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EFFECTIVENESS OF SIMPLE COMPUTATIONAL SCHEMES FOR TRANSIENT TEMPERATURE DISTRIBUTIONS IN STRUCTURAL COMPONENTS

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SUMMARY

The majority of two-dimensional thermal analysis problems in the design of nuclear power plant structural components can be classified as one of the following four categories: (1) linear steady-state, (2) linear transient, (3) piecewise linear transient and (4) non-linear steady state and transient. A reduction of computation costs has been achieved by implementing four finite-element computer programs for thermal analysis, each corresponding to one of the above categories, using the simplest yet sufficiently accurate analysis method while preserving the convenience of a common data base. This commonality also applies to the structural analysis and graphic interpretation programs with which these four programs interface.

The computer programs are based on the finite element discretization of plane and axisymmetric solids, the linear steady state, linear transient and non-linear transient matrix formulations, and the time integration scheme, all developed by Wilson. In addition to the boundary conditions of convection, radiation and liquid flow, the effect of heat generation by gamma ray energy deposition is considered.

In cases where a linear treatment of a problem is too inaccurate and a non-linear one is too expensive the piecewise linear computational scheme is successfully applied. The time history of effective thermal forces is subdivided in segments in which material and boundary properties of the problem can be approximated as being constant. Within such a segment the time step remains constant, too, and thus the effective conductivity matrix has to be triangularized only once.

Six example problems have been selected from a large number of applications of these programs. They have been divided into four categories as defined above.

1. INTRODUCTION

In the analysis of two-dimensional temperature distribution problems the selection of the proper computational scheme, and the interface of the temperature analysis programs with graphics and structures programs, are important for a successful data production. Most of the temperature distribution problems are transient and, more or less, non-linear in nature. The finite element analysis methods for the most general class of thermal problems have been available for some time [1] and the computer programs developed by Wilson et al. [2][3] have been used widely in the nuclear industry.

In an attempt to economize computer costs and at the same time automate the data transfer from the temperature analysis programs to the corresponding graphics and structures programs, four individual temperature distribution analysis programs, TEMP1 through TEMP4, were developed at KWU following Wilson's earlier theoretical and computational foundations. These programs are each applicable to a distinct class of problems. In this paper numerical examples are presented for each one of them. It appears from comparison with analytical results that simplified analysis schemes are quite effective in handling the various types of transient conditions.

2. THEORETICAL BACKGROUND

The two-dimensional (plane or rotationally symmetric) finite element model for the analysis of transient temperature distributions leads to a matrix equation, in which the nodal point temperatures, $\underline{T}$, are the unknowns,

$$\underline{C}\,\dot{\underline{T}} + \underline{K}\,\underline{T} = \overline{\underline{Q}} \; . \qquad (1)$$

In eq.(1) $\underline{C}$ is the capacity matrix, $\underline{K}$ is the conductivity matrix and $\underline{Q}$ is the vector of nodal heat flow input. The time derivative is denoted by $\dot{(\;)}$.

2.1 LINEAR STEADY STATE ANALYSIS

In a linear steady state problem the first term of eq.(1) is dropped. In addition the system conductivity matrix $\underline{K}$ is considered constant as the temperature changes. The equation reduces to

$$\underline{K}\,\underline{T} = \overline{\underline{Q}} \; . \qquad (2)$$

Two boundary conditions are possible in this equation: an insulated boundary, in which the surface heat flow $\overline{Q}_b$ is equal to zero, and a boundary with specified (guided) temperature $\overline{T}_b$, which is converted into a heat flow by a large artificial boundary conductivity constant, k_{bb},

$$\overline{Q}_b = k_{bb}\,\overline{T}_b \; . \qquad (3)$$

The conductivity constant k_{bb} is also added to the diagonal element of system conductivity matrix. Eq.(2) represents a set of linear equations with a positive-definite and symmetric coefficient matrix readily to be solved.

2.2 LINEAR TRANSIENT ANALYSIS

In a linear transient analysis eq.(1) is solved with a time integration scheme based on the assumption that all material properties of the model do not change with time and temperature. This applies to the heat capacitiv matrix $\underline{C}$, the conductivity matrix $\underline{K}$, the boundary heat transfer coefficients h and the artificial boundary conductivity coefficients k_{bb} used for imposing guided temperatures. At time t=0 an initial temperature distribution is assumed to be present.

The integration scheme implemented assumes that the time step Δt does not change during the integration process. The variation of temperatures within the time step are assumed to be linear. Consider eq.(1) at time step n+1 corresponding to t+Δt,

$$\underline{C}\,\dot{\underline{T}}_{n+1} + \underline{K}\,\underline{T}_{n+1} = \overline{\underline{Q}}_{n+1} \; . \tag{4}$$

The first derivative is replaced by

$$\dot{\underline{T}}_{n+1} = \frac{1}{\Delta t}(\underline{T}_{n+1} - \underline{T}_n) \; , \tag{5}$$

and eq.(4) is then rewritten as

$$[\frac{1}{\Delta t}\,\underline{C} + \underline{K}]\,\underline{T}_{n+1} = \overline{\underline{Q}}_{n+1} + \frac{1}{\Delta t}\,\underline{C}\,\underline{T}_n \; . \tag{6}$$

Eq.(6) represents a set of linear equations

$$\hat{\underline{K}}\,\underline{T}_{n+1} = \hat{\underline{Q}}_{n+1} \; , \tag{7}$$

in which $\hat{\underline{K}}=\frac{1}{\Delta t}\,\underline{C}+\underline{K}$ is the effective conductivity matrix for all time steps, and $\hat{\underline{Q}}_{n+1}=\overline{\underline{Q}}_{n+1}+\frac{1}{\Delta t}\,\underline{C}\,\underline{T}_n$ is the effective heatflow vector for time step n+1. In this scheme the decomposition of $\hat{\underline{K}}$ needs to be carried out only once.

2.3 PIECEWISE LINEAR TRANSIENT ANALYSIS

The analysis of a problem in which the temperature and convection boundary conditions and material properties change with time and temperature is carried out with an integration scheme derived from the one for linear transient analysis. The entire time domain of integration is subdivided into "segments", each consisting of several time steps. Within each segment the the time step Δt and the effective system conductivity matrix $\hat{\underline{K}}$ are assumed to be constant. Also the heat transfer coefficients h of the convection boundary conditions, and the artificial conductivity coefficients of the specified boundary temperatures, are considered constant within each segment. All temperatures, heat sources and other loading conditions are assumed to vary linearly with time within a time step.

From the governing matrix equation, eq.(4), taken at time steps n and n+1, an incremental equation is derived as

$$[\frac{1}{\Delta t}\,\underline{C} + \underline{K}]\,\Delta\underline{T}_{n+1} = \Delta\overline{\underline{Q}}_{n+1} + \frac{1}{\Delta t}\,\underline{C}\,\Delta\underline{T}_n \; , \tag{8}$$

in which

$$\Delta\underline{T}_{n+1} = \underline{T}_{n+1} - \underline{T}_n \; , \text{ and } \Delta\overline{\underline{Q}}_{n+1} = \overline{\underline{Q}}_{n+1} - \overline{\underline{Q}}_n \; . \tag{9}$$

The incremental equation is summarized as

$$\hat{\underline{K}}\,\Delta\underline{T}_{n+1} = \Delta\overline{\underline{Q}}_{n+1} + \frac{1}{\Delta t}\,\underline{C}\,\Delta\underline{T}_n \; . \tag{10}$$

Again, during the sequence of iteration steps, the effective conductivity matrix has to be triangularized only once. This affects considerable savings of computer time, particularly in a situation in which steep time gradients of temperature distribution are followed by long periods of small variations, allowing a drastic increase of step size Δt.

The incremental integration scheme starts at time step 0 at t=0 as follows

$$\hat{\underline{K}}\ \underline{T}_1 = \underline{\bar{Q}}_1 + \frac{1}{\Delta t}\ \underline{C}\ \underline{T}_0 \ , \tag{11}$$

where $\underline{T}_0$ is the initial temperature distribution. In order to be sure of the initial computations two more steps should be carried out using eq.(8) with $\underline{T}_0$, resulting in a steady state situation.

2.4 NONLINEAR TRANSIENT ANALYSIS

The computational scheme for a nonlinear analysis assumes a linear variation of temperatures between time steps n and n+1. While the capacity matrix $\underline{C}$ is assumed to be constant the system conductivity matrix $\underline{K}$ changes with temperature and time. So do the boundary conditions. The effective stiffness matrix has to be recomputed and triangularized frequently. The computational scheme has to achieve thermal equilibrium at the end of each time step. This is done by iterating on the equation

$$\hat{\underline{K}}^{(j)}_{n+1}\ \underline{T}^{(j)}_{n+1} = \underline{\bar{Q}}^{(j)}_{n+1} + \frac{1}{\Delta t}\ \underline{C}\ \underline{T}_n \tag{12}$$

until a norm of $\underline{T}^{(j)}_{n+1}$ converges to a given tolerance as the iteration step j increases.

2.5 NONLINEAR BOUNDARY CONDITION

The boundary conditions of convection and liquid flow have been presented in ref.[3], in which the medium surrounding the body under consideration is semi-infinite. For a liquid flow between two bodies close to each other the usual liquid flow recurrence relation is modified as follows. Let the flow of the liquid be bounded by the two opposing surface line segments, a-b in body 1, and c-d in body 2, and let A_1 and A_2 be the surface areas and h_1 and h_2 the heat transfer coefficients of the two bodies, respectively. With the mass density ϱ, the specific heat c and volume flow rate of the liquid v, the change of temperature in the liquid while moving past the two bodies from the nodes a and c (index "s" = start) to the nodes b and d (index "e" = end) is given by

$$T_s - T_e = \gamma_1(T_s+T_e-T_a-T_b) + \gamma_2(T_s+T_e-T_c-T_d) \ , \tag{13}$$

where $\gamma_1=A_1h_1/2\varrho c v$ and $\gamma_2=A_2h_2/2\varrho c v$. The recurrence relation to determine the temperature in the liquid is then

$$T_e = \frac{(1-\gamma_1-\gamma_2)T_s+\gamma_1(T_a+T_b)+\gamma_2(T_c+T_d)}{1+\gamma_1+\gamma_2} \tag{14}$$

The use of these boundary conditions will be explained in the last one of the example problems.

3. COMPUTER PROGRAM IMPLEMENTATION

Corresponding to the four computational schemes for thermal analysis four computer programs TEMP1 through TEMP4 have been set up. Each of these programs interfaces with the data preprocessing program having graphics capability for data checks. Geometry and topology data decks are compatible with the input requirements of several structural analysis programs for two-dimensional problems. The temperature analysis results of all four programs are interpreted by the graphics program in a variety of display modes such as isothermal contour lines and diagrams.

4. NUMERICAL EXAMPLES

In this section six examples are presented of computations carried out by all four of the programs. The first two examples are linear steady state problems, the third a linear transient, the fourth a piecewise linear transient, the fifth a non-linear steady-state, and the last one a non-linear transient problem.

4.1 EXAMPLE 1: FEEDWATER DISTRIBUTION BOX

The temperature transients of the structural components in the vicinity of the feedwater nozzles of the steam generator (SG) of the pressurized water reactor (PWR) Grafenrheinfeld (Fig.1) have been analyzed for a transition through several levels of plant load, starting at 3 and ending at 100 percent load. The purpose of the thermo-analysis is a subsequent structural fatigue analysis. The changes between the different levels of load are so slow that each condition can be assumed stationary. Each load level can be analyzed by the linear steady state program TEMP1. Fig.2 shows results for two finite element models of the feedwater distribution box. The model sizes are the following. Model 1 has 843 nodes, 762 elements and the max. difference of node numbers is 16. Model 2 has 1210 nodes, 1070 elements and a max. node number difference of 34. The isothermal lines on Fig.2 are presented for the 20% operating load level, which is the most severe one.

4.2 EXAMPLE 2: CORE SHROUD OF A BOILING WATER REACTOR (BWR)

The steady state analysis of the core shroud subjected to heat generation by gamma ray energy deposition was carried out with the model shown on Fig.3. The gamma ray energy deposition was computed from

$$q(x) = q_o e^{-kx} \qquad (15)$$

where k is the attenuation coefficient of the gamma rays and x is measured from the inside surface of the core shroud. The transfer coefficients on the inside (upward core flow) and the outside (downward return flow) of the core shroud are shown on Fig.3, together with the temperature distribution computed for the model with TEMP1. The results compare favorably with analytical results.

4.3 EXAMPLE 3: THERMO-SHOCK IN A PRESSURE VESSEL WALL

The computation of transient temperature distributions in a reactor vessel wall when the water temperature suddenly drops as a consquence of a reactor cooling pump failure is presented on Fig.4. This is a thermo-shock problem in which the reactor vessel is heat-insulated externally and has a higher temperature on the outer wall surface caused by the heat generated within the steel wall by gamma ray energy deposition. The problem has been analytically solved by Eberwein [4]. The same input data as in this Ref. were assumed. The problem was solved with a 17-element model using the linear transient program TEMP2. Good agreement with the analytical results was obtained.

4.4 EXAMPLE 4: FEEDWATER NOZZLE SUBJECTED TO COLD SLUG

The feedwater nozzle of the SG, PWR Grafenrheinfeld (Fig.1), was subjected to a cold water feed change-over to a second main feed water pump via high pressure feedwater heater by-pass. As indicated on Fig.5, the température drops from 218 C to 60 C within 5 seconds as the cold slug moves through, and returns to 218 C after the slug has passed. This is a typical problem for a piecewise linear transient analysis with TEMP3. As indicated on the diagram in Fig.5 the time steps used are Δt_2=1s; Δt_3=4s; Δt_4=40s and Δt_5=280s, the indices referring to the segment numbers. Temperature distributions for time step 3 (initial condition) and step 23 (end of passage of cold slug) are presented. The finite element model used has 2000 nodes, 1356 elements and a max. node number difference of 31.

4.5 EXAMPLE 5: FREE CONVECTIVITY PROBLEM

A non-linear steady state analysis of the two concrete cylinders surrounding the reactor pressure vessel was carried out with TEMP4. Between the two concrete cylinders, which are 1.4 m apart air is pumped at a velocity calling for the mathematical modeling of the interfaces 2 and 3 as free convection boundary conditions. Between the inner concrete wall and the pressure vessel insulation a gap of 67 mm exists through which air is forced at a high velocity. At interface 1 a forced convection boundary condition exists. The outer cylinder interface 4 is adjacent to water. The input data are summarized on Fig.6. Two finite element models with 20 elements were used to compute the temperature distribution shown in the diagram. The results compare favorably with those from an analytical solution.

4.6 EXAMPLE 6: ANALYSIS OF A CONTROL ROD DRIVE DURING SCRAM

The control rod drive shaft with all its internal components is depicted on Fig.7. A finite element model was developed which contains the various cavities and gaps filled with water (W) and air (A). In the gap outside the innermost hollow piston there is a liquid flow during scram as the body is cooled and the temperature in the liquid rises. The liquid flow is indicated by arrows. The introduction of this gap into the finite element model is accomplished via the modified non-linear boundary condition summarized in eq.(14). The levels 1 through 10 in Fig.7 mark the locations at which analytical solutions were computed.

5. CONCLUSIONS

Four distinct computational categories have been established for the vast variety of temperature distribution analysis problems in nuclear plant components. Four corresponding computer programs are described, each applicable to one of these categories. Solutions to example problems indicate the computational effectiveness of the computer programs. The program solutions were checked against known analytical solutions. They demonstrate the validity of the simplifications made in the solution algorithms. The piecewise linear solution algorithm is the most versatile and at the same time most effective one, while retaining the simplicity of the linear code from which it was developed.

6. ACKNOWLEDGEMENT

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7. REFERENCES

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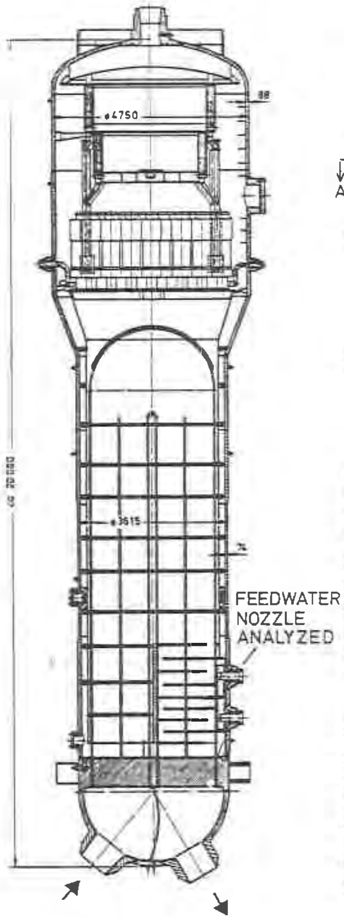

Fig. 1 - Steam Generator

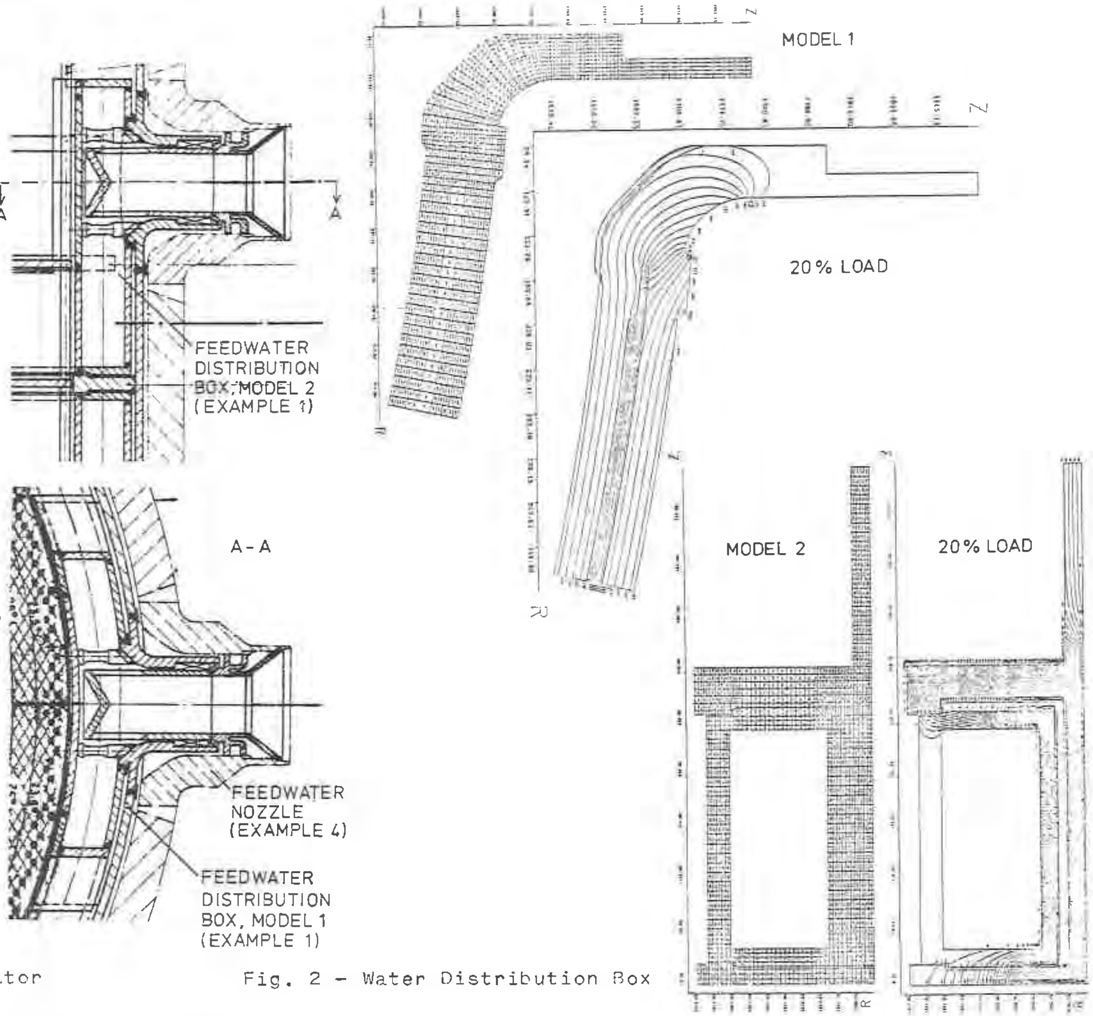

Fig. 2 - Water Distribution Box

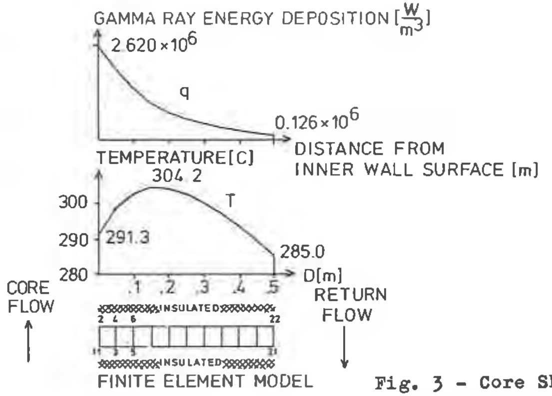

Fig. 3 - Core Shroud

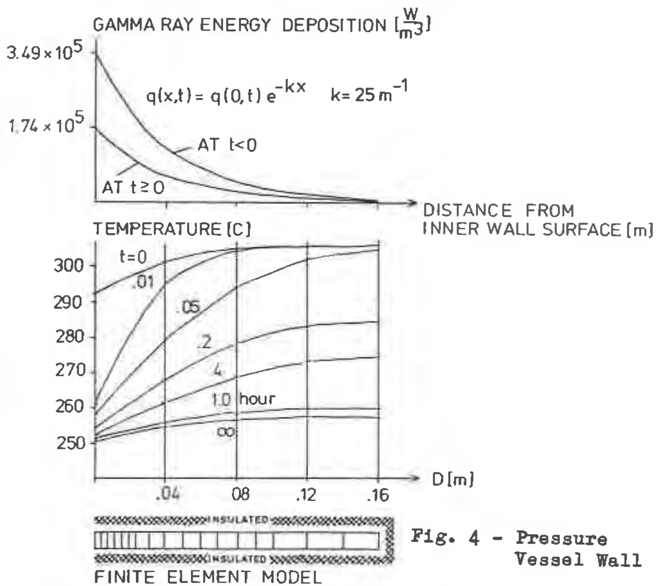

Fig. 4 - Pressure Vessel Wall

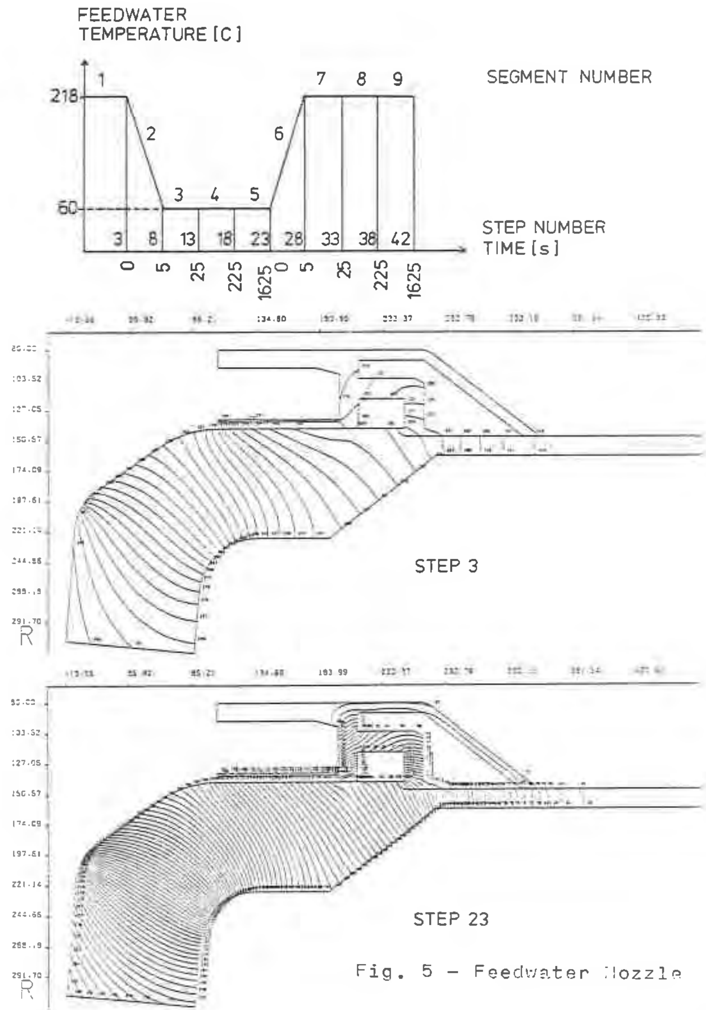

Fig. 5 - Feedwater Nozzle

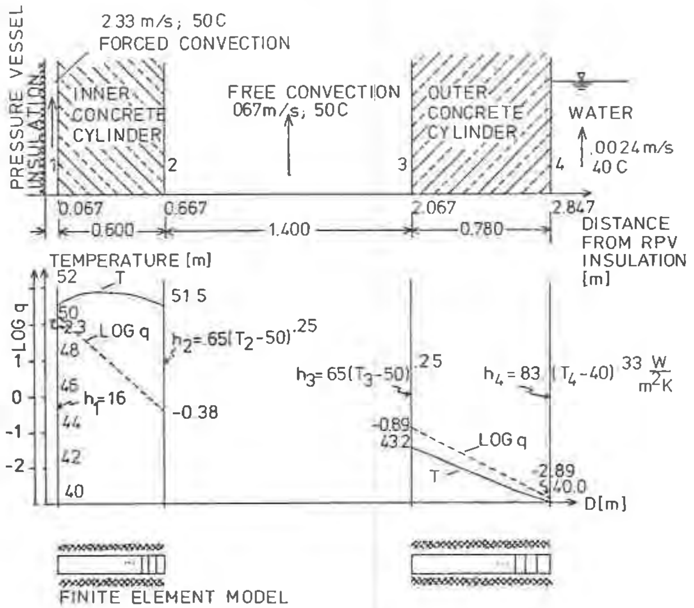

Fig. 6 - Free Convection between Concrete Cylinders Surrounding Pressure Vessel

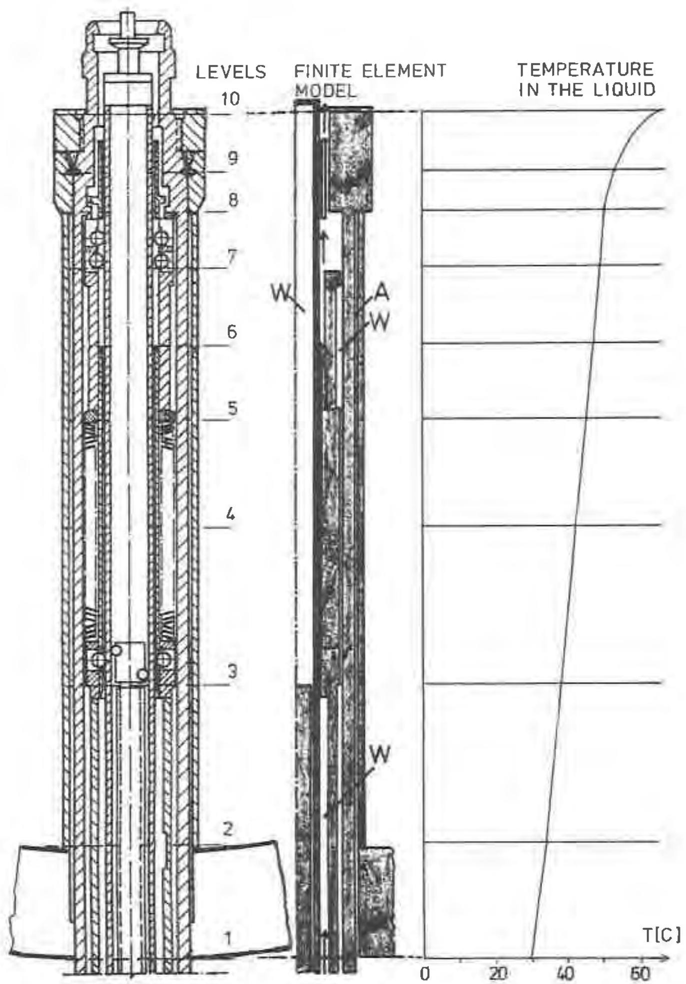

Fig. 7 - Control Rod