds to the fundamental period, and the yielding strength of the structure. The yielding strength is specified equal to $0.15G \times m$ (G: the acceleration of gravity). The fundamental period of the structure is taken in the range of 0.1 to 1.6 seconds, and the intensity of excitation is varied so as the resultant inelastic responses of structure to be 1.0 to 6.0 in the term of the so-called ductility factor.

2.3 Equilibrium of energies dissipated by and applied to the structure

The equilibrium of energies dissipated by and applied to the structure during an earthquake excitation is expressed by:

$$\int_0^t m\ddot{x}\,\dot{x}\,dt + \int_0^t c\dot{x}\,\dot{x}\,dt + \int_0^t F\dot{x}\,dt = -\int_0^t m\ddot{y}_0\,\dot{x}\,dt \qquad (2)$$

in which the terms in the left hand side of Eq. (2) represent the energies E_I, E_D and E_H dissipated by the structure due to the inertia, damping and hysteresis characteristics, respectively, and the term in the right hand side designates the energy E_T applied to the structure during an earthquake action (Akiyama 1980 and Kubo et al. 1994).

3 ENERGY DISSIPATED BY THE HYSTERESIS CHARACTERISTICS

3.1 Ratio of the energy dissipated by the hysteresis characteristics compared to the entire energy dissipated by a structure

Based on the distinct load-deflection characteristics for reinforced concrete structures examined herein, the energy dissipated by the inelastic hysteresis characteristics can be directly related to the maximum response with the assumption that cyclic hysteresis responses with small amplitudes dissipate negligibly small amount of energies. Within the discussion hereinafter, the correlation between the maximum response of structure and the energy dissipated by the hysteretic characteristics is examined.

Figure 1 represents the ratio of the energy dissipated by the hysteretic characteristics E_H compared to the entire energy dissipated by the structure E_T associated with the variation of inelastic responses developed. The axes x and y denote the inelastic response of structure expressed in term of the ductility factor and the ratio of the dissipated energies E_H/E_T, respectively. Figures (a) and (b) show the results of the structure of the flexure failure with the Takeda model and of the shear failure with the Origin-oriented model, respectively. It is worth pointing out that for the structures with the Takeda model the ratios lie within a small variation associated with the initial periods, while for those with the Origin-oriented model the ratios are scattered widely associated with the initial periods of the structure.

3.2 Ratio of the hysteresis dissipated energy associated with the inelastic responses

Among structures having various initial periods in the range of 0.1 to 1.6 seconds, those of 0.4 and 1.2 seconds are taken to represent the structures having short and medium periods. Figures 2(a) through (c) show the ratios obtained for the structures both with the Takeda and Origin-oriented models when subjected to eight components of earthquake motion. Since for the structures of the Origin-oriented model the ratios are varied associated with the initial periods, the ratios for the structures of 0.4 and 1.2 seconds are separately illustrated in Figs. 2(b) and (c).

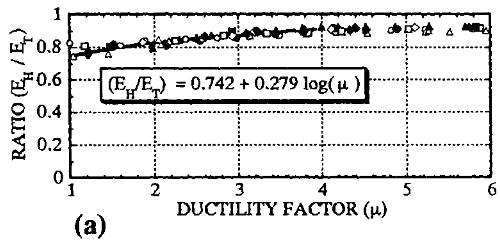
The regression curves of the ratio E_H/E_T on the ductility factor μ are established in the range of μ less than 4. For the responses obtained for structures of the Takeda models representing flexural failure, the mean-square regression curve is given by:

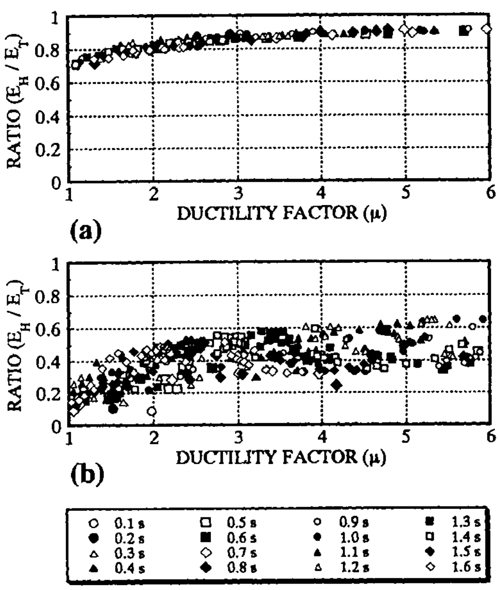
$$(E_H/E_T) = 0.742 + 0.279 \log(\mu) \qquad (3)$$

The regression curves for the responses of Origin-oriented model representing shear failure with initial periods of 0.4 and 1.2 seconds are respectively given by:

$$(E_H/E_T) = 0.066 + 0.549 \log(\mu) \qquad (4\text{-}1)$$

$$(E_H/E_T) = 0.170 + 0.592 \log(\mu) \qquad (4\text{-}2)$$





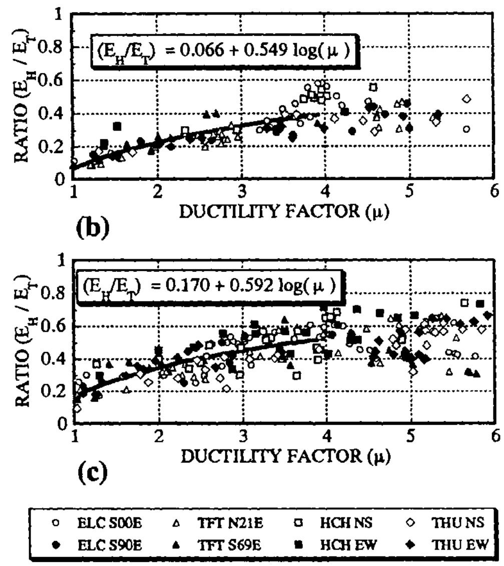


Figure 1 Ratios of the energy dissipated by the hysteresis characteristics to the entire dissipated energy obtained when subjected to the El Centro S00E component: (a) ratios E_H/E_T for the Takeda model structures; and (b) ratios E_H/E_T for the Origin-oriented model structures.

Figure 2 Ratios of the energy dissipated by the hysteresis characteristics to the entire dissipated energy: (a) ratios E_H/E_T for the Takeda model structures; and (b) and (c) ratios E_H/E_T for the Origin-oriented model structures with initial periods of 0.4 and 1.2 seconds, respectively.

4 CORRELATION OF THE ENERGY DISSIPATED BY THE HYSTERESIS CHARACTERISTICS WITH THE INELASTIC RESPONSE OF A STRUCTURE

4.1 Normalization of the energy dissipated by the hysteresis characteristics

Within the study presented herein, since the initial period of structure is taken as one of analytical parameters with the yielding strength kept constant, the energies dissipated by the hysteresis characteristics are different with each other even in the case when the inelastic responses expressed in term of the ductility factor are identical with each other. The energies dissipated by the hysteresis characteristics, hence, are normalized by the so-named yielding potential energy defined in Fig. 3 (Kubo et al.: 1995). Normalizing the dissipated energy, it becomes possible to examine the inelastic responses on the common base regardless of the structural properties such as the initial period and yielding strength.

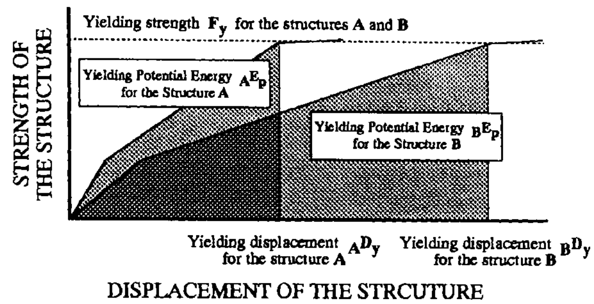


Figure 3 The yielding potential energy for structures with various initial periods, which is utilized for the normalization of the energy dissipated by the hysteresis characteristics.

4.2 Correlation of the normalized energies with the inelastic responses

Figures 4 and 5 show the correlation between the normalized energies dissipated by the hysteresis characteristics and the normalized inelastic responses expressed in terms of the ductility factor, which illustrate the results obtained when subjected to the El Centro S00E component and the Tohoku University EW component, respectively. Figures (a) and (b) in the both figures illustrate the correlation with the responses obtained for structures whose inelastic hysteresis model are specified by the Takeda and Origin-oriented models,

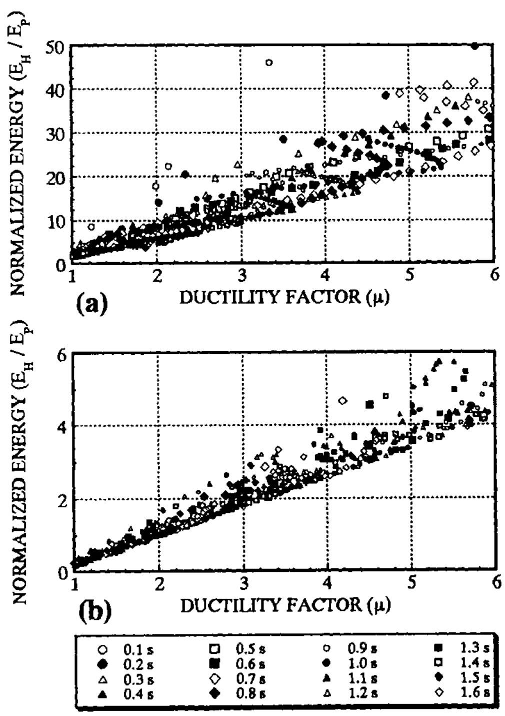


Figure 4 Correlation of the normalized energies E_H/E_p with the responses obtained when subjected to the El Centro S00E component: (a) structures of the Takeda model; and (b) structures of the Origin-oriented model.

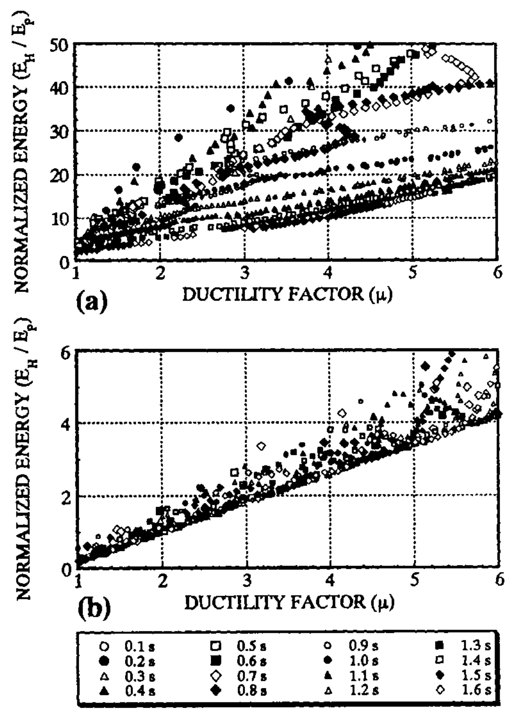


Figure 5 Correlation of the normalized energies E_H/E_p with the responses obtained when subjected to the Tohoku University EW component: (a) structures of the Takeda model; and (b) structures of the Origin-oriented model.

respectively. The axis x denote the response ductility factor, and the axis y designates the normalized energy.

Since the hysteresis rules of both the Takeda and Origin-oriented models reveal the peak-oriented cyclic characteristics, a linear correlation between the inelastic responses and normalized energies will be expected, the general tendency of which is realized in both Figs. 4 and 5. Comparing the results shown in Figs. 4 and 5 with each other, the correlation can be summarized as follows: (i) The normalized energies (E_H/E_P) with variation of initial periods of structure correlated to the ductility factor (μ) show greater variation for the Takeda hysteresis model than those for the Origin-oriented model. It can be realized that the lowest limit will be clearly established for the responses of the Origin-oriented hysteresis model structures; (ii) The value of the normalized energy for the Origin-oriented models corresponding to the specific ductility factor is less than that for the Takeda models, the feature of which can be indicated by the evidence that the energy dissipation capacity of a shear failure structure represented by the Origin-oriented model is significantly inferior to that of a flexural failure structure represented by the Takeda model; and (iii) The greater variation observed in the Takeda model structures can be interpreted in the way that cyclic hysteresis loops with small amplitudes, which are not explicitly taken into consideration in the stage to evaluate the correlation between the dissipated energy and the maximum responses, can dissipate a fairly large amount of energies.

For the results obtained with the variation associated with ground motion records illustrated in Figs. 6 and 7, the correlation between the normalized energies and response ductility factors is found. For the Takeda model structures, the mean-square regression lines are established with parameters of initial periods of structure as follows:

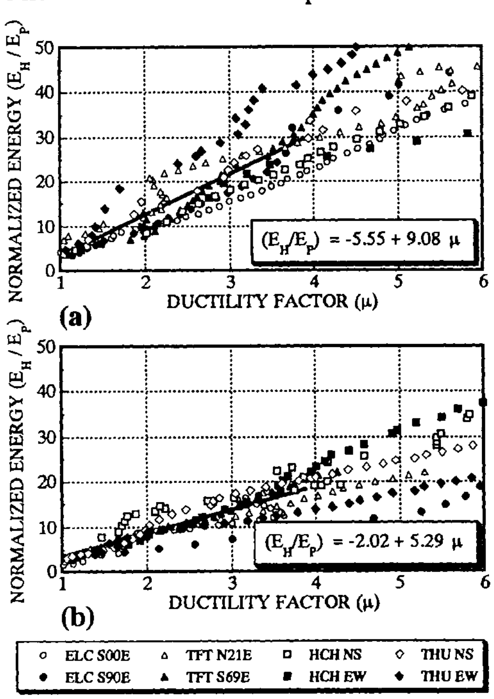


Figure 6 Correlation of the normalized energy E_H/E_P with the responses for the structure with the load-deflection characteristics specified by the Takeda model: (a) structures whose initial period equals 0.4 second; and (b) structures whose initial period equals 1.2 second.

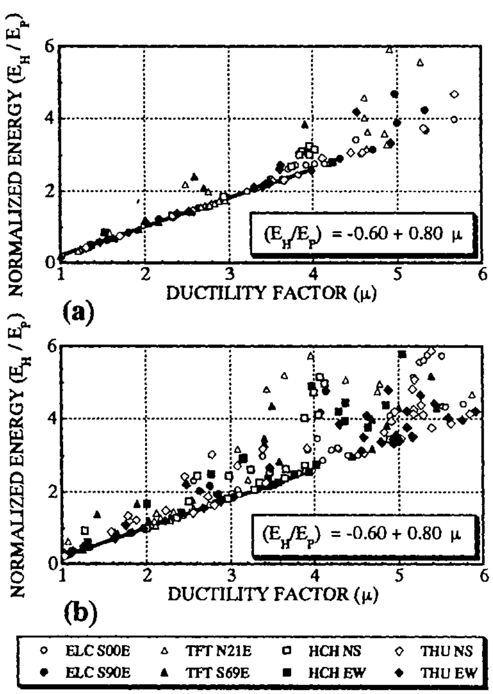


Figure 7 Correlation of the normalized energy E_H/E_P with the responses for the structure with the load-deflection characteristics specified by the Origin-oriented model: (a) structures whose initial period equals 0.4 second; and (b) structures whose initial period equals 1.2 second.

$(E_H/E_P) = -5.55 + 9.08\,\mu$ for initial period $T_o = 0.4$ second (5-1); and
$(E_H/E_P) = -2.02 + 5.29\,\mu$ for initial period $T_o = 1.2$ second (5-2)

For the Origin-oriented model structures, the line determining the lowest limit of energies associated with the response ductility factor is given by:

$(E_H/E_P) = -0.60 + 0.80\,\mu$ (6)

that is determined invariant with the initial period of structure.

5 SUMMARY AND CONCLUSIONS

Correlation between the energies dissipated by inelastic load-deflection hysteresis characteristics and the inelastic responses of structure is examined for reinforced concrete structures developing both flexural failure mode and shear failure mode. The conclusions obtained hereby are summarized as follows:

(i) The ratios of the energies dissipated by the hysteresis characteristics compared to the entire energies applied during strong earthquake motions are correlated with the inelastic responses generated within the structure. The correlation is found invariant with variation of earthquake excitation. It, however, varies with the initial period of the structure when the inelastic load-deflection is determined by the Origin-oriented model.

(ii) The hysteresis dissipated energies are revealed well correlated to the inelastic responses expressed by terms of the ductility factor when the energies are normalized by the so-named yielding potential energy. Normalizing the dissipated energy by the yielding potential, the correlation is established variant with the initial period of structure for the responses obtained from the Takeda hysteresis model.

Based on the results obtained within this study, one can examine the seismic capacity of a structure through the processes described in the following.

(i) First one determines the inelastic deformation in terms of ductility factor (μ) that can be allowed to produce during seismic excitation within the structure;

(ii) The normalized dissipated energy (E_H/E_P) is evaluated by use of Eq. (5) or (6) depending the possible failure mode and the initial period of the structure;

(iii) The entire energy applied to the structure (E_T) can be evaluated from the prescribed inelastic response by use of Eq. (3) or (4); and

(iv) The obtained entire energy will be compared to the energy applied during the strong ground shaking. When the former quantity is greater than the latter, the structure is judged safe against the ground excitation, and when not, the structure produces an inelastic response greater than permitted, possibly leading to failure.

Following the processes introduced above backward, one can utilize the results obtained in this study for a seismic design process determining the excitation as applied energy during an earthquake excitation and the seismic performance of the structure as an energy dissipation capacity established based on the ductility of structure.

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