DYNAMIC NONLINEAR ANALYSIS OF SHELLS OF REVOLUTION

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SUMMARY

Over the past few years a series of finite element computer programs have been developed at Texas A & M University for the static and dynamic nonlinear analysis of shells of revolution. This paper will discuss one of these, DYNAPLAS, which is a program for the transient response of ring stiffened shells of revolution subjected to either asymmetric initial velocities or to asymmetric pressure loadings. Both material and geometric nonlinearities may be considered.

The present version, DYNAPLAS II, is the result of several years of work which began with the programs SAMMSOR and DYNASOR. As is the case for the earlier programs, a driver program, SAMMSOR III, generates the stiffness and mass matrices for the harmonics under consideration. A highly refined meridionally curved axisymmetric thin shell of revolution element is used in conjunction with beam type ring stiffeners in the circumferential direction. The shell element uses a cubic displacement function for u, v and w and through static condensation a basic eight degree of freedom element is generated. The shell material may be isotropic or orthotropic.

DYNAPLAS II uses the "displacement" method of analysis in which the nonlinearities are treated as pseudo loads on the right-hand side of the equations of motion. The equations are written for each Fourier harmonic used in representing the asymmetric loading components, and although the left-hand side of the equations is uncoupled, the right-hand side is coupled by the nonlinear pseudo loads. This method of formulation results in a large savings in computer time since the stiffness matrix remains unaltered during the transient response calculations.

The strain displacement equations of Novozhilov are used and the incremental theory of plasticity is used with the von Mises yield condition and associated flow rule. Either isotropic work hardening or the mechanical sublayer model (an assemblage of elastic perfectly plastic layers) may be used. In addition, strain rate effects may be included. Two temporal integration methods are available: either the explicit central difference method or the implicit Houbolt method. This allows the user to choose a method appropriate to the problem on hand.

There has been an extensive effort made to check the program. In particular, a closed form solution for the elastic-plastic behavior of rings subjected to axisymmetric impulsive loading was used as a check problem. Additionally, the program has been checked by comparing results with other programs, such as REPSIL, SHORE and JET-3. And whenever possible, experimental results have been used as check problems. These have included an explosively loaded clamped circular plate and a circular ring subjected to a uniform inward initial radial velocity.

The program has found extensive use in aerospace, mechanical and civil engineering applications. One example has been its use in the analysis of containment vessels for light water reactors. The loading in this case consisted of a uniform internal pressure with a subsequent localized dynamic pressure load.

1. Introduction

Nonlinear structural analysis has become increasingly more important over the last few years. This is, in part, due to the fact that materials are being loaded closer to their design limits and also because the consequences of severe loading conditions, such as hypothetical accidents, must be analyzed. At Texas A&M University, College Station, work began several years ago on the development of a computer program [1] for the static analysis of shells of revolution considering geometric nonlinearities (the so-called large deflection problem). This work led to a series of programs which perform both static and dynamic analyses of ring stiffened shells of revolution subjected to asymmetric loadings. Both geometric and material nonlinearities are permitted. One of the primary objectives of the research has been to develop programs that do not use excessive amounts of computer time. This paper is concerned with the dynamic analysis program DYNAPLAS II [2]. Recent work on extension of this program to include thermal loading and its effects is discussed by Haisler and Stricklin [3].

A Lagrangian formulation with the nonlinearities treated as pseudo forces on the right hand side of the equations of motion is used. The formulation for the spatial discretization of the shell and ring stiffeners is by the finite element (displacement) method while the pseudo forces are calculated by finite differences.

A "driver program" SAMMSOR IV [4] generates the stiffness and mass matrices for the Fourier harmonics under consideration. These matrices are then read by the program DYNAPLAS II. A restart provision is included.

The next section discusses the formulation and features of the program. Greater detail may be found in references [2,4,5].

2. Formulation

2.1 Element Description

2.1.1 Shell

The shell of revolution is idealized by a sequence of meridionally curved elements, Figure 1. The slope ϕ of the element is represented by a second order polynomial as a function of the meridional distance s

$$\phi = a_0 + a_1 s + a_2 s^2 \qquad (1)$$

The values for a_0, a_1, a_2 and s are determined by the nodal coordinates and through user input of the slope of the shell at the nodal points. It should be noted that this is the element used for the calculation of the linear stiffness matrices. The nonlinear contribution, in the form of pseudo forces, is calculated on the basis of a conical frustum with straight sides.

2.1.2 Ring Stiffeners

Ring stiffeners which are assumed to have zero products of inertia may be included. They may be eccentrically located with respect to the midsurface of the shell and current provisions allow for three rectangular flange members. The effect of the stiffener is obtained by adding its strain energy to that of the shell element and hence the stiffener is "smeared" over the length of the element.

2.2 Displacement Functions

In order to allow for asymmetric loading, a displacement (v) in the circumferential direction must be included in addition to displacements in the normal (w) and meridional (u) directions. The assumed displacement functions are

$$w = \sum_{i=o}^{N} \left[\alpha_1^i + \alpha_2^i s + \alpha_3^i s^2 + \alpha_4^i s^3\right] \cos i\theta$$

$$u = \sum_{i=o}^{N} \left[\alpha_5^i + \alpha_6^i s + \beta_1^i s(s-L) + \beta_2^i s^2(s-L)\right] \cos i\theta \qquad (2)$$

$$v = \sum_{i=o}^{N} \left[\alpha_7^i + \alpha_8^i s + \beta_3^i s(s-L) + \beta_4^i s^2(s-L)\right] \sin i\theta$$

where

i = Fourier harmonic number

L = meridional length

α, β = generalized coefficients

θ = circumferential angle

The coefficients β_1, β_2, β_3 and β_4 are eliminated by static condensation for the rigid body harmonics, 0 and 1, resulting in an 8x8 element stiffness matrix. The inclusion of the β coefficients has improved the rate of convergence of the element. Only loads which are symmetric about $\theta = 0^o$ are allowed. Furthermore, the loading must be fairly smooth in the circumferential direction; as the number of harmonics required to represent the circumferential variation of the loading increases, the efficiency and capacity (number of elements) of the program decreases.

2.3 Strain-Displacement Equations

The strain displacement equations used in DYNAPLAS are those based on the theory presented by Novozhilov [6].

$$\varepsilon_s = \hat{e}_s + n\hat{\kappa}_s + \frac{1}{2}\hat{e}_{13}^{\,2}$$

$$\varepsilon_\theta = \hat{e}_\theta + n\hat{\kappa}_\theta + \frac{1}{2}\hat{e}_{23}^{\,2} \qquad (3)$$

$$\varepsilon_{s\theta} = \hat{e}_{s\theta} + n\hat{\kappa}_{s\theta} + \hat{e}_{13}\hat{e}_{23}$$

where

ε_s = total strain in the meridional direction

ε_θ = total strain in circumferential direction

$\varepsilon_{s\theta}$ = total shear strain

$$\hat{e}_s = \frac{\partial u}{\partial s} - w\frac{\partial \phi}{\partial s}$$

$$\hat{e}_\theta = \frac{1}{r}\left[\frac{\partial v}{\partial \theta} + u \sin \phi + w \cos \phi\right]$$

$$\hat{e}_{s\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}\sin \phi + \frac{\partial v}{\partial s}$$

$$\hat{e}_{13} = \frac{\partial w}{\partial s} + u\frac{\partial \phi}{\partial s}$$

$$\hat{e}_{23} = \frac{1}{r}\frac{\partial w}{\partial \theta} - \frac{v \cos \phi}{r}$$

$$\hat{\kappa}_s = -\frac{\partial \hat{e}_{13}}{\partial s}$$

$$\hat{\kappa}_\theta = -\frac{1}{r}\frac{\partial \hat{e}_{23}}{\partial \theta} - \frac{1}{r}\sin \phi\, \hat{e}_{13}$$

$$\hat{\kappa}_{s\theta} = -\frac{1}{r}\frac{\partial \hat{e}_{13}}{\partial \theta} + \frac{\sin \phi}{r}\hat{e}_{23} - \frac{\partial \hat{e}_{23}}{\partial s}$$

n = outward normal measured from shell mid-surface

These equations are valid for moderate rotations only.

2.4 Plasticity

The incremental theory of plasticity is used with the Von Mises yield condition and associated flow rule. Studies at Texas A&M University [7] resulted in the adoption of both the isotropic hardening and mechanical sub-layer models. This latter model is an assemblage of elastic-perfectly plastic layers arranged in parallel. Detailed descriptions of these and other models and their applicability for different materials and for different loading paths is given in reference [7].

2.5 Equations of Motion

Using the matrix displacement method the equations of motion are obtained from Lagrange's equation for the ith harmonic as:

$$\left[M^i\right]\left\{\ddot{q}^i\right\} + \left[K^i\right]\left\{q^i\right\} = \left\{P^i\right\} + \left\{Q^{I^i}\right\} + \left\{Q^{NL^i}\right\} \qquad (4)$$

where

M = mass matrix

K = stiffness matrix

q = generalized displacements

P = generalized forces due to the applied loads

Q^I = pseudo forces due to the plastic strains

Q^{NL} = pseudo forces due to the geometric nonlinearities

Note that the formulation treats the nonlinearities as pseudo forces. Also, it should be noted, that although the left hand side of the equations are uncoupled, the Fourier coupling is retained through the nonlinear pseudo forces on the right hand side. The nonlinear forces can be expressed in terms of the nodal displacements and the equations may be solved for any harmonic i and the individual components superimposed. This type of pseudo force formulation does not require recomputation of the left hand side and hence there is a considerable savings in computer time.

The equations of motion are solved by either the Houbold method (a third order backwards difference implicit expression) or by the explicit central difference method. Both methods have their merits and further discussion on this topic may be found, for example, in reference [8]. For linear analyses the Houbolt method is unconditionally stable, however, this is not necessarily true for the case when nonlinearities are present. The Houbolt method was selected on the basis of using less computer time (for many problems) and because its artificial damping works well in conjunction with the pseudo force method.

3. Program Features

There are several other features in the program, which will only be briefly described herein; details are given in references [2,4]. Strain rate effects are incorporated through the use of a model suggested by Cowper and Symonds [9]. To model either incomplete shells of revolution or a shell with a cutout, springs may be attached to the shell at discrete points. However, considerable judgement may be required to obtain the proper boundary conditions.

The program is dynamically dimensioned for many of the arrays thus allowing for optimum use of core storage and also to permit varying size problems to be executed without changing any FORTRAN statements. A restart feature is included which is useful for either continuing calculations or for, say, increasing the time step size. Through the artificial damping feature of the Houbolt method a static solution may be obtained; this could be considered to be a form of dynamic relaxation.

4. Program Limitations

In addition to being limited to moderate rotations and to the plasticity theory described there are several limitations of the program; these include:

a) input of specific damping values are not permitted

b) multi-layered and branched shells are not permitted (currently the program is being extended to include this capability)

c) thermal loading is not permitted (reference [3] discusses a recent extension which permits thermal loading)

d) follower forces are not allowed.

5. Numerical Results

There have been many comparisons made between the results obtained by the DYNAPLAS program and with results from other computer programs or with results from experiments. These have included an explosively loaded circular plate, a clamped circular ring subjected to an impulse over an angle of 120° and a frustum of a cone clamped at both ends and subjected to a cosine impulse over half its circumference. In the latter two cases comparisons were made with the results from the computer programs REPSIL [13], SHORE [14], and JET-3 [15]. These are described in detail in references [2,5]. Additionally the following results have been obtained.

5.1 Ring Loaded in Compression

The first example to be discussed is a circular 6061-T6 aluminum ring which was loaded experimentally by a very short duration (10 μ sec.) pulse. The ring had an outer diameter of 6.00 in. and a thickness of 0.100 in. and was 0.50 in. wide. The impulse delivered to the ring in the experiment [10] was 3900 dyne-sec/cm^2 and is sufficient to cause a permanent deformation of the ring. The stress-strain curve for the material is shown in Figure 2. Results of calculations using both the isotropic hardening model and the mechanical sublayer model are shown in Figure 3, along with the experimental result. Four points were used to represent the stress-strain curve in the DYNAPLAS calculations. The calculated initial response for both models agrees well with the experimental result. However, when unloading occurs the agreement is not as good. The reverse loading behavior is dependent on the magnitude of the loading and except for problems in which the loading is uniform would be difficult to incorporate in any practicable model.

5.2 Ring Loaded in Tension

The second example is similar to the first except the ring is 1.00 in. wide and the load of 5740 dyne-sec/cm^2 is applied to expand the ring. Again both the isotropic hardening model and the mechanical sublayer model were used in DYNAPLAS. Results are shown in Figure 4. The experiment is described in reference [11].

5.3 Cylinder

The third example is a free 6.00 in. long 2014-T651 aluminum cylindrical shell which is loaded by a cosine impulse imparted over half of its circumference (half-cosine load). Details of the experiment are given in reference [12]. Comparison between the results obtained by DYNAPLAS, using 7 Fourier harmonics, and the experiment is shown in Figure 5. The cylinder had an outer diameter of 6.865 in. and a thickness of 0.166 in. The impulse delivered to the cylinder was 1670 dyne-sec/cm^2 and the stresses remained within the elastic region. Agreement is seen to be excellent.

5.4 Reactor Containment Vessel

DYNAPLAS has been used in many applications, in part, due to its

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relatively fast computation time. One example is the modeling of a concrete containment vessel for a light water reactor shown in Figure 6. The analysis consisted of performing both static and dynamic calculations in addition to calculating the natural frequencies and mode shapes. The static analysis considered internal pressure loading, the dead load, and a thermal gradient of 115F° through the thickness of the wall. The internal pressure is 47.0 psig and is the design basis accident pressure loading. Various configurations and base boundary conditions were used. In addition, a postulated dynamic overload pressure equal in magnitude to 1.5 times the design pressure with a rise time of 0.001 sec to peak pressure and a decay time of 0.005 sec. was used in the dynamic calculations. The results of one of the calculations is shown in Figure 7 which is a plot of the meridional stress at the outer surface vs. time. The stress at zero time is due to the static loadings.

The static calculations were done by the program SNASOR [1] and the frequency analysis by FAMSOR [16].

6. Acknowledgments

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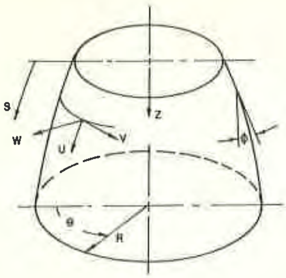

Fig. 1 Shell Element

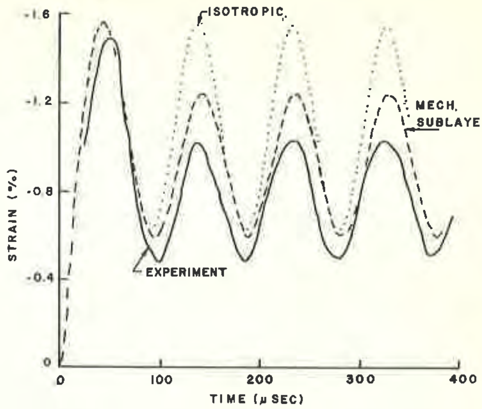

Fig. 3 DYNAPLAS and Experimental Comparisons (Ring Loaded in Compression)

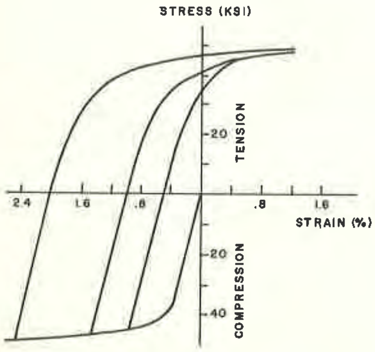

Fig. 2 Reverse Loading Behavior for 6061-T6 Aluminum

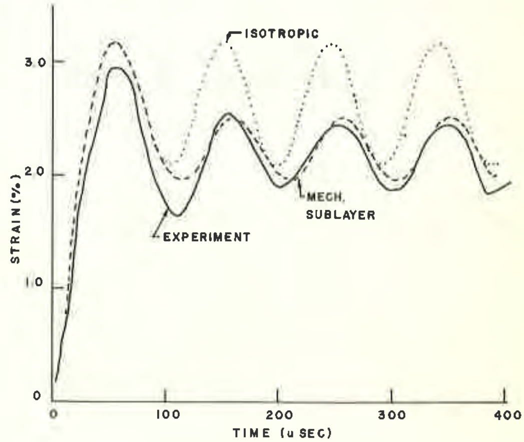

Fig. 4 DYNAPLAS and Experimental Comparisons (Ring Loaded in Tension)

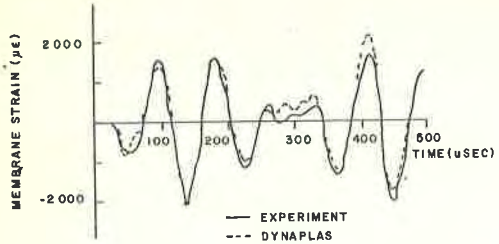

Fig. 5 Comparison of Experimental and DYNAPLAS Results (Membrane Strain at 180°)

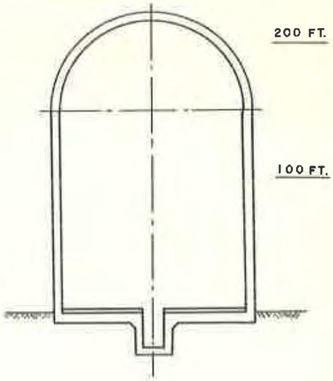

Fig. 6 Secondary Containment Vessel

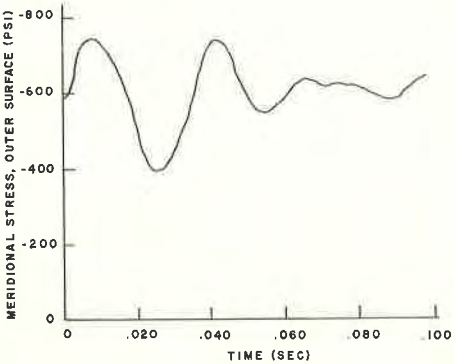

Fig. 7 Meridional Stress vs. Time