

PERFORMANCE EVALUATION OF THE CONCURRENT TOKEN PASSING PROTOCOL

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ABSTRACT

A concurrent token bus protocol for a local area network is proposed in this paper. The main feature of the protocol is a reduction of the token passing time and the improvement of the network throughput. The concurrence is achieved by using a sub-channel to carry the control token. The token is released one slot after the start of a transmission which is in contrast to the IEEE Token Ring protocol type I and II in which the token is released at the end of the transmission. The simulation comparisons, between the C-token protocol and the type II Token Ring protocol, show that the mean response time is significantly reduced and a better throughput is obtained. The theoretical expressions for the throughput and delay are presented.

1. INTRODUCTION

A local area network [5] consists of a number of stations which share one high bandwidth transmission medium. The stations are located in a small geographical area (e.g., in the same building, campus etc.). The performance of the network depends highly upon the management of users' accesses to the channel.

This subject, Medium Access Control, has been studied and a number of protocols have been proposed in the past decade. In general, we can classify the control procedures into two main categories: contention schemes and conflict-free schemes. The contention schemes, for example CSMA/CD [2], have attractive performance under light traffic loads but the collision problem cannot be avoided completely. Users may suffer a potentially unlimited response time in heavy load conditions. In a conflict-free control scheme, the channel access is controlled by passing a permit token to the stations of the network [3]. Only the station who owns the token can use the channel. Since there is

only one token in the network, collisions will never occur. The response time at any station is bounded since the token will come back to a station within a predictable period of time. However, the token searching time, which is the time needed to find an active station, is an unavoidable overhead.

As the capacity of the transmission medium increases the transmission time for a message or packet decreases significantly; however, the overhead of token searching time does not change at all since the token searching time depends on propagation time in the network. The ratio, denoted by "a", of the propagation delay to the transmission time of a packet represents the relationship between the fraction of time that the channel is used by the data transmission and by the scheduling process. For, "a" \ll 1, today's medium access control protocols work properly. In the environment of "a" $>$ 1, some of them do not perform well; for example, CSMA/CD can only achieve throughput of $1/e$ if $a=0.5$. Therefore, some alternatives have proposed modifying the existing protocols in order to survive at high speed operations.

To divide a high speed channel into several lower speed channels, see [9], is a possible way to adopt CSMA/CD in a high speed environment. However, in order to fully utilize a group of available subchannels will require the fragmentation and reassembly procedures at the sending and receiving sites. A certain degree of complexity will be increased for the control procedures at the transport layer. For conflict-free control protocols, the protocols are designed to train all the waiting packets at different stations in order to increase the transmission times on the channel at the same time, see [10]. The training concept similar to slotted ring will employ small packet size in order to give a fair scheduling scheme. This will result in more software execution at higher level

layers. Another disadvantage is that the logical order of transmission is corresponding to the physical location of stations. The system gives the preference to the stations close to the stations which generate cycles. Based upon the facts of extremely high transmission speed and the arguments for the reduction of scheduling time, we propose a concurrent token protocol.

In this paper, we are interested in a token controlled local area network which uses a control token to schedule the accesses to the shared channel. We would like to eliminate the search time such that the throughput of the system and response time of a packet can be improved. A typical token bus control procedure consists of a sequence of events. An active station waits for the control token and starts transmitting as soon as it receives the token. Once the station finishes the transmission, it releases the token to the next station. The token will be passed around the network to grant right of access to the channel to an active station. During this period the channel is idle and a portion of channel capacity is lost. If the token can be released at the beginning of a transmission, the search time can be overlapped with the transmission period. We also allow increasing the amount of data that a station can transmit at one time in order to reduce the number of times of token passing. In order to achieve this concurrence we have to move the control signals, the control token, from the data channel and permit another medium to carry it. Therefore, we propose a dual-bus network which consists of one data channel and one token channel. The data channel is a high transmission capacity medium used by data information and the token channel is a lower bandwidth channel used to accommodate the system signalings in order not to interfere with the data information. This is the basic idea behind the design of a concurrent token bus protocol (C-Token Bus) which controls the

accesses to the data channel by the control token in the token channel.

With C-token protocol, we can obtain a fairer scheduling and reduce the logical distance between two consecutive stations. Refer to Figure 1; S_1 and S_4 are the two active stations at the concurrent slot. S_4 needs to wait a transmission time and a propagation delay to start its transmission according to the type II token ring protocol. However, it only takes a transmission time, assuming that transmission time is greater than the propagation delay between S_1 and S_4 , if C-token protocol is used. Therefore, a smaller possibility is achieved in C-token protocol for S_2 and S_3 to generate packets and use the token before S_4 . Furthermore, application of clear channel technique simplifies the operations needed at the nodes since we separate the control signals and data information in two channels.

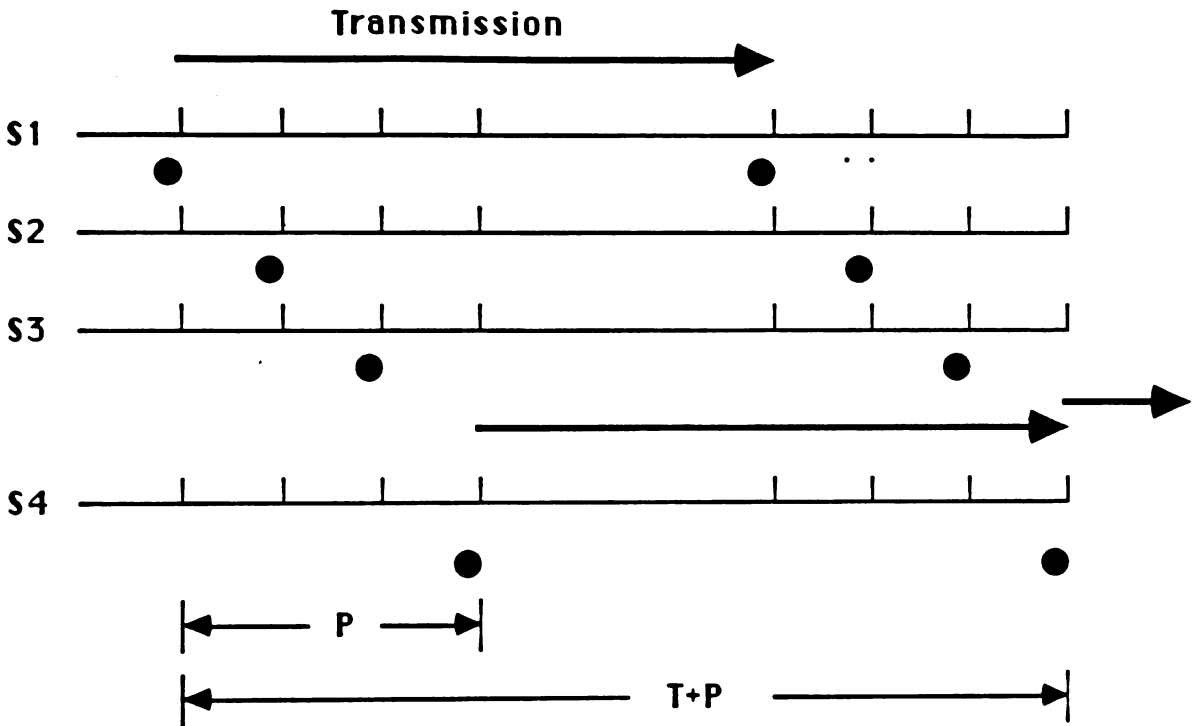


Figure 1. Comparisons of token passing for two different protocols.

2. THE MODEL AND PROTOCOL OF THE C-TOKEN BUS

A dual-bus network with M stations has a main channel to carry data packets and a token subchannel. Both channels are assumed to be time slotted with slot size, τ , equal to the average propagation delay between any two consecutive nodes in the passing list. This is distinct with the common assumption made in conventional slotted system in which τ is equal to a round trip propagation delay. The stations are synchronized and enforced to start transmission at the beginning of a time slot. As in the IEEE token bus control scheme [4], there is only one control token in the network that grants right of

access to the data channel; only one user can access the data channel at any time. This assumption conserves the conflict-free character of the protocol.

We study the model under the typical assumption [6] that each station will generate a new packet with a constant probability, σ , during a time slot. Once it generates a packet, the packet will be stored in the station buffer until it is successfully transmitted. While in this waiting state, the station is considered to be a backlogged station and no longer generates packets. The channels are assumed to be error-free. The protocol is described as follows:

- [a] The backlogged stations have to wait for the token and listen to the data channel when the token is received. If the data channel is sensed busy, the station will keep the token and persists until the data channel becomes idle.
- [b] Once the station which possesses the token finds the data channel idle it starts the transmitting on the data channel.
- [c] One time slot after the beginning of the transmission, the station releases the token to the next station on the token channel. This ensures that the rest of stations are aware of the activity on the data channel.
- [d] An idle station which receives the token simply passes it to the next station.

For a network as described above each station may be viewed as in Figure 3.2. Each station has two taps, a writing tap on the data channel and a token-receiving tap on the token channel. Here, we assume that any delay caused by software or hardware in the interface is negligible and the token takes one time slot to travel from one station to

another station.

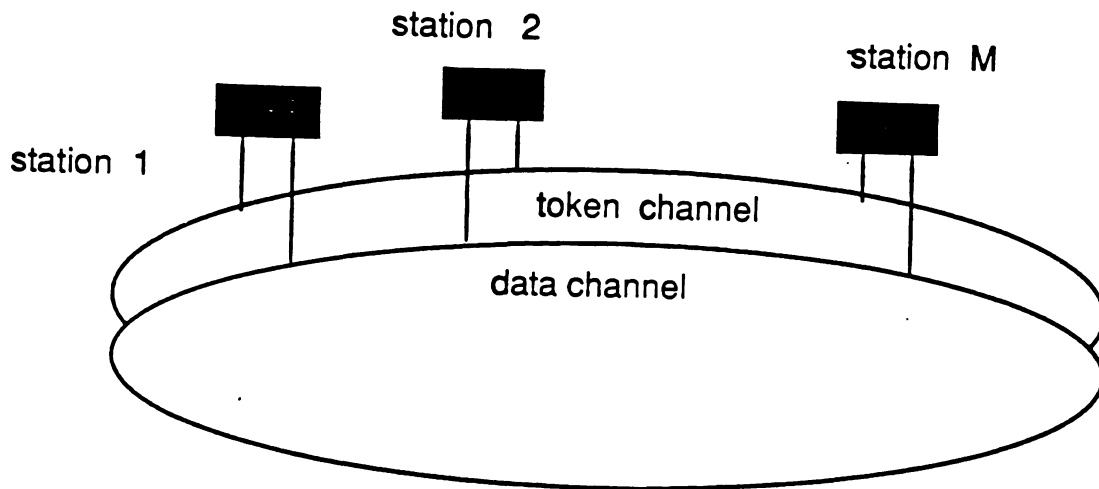


Figure 2. Configuration of a C-token ring local area network

3. ANALYSIS

The behavior of the system can be described by a cycle period which consists of an idle period and a busy period, Figure 2. An idle period is defined when no station in the system is transmitting; there may be some backlogged stations but none of them are allowed to send messages. A busy period means that the data channel is being used. Due to the protocol we proposed and the evolution of the number of the backlogged stations in the system, we can determine the stationary probability of the number of backlogged stations at the beginning of an idle period; which we denote by t_e .

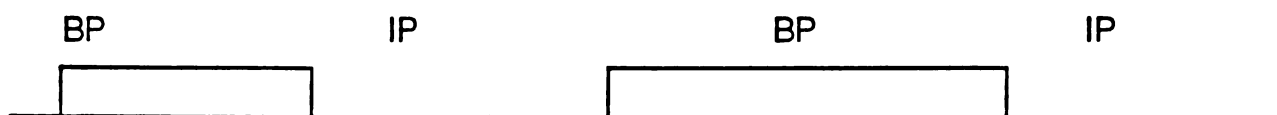


Figure 3. Periodic behavior of the data channel

We assume that total number of stations in the network is less or equal to the packet size in term of slots in 3.1 and 3.2. According to the protocol, the transmitting station releases the token one time slot after the beginning of the transmission, therefore, there are surely no backlogged stations in the system when the last transmission station in a busy period releases the control token. On the other hand, if there is any station in an active state when the token is released, one of them will receive the token before the termination of the current transmission and starts another transmission. In this case the busy period will continue, and the current transmitting station is not the last station that sends packet in the busy period. Based upon this argument we derive an expression for the distribution of backlogged stations at the beginning of an idle period.

A similar technique is used to derive the distribution of backlogged stations at the beginning of a busy period. The notation used in this section is summarized as follows;

t_e : beginning of the first slot of an idle period.

I : length of an idle period measured in slots.

N' : number of backlogged stations in the system at the beginning of t^{th} slot.

T : fixed packet size measured in slots.

M : number of stations in the system.

σ : probability for an idle station to generate a packet in a time slot.

π_i : probability of i backlogged stations in the system at time t_e

π_i^* : probability of i backlogged stations in the system at one time slot after the beginning of a busy period.

The assumption, $T \geq M$, may not be valid if the transmission speed of the data channel increases on the number of station goes up. Therefore, we provide the analysis for $T < M$ in 3.2. In this section the calculations for the distribution of the number of backlogged stations at the beginning of a busy period and an idle period are obtained. The average length of a busy period and an idle period are given in section 3.3 such that we can compute the system throughput. In section 3.4 the average backlogged station in the system is analyzed such that we can use Little's result to obtain the average waiting time for a packet in a node.

3.1. THE DISTRIBUTION OF BACKLOGGED STATIONS AT t_e FOR $T \geq M$

Without loss of generality, the stations are named 1 to M according to the token passing sequence. Thus station 1 passes the token to station 2 and station 2 passes to station 3, and so on. Finally, station M passes the token to station 1 and the passing sequence is repeated. Assuming that station 1 is the last station that transmits a packet in

a particular busy period, then, at the moment when station 1 releases the token the other $M-1$ stations must be in the idle state; there are no backlogged stations in the system. For the given condition that station 1 is the one to end the busy period the number of time slots in which the rest of the stations, stations 2 to M , can generate new packets is $(T+1-j) \bmod M$ according to their number respectively, $j=2,\dots,M$. Due to the assumption of $T \geq M$, we develop two expressions for the computation of the distribution of the number of backlogged stations at the beginning of an idle period.

CASE I : T is a multiple of M

$$\pi_0 = \prod_{j=1}^{M-1} (1-\sigma)^{(T-j) \bmod M} \quad (1)$$

$$\pi_{l \neq 0} = \sum_{E_l} \left\{ \prod_{a=1}^l 1 - (1-\sigma)^{(T-i_a) \bmod M} \prod_{b=l+1}^{M-1} (1-\sigma)^{(T-i_b) \bmod M} \right\} \quad (2)$$

$$i_a, i_b \in \{1, 2, \dots, M-1\}$$

E_l is any combination of l stations chosen from stations 2 to M . In this case, it is not possible for station 1 to be a backlogged station at t_e

CASE II : T is not a multiple of M

$$\pi_0 = (1-\sigma) \prod_{j=1}^{M-1} (1-\sigma)^{(T-j) \bmod M} \quad (3)$$

$$\pi_{l \neq 0} = (1-\sigma) \{ \pi_l \in \text{case I} \} + \sigma \{ \pi_{l-1} \in \text{case II} \} \quad (4)$$

In this case, station 1, which is assumed to initiate the busy period, returns to an idle state at the beginning of the $(t_e - 1)^{\text{th}}$ slot; it may become an active station at t_e .

The proof of the derivation of π_l is straightforward but the numerical computation requires some precaution (an explicit calculation seems to be hopeless). To overcome this numerical difficulty, we present a convolution type algorithm [7] in Appendix A for the computation of π_l 's. The proposed algorithm has a complexity of $O(M^2)$. The proof of π_0 is based upon the argument that given a busy period will be terminated by station 1, the number of slots during which station j , $j=2, 3, \dots, M$, may generate a new packet and become an active station is $(T+1-j) \bmod M$. Therefore the probability that station j does not generate a new packet in this period is $(1-\sigma)^{(T+1-j) \bmod M}$, i.e. $(1-\sigma)^{(T-j) \bmod M}$, for $j=1, 2, \dots, M-1$. The expression for π_l , $l \neq 0$, can be obtained by a similar argument that probability for stations 2 to M to become an backlogged station at the end of a busy period, the beginning of an idle period, is $1-(1-\sigma)^{(T-j) \bmod M}$. Therefore, the probability of l stations become backlogged during this period is expressed by (2).

For case I, T is a multiple of M , the beginning of the last slot of a busy period, station M sends the token back to station 1 which is the station initiates the busy period. Station 1 does not have any possibility to become a backlogged station under the assumption that this busy period will be terminated. In case II, T is not a multiple of M , station 1 returns to an idle state at the beginning of the last slot such that it can be one of the possible candidates to be a backlogged station in a time slot. That is the reason $1-\sigma$ and σ shown in (4) to represent the probability that station 1 does or does not generate a new packet in one time slot.

3.1.1. THE BACKLOG DISTRIBUTION, π_l^* , AT $t_e + I$

Consider a particular cycle, see Figure 4, which consists with an idle period starting at t_e and a busy period starts from $t_e + I$. The number of backlogged stations at the

beginning of a busy period depends on the number of backlogged stations at the beginning of the previous idle period and new arrivals at those stations which are in idle states during this period. The token is released at t_e and starts an idle period; from this point, the token is passed station by station until an active station is found and a new busy period starts. The computation of the distribution is based upon the transition probability of these two special time instants. The active station who will initiate a new busy period can be either an old backlogged station or a newly generated backlogged station. An old backlogged station means that it already had a packet waiting for transmission at time t_e . A newly backlogged station is a station that was idle at t_e and a new packet is generated at the station during the idle period. Thus, the number of backlogged stations at $t_e + I$, the beginning of a busy period, must be greater than or equal to one less than the number of backlogged stations at t_e depending upon whether the station which starts the busy period is a newly backlogged station or an old backlogged station.

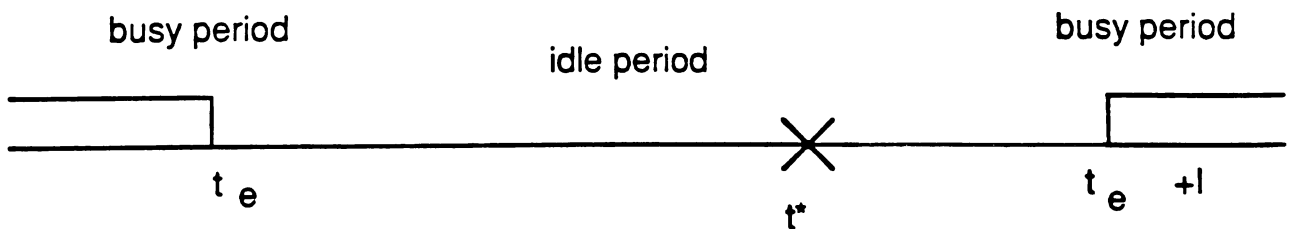


Figure 4. A snapshot on the data channel; system is not completely idle at t^*

The notation that is used in the derivation is as follows;

$P_{s,s+l,n}$: probability of having s backlogged stations in the system at t_e and $s+l$ backlogged stations at t_e+l given that a newly generated backlogged station starts the busy period, where $l \geq 0$.

$P_{s,s+l,o}$: similar to the above except that an old backlogged station starts the busy period, where $l \geq -1$.

l_1 : newly backlogged stations who have been visited by the token during the idle period.

l_2 : newly backlogged stations which have never been visited by the token during the idle period.

where

$$\begin{cases} l_1+l_2=l & l \geq 1 \\ l_1=l_2=0 & l = -1 \text{ or } 0 \end{cases}$$

For the case of $s \neq 0$, we easily obtain

$$P_{s,s+l,n} = \sum_{i=1}^{M-s-1} \frac{\binom{M-i-1}{s}}{\binom{M-1}{s}} (1-(1-\sigma)^i) \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{l=l_1+l_2} \sum_{E_{l,i}, a=1}^{l_1} \prod [1-(1-\sigma)^{i_a+1}] \quad (5)$$

$$\prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} \binom{M-s-i-1}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2-1)}$$

$$P_{s,s+l,o} = \sum_{i=1}^{M-s} \frac{\binom{M-i-1}{s-1}}{\binom{M-1}{s}} [1-(1-\sigma)^i] \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{l=l_1+l_2} \sum_{E_{l,i}, a=1}^{l_1} \prod [1-(1-\sigma)^{i_a+1}] \quad (6)$$

$$\prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} \binom{M-s-i-1}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2-1)}$$

where $\sum_{l=l_1+l_2}$ represents any possible combination of (l_1, l_2) with $l_1+l_2=l$

$E_{l_1, j}$ represents any possible set of l_1 new backlogged stations from those i stations. Each station is indexed by a number from 1 to i and station j , $j=1 \dots i$, can generate a packet in j slots. A computational algorithm is developed in Appendix A to compute

$$\sum_{E_{i,a}=1}^{I_1} \prod [1-(1-\sigma)^{i_a+1}] \prod_{b=I_1+1} (1-\sigma)^{i_b+1}$$

For the case of $s=0$, we need one more step to compute the transition probabilities, $p_{s=0,l,n}$ and $p_{s=0,l,o}$. Consider an idle period that starts with no backlogged stations in the system. We release the token and observe the number of backlogged stations in the system at the beginning of each slot. Once the number of backlogged stations is no longer zero then, we mark the time instant as t_e^* .

Clearly there are no backlogged stations in the system at the beginning of slot t_e^*-1 , $N^{t_e^*-1}=0$. We define $p_j^* = P\{N^{t_e^*}=j \mid N^{t_e^*-1}=0, j=1..M\}$. Obviously, those j backlogged stations must be generated during the $(t_e^*-1)^{th}$ slot. The probability of generating j backlogged stations in one slot can be expressed as:

$$\binom{M}{j} (1-\sigma)^{M-j} \sigma^j$$

Therefore, p_j^* can be expressed as:

$$p_j^* = \binom{M}{j} \sum_{l=1}^{\infty} (1-\sigma)^{Ml-j} \sigma^j$$

We have

$$p_j^* = \binom{M}{j} \sigma^j \frac{(1-\sigma)^{M-j}}{1-(1-\sigma)^M}, \quad j = 1..M. \quad (7)$$

The transition probabilities, $P\{N^{t_e^*} = s \mid N^{t_e^*+l} = s+l\}$, which are denoted by $p_{s,s+l,n}^*$ and $p_{s,s+l,o}^*$ for a new and an old backlogged station respectively can be obtained as;

$$P_{s,s+l,n}^* = \sum_{i=1}^{M-s-1} \frac{\binom{M-i-1}{s}}{\binom{M}{s}} [1-(1-\sigma)^i] \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{l=l_1+l_2} \sum_{E_{i,j}, a=1}^{l_1} \prod [1-(1-\sigma)^{i_a+1}] \quad (8)$$

$$\prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} \binom{M-s-i-1}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2-1)}$$

$$P_{s,s+l,o}^* = \sum_{i=0}^{M-s} \frac{\binom{M-i-1}{s-1}}{\binom{M}{s}} \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{l=l_1+l_2} \sum_{E_{i,j}, a=1}^{l_1} \prod [1-(1-\sigma)^{i_a+1}] \quad (9)$$

$$\prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} \binom{M-s-i}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2)}$$

Therefore the distribution of the number of backlogged stations at the beginning of a busy period, denoted by π_i^* , can be obtained from;

$$\pi_i^* = \sum_{j=1}^{i+1} \pi_0 p_j^* p_{j,i}^* + \sum_{k=1}^{i+1} \pi_k p_{k,i} \quad , \quad 0 \leq i \leq M-1 . \quad (10)$$

where

$$p_{j,i}^* = p_{j,i,n}^* + p_{j,i,o}^*$$

$$p_{k,i} = p_{k,i,n} + p_{k,i,o} .$$

3.2. FOR THE CASE OF $T < M$

In 3.1 we analyze the system that the packet transmission time in slots is bigger than the number of stations in the system such that the token can search all the stations in the network in a transmission time. Therefore we have a determinable point which has been used to calculate the distribution of the backlogged station at the beginning of an idle period; thus, the time instant which is M time slots before the end of the busy period.

We have exact information of the system at this point; all the stations in the network must be in idle states, otherwise, the busy period will not stop.

The assumption above will not be valid if the transmission speed of the data channel increases to a certain degree such that token can not visit all the stations within a transmission time. In this case, $T < M$, no such time instant we can use to analyze the system. If we investigate a system of a busy period, the time instants that a transmitting station releases the token, one time slot after its transmission, can be recognized as the regeneration points in the busy period. The probability distribution of the backlogged stations at those points are identical from which onward to the future of the evolution of the number of backlogged stations is a probabilistic replica of the previous one. Such characteristic can be used to obtain the distribution of backlogged stations at those regeneration points. The distribution of backlogged stations at one time slot after the beginning of a busy period, π_i^* , is considered to be identical to the distributions at any other point in the same busy period.

We will analyze the behavior of the system in the busy period such that all considerations of the analysis is based upon the given condition that all events are in the same busy period. The conditional transition probabilities q_{ij} 's are defined as the probability that a transmitting station releases the token with i backlogged stations in the system and j backlogged stations in the system when the next transmitting station releases the token which is $T+1$ slots after the previous event. Obviously, the given condition is that token must find an active station within T time slots in order to be able to continue the busy period.

We obtain the q_{ij} as follows;

$$q_{0j} = \frac{1}{1 - \prod_{n=1}^T (1-\sigma)^n} \sum_{i=1}^T \frac{\binom{M-i-1}{s}}{\binom{M-1}{s}} (1-(1-\sigma)^i) \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{j=l_1+l_2} \sum_{E_{l_1, l_2} a=1}^{l_1} [1-(1-\sigma)^{i_s+1}] \quad (11)$$

$$\prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} \binom{M-s-i-1}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2-1)}$$

$$q_{ij} = \frac{1}{1 - \frac{\binom{M-T-1}{i}}{\binom{M-1}{i}} \prod_{n=1}^T (1-\sigma)^n} \left\{ \sum_{i=1}^T \frac{\binom{M-i-1}{s}}{\binom{M-1}{s}} (1-(1-\sigma)^i) \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{j-i=l_1+l_2} \sum_{E_{l_1, l_2} a=1}^{l_1} [1-(1-\sigma)^{i_s+1}] \right.$$

$$\left. \prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} \binom{M-s-i-1}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2-1)} \right\} \quad (12)$$

$$+ \sum_{i=1}^T \frac{\binom{M-i-1}{s-1}}{\binom{M-1}{s}} [1-(1-\sigma)^i] \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{j-i=l_1+l_2} \sum_{E_{l_1, l_2} a=1}^{l_1} [1-(1-\sigma)^{i_s+1}]$$

$$\left. \prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} \binom{M-s-i-1}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2-1)} \right\}$$

where $\sum_{j-i=l_1+l_2}$ represents any possible combination of (l_1, l_2) with $l_1+l_2=j-i$, however,

$j-i = 0$ or -1 , then, $j = i = 0$.

The definition of $E_{l,i}$ is the same as in 3.1.

The first term in (12) represents the condition that the next transmission station is a new backlogged station and the condition of a old backlogged station transmitting next is represented in the second term. The consideration of q_{ij} is similar to the derivation of (5)(6).

Now, the balance equations for the regeneration points in the busy period can be expressed as follows;

$$\begin{aligned}
 \pi_0^* &= \pi_0^* q_{00} + \pi_1^* q_{10} \\
 \pi_1^* &= \pi_0^* q_{01} + \pi_1^* q_{11} + \pi_2^* q_{21} \\
 &\vdots \\
 &\vdots \\
 \pi_n^* &= \sum_{i=0}^{n+1} \pi_i^* q_{in} \\
 &\vdots \\
 &\vdots \\
 \pi_{M-1}^* &= \sum_{i=0}^{M-1} \pi_i^* q_{iM-1}
 \end{aligned} \tag{13}$$

In matrix form we have

$$\Pi^* = \Pi^* Q \tag{14}$$

then, Π^* can be obtain.

Based upon the π_i^* obtained in (14) we are able to calculate π_i ; the distribution of the backlogged stations at the beginning of an idle period. Again, the transition probability is calculated given that the transition will change the state of the system from busy period to idle period.

From now on we change the definition of q_{ij} to the conditional probability that the last transmitting stations releases the token with i backlogged stations in the system and after T slots of time no active station is found while j backlogged stations in the system. Clearly, if $i > M - T$, the transition probability is zero since token will find an active station in T time slots with probability one.

$$q_{ij} = \frac{1}{\binom{M-T-1}{i} \prod_{n=1}^T (1-\sigma)^n} \sum_{i=1}^T \frac{\binom{M-i-1}{s}}{\binom{M-1}{s}} (1-(1-\sigma)^i) \prod_{j=1}^{i-1} (1-\sigma)^j \sum_{j-i=l_1+l_2} \sum_{E_{i,T} a=1}^{l_1} \prod [1-(1-\sigma)^{i_a+1}] \quad (15)$$

for $i < M - T$ and $i \leq j$.

Then, π_n can be obtained as follows;

$$\pi_n = \sum_{i=0}^n \pi_i^* q_{in} \quad (16)$$

The principle behind the derivation in this section is that the distributions of the backlogged station at the regeneration points are identical. The first transmission in a busy period is considered to be exactly the same as the rest of the regeneration points, even though, the transitions into the states at this point are from an idle period. The same consideration is used for the last transmission in the busy period even though the transitions out from the states at this point are not in the busy period any more.

3.3. THROUGHPUT ANALYSIS

Let S be the stationary throughput of the channel, i.e., S is defined as the fraction of channel time occupied by successful transmissions. A standard result from renewal theory [8] tells us that S can be obtained as the ratio of time the channel is carrying successful transmission during a cycle averaged over all cycles, to the average cycle length.

Therefore, we have

$$S = \frac{T \sum_{i=0}^{M-1} \pi_i^* R_i}{\sum_{i=0}^{M-1} \pi_i I_i + (T+1) \sum_{i=0}^{M-1} \pi_i^* R_i} \quad (17)$$

where I_i is the expected length of an idle period, given $N^{t_e} = i$ and R_i is the expected number of transmissions in a busy period given $N^{t_e + I} = i$. These values are derived in the next section.

3.3.1. THE EXPECTED IDLE LENGTH I_k

According to the number of backlogged stations at the beginning of an idle period, we have two different sets of expressions for I_k .

CASE I : $k \neq 0$

$$I_k = \sum_{i=1}^{M-k} i \left\{ \frac{\binom{M-i-1}{k-1}}{\binom{M-1}{k}} \prod_{j=1}^{i-1} (1-\sigma)^j + \frac{\binom{M-i-1}{k}}{\binom{M-1}{k}} [1 - (1-\sigma)^i] \prod_{j=1}^{i-1} (1-\sigma)^j \right\} \quad (18)$$

Since the number of the backlogged stations, k , at t_e is not zero, the token will find an active station in at most $M-k$ slots. This equation can be interpreted as follows; the first part in the bracket represents an old backlogged station that terminates the idle period while the second part represents the event that a new busy period is started by a new backlogged station.

CASE II : $k = 0$

In this case, we need to add another period in which the number of backlogged stations in the system changes from zero to non-zero. This event follows the geometric distribution with parameter $\lambda=1-(1-\sigma)^M$, the probability of at least one new packet has been generated in the system during one time slot. The expected length of this period can be expressed as $\frac{1}{1-(1-\sigma)^M}$.

The expected length to reach the beginning of the next busy period, which is denoted by I_k^* , can be obtained by

$$I_k^* = \sum_{i=0}^{M-k} i \left\{ \frac{\binom{M-i-1}{k-1}}{\binom{M}{k}} \prod_{j=1}^{i-1} (1-\sigma)^j + \frac{\binom{M-i-1}{k}}{\binom{M}{k}} [1-(1-\sigma)^i] \prod_{j=1}^{i-1} (1-\sigma)^j \right\} \quad (19)$$

Therefore, the expected idle period given that there are zero backlogged stations at the beginning of the idle period, $I_{k=0}$, can be obtained as

$$I_{k=0} = \frac{1}{1-(1-\sigma)^M} + \sum_{k=1}^{M-1} I_k^* P_k^* \quad (20)$$

3.3.2. COMPUTATION OF R_i

In order to calculate the number of successful transmissions, R_i , in a busy period we introduce a state descriptor (j, m, l) [7] which describes the system at the instant the token is released. The state descriptor can be interpreted as follows; station j , the station which starts the busy period is named 1, releases the token in the m^{th} slot in the current transmission and l backlogged stations exist in the system at this moment. We define the variable $Y(j, m, l)$'s as the expected number of successful transmissions in the period from the observed point, system is in state (j, m, l) , to the end of the busy period. The third parameter, l , indicates that there are l backlogged stations accumulated in the

system at the moment that station j releases the token such that $l \in \{0,1,\dots,M-1\}$ for $j=1$ and $\in \{0,1,2,\dots,M-2\}$ for $j \neq 1$. The variables Y 's satisfy the following linear equations

$$Y(j, m \neq 1, l) = \sum_{m', l'} Y(j+1, m', l') P \{(j, m, l) \rightarrow (j+1, m', l')\} \quad (21)$$

$$Y(j, m=1, l) = 1 + \sum_{m', l'} Y(j+1, m', l') P \{(j, 1, l) \rightarrow (j+1, m', l')\} \quad (22)$$

where $P \{(j, m, l) \rightarrow (j+1, m', l')\}$ represents the transition probabilities between two consecutive observed points at which the system is in states (j, m, l) and $(j+1, m', l')$ respectively. We need $Y(1, 1, l)$'s, for $l=0, 1, \dots, M-1$. For stationary consideration Y 's must be independent of j . We write the linear equations in matrix form as follows:

$$Y = YB + D$$

where

$$Y = \begin{bmatrix} Y(1,0) \\ Y(1,1) \\ \vdots \\ \vdots \\ Y(1,M-1) \\ Y(2,0) \\ \vdots \\ \vdots \\ Y(2,M-2) \\ \vdots \\ \vdots \\ Y(T,0) \\ T(T,1) \\ \vdots \\ \vdots \\ Y(T,M-2) \end{bmatrix} \quad D = \begin{bmatrix} 1_0 \\ 1_1 \\ 1_2 \\ \vdots \\ \vdots \\ 1_{M-1} \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \quad (23)$$

The transition $\{(j,m,l) \rightarrow (j,1,l')\}$ indicates that the token is released when the state of the system is (j,m,l) and the next station which will receive the token happens to be an active station. Therefore, the token will be released in the next transmission by station $j+1$; one time slot after the transmission. The components of the matrix B represent the transition probability from state (m,l) to state (m',l') associated with two consecutive token releasing events. The sequence of the B matrix is the same as the Y matrix along the row and the column dimension. The expressions for these elements are explicitly given in Appendix B. Then, the Y matrix can be obtained from

$$Y = (I - B)^{-1}D \quad (24)$$

Obviously, the information we want, R_i 's, are equal to $Y(1,i)$'s since we assume that the station which initiates the observed busy period is named as 1.

3.4. DELAY ANALYSIS

Let N be the average channel backlog, obtained as the ratio of the expected sum of backlogged stations over all slots in a cycle (average all cycles), to the average cycle length. Therefore we have

$$N = \frac{\sum_{i=0}^{M-1} \pi_i B_i^I + \pi_i^* B_i^B}{\sum_{i=0}^{M-1} [\pi_i I_i + \pi_i^* R_i (T+1)]} \quad (25)$$

where

B_i^I : expected sum of backlogged stations over all slots in an idle period given $N^{t_c=i}$.

B_i^B : expected sum of backlogged stations over all the slots in a busy period given $N^{t_c+i}=i$.

The notation used in the following section is summarized as follows;

$BK_{s,0}^{s+l}$: total number of backlogged stations over all the slots in an idle period, given $N^{t_c=s}, N^{t_c+l}=s+l$ and an old backlogged station starts the new busy period.

$BK_{s,n}^{s+l}$: same definition as above except that a new backlogged station starts the busy period.

$\overline{BK}_{s,0}^{s+l}$: total number of backlogged station over all the slots in the period between t_c^* and t_c^*+l , given that one of the backlogged stations at t_c^* starts the busy period.

$\overline{BK}_{s,n}^{s+l}$: same as the definition above except that the station which starts the busy period

is not one of the backlogged stations in the system at t_e^* .

3.4.1. COMPUTATION OF B_i^l

Consider an idle period with $N^{t_e^*} = s$ and $N^{t_e^*+l} = s+l$. The total number of backlogged stations over all slots in this idle period can be categorized into two cases. Firstly, it is an old backlogged stations that terminates the idle period. This implies $l+1$ backlogged stations are newly generated in this idle period, then

$$BK_{s,\sigma}^{s+l} = \sum_{i=1}^{M-s} \frac{\binom{M-i-1}{s-1}}{\binom{M-1}{s}} \prod_{j=1}^{i-1} (1-\sigma)^j \left\{ s \times i + \sum_{l=l_1+l_2} \sum_{E_{l,i}, a=1}^{l_1} \prod [1-(1-\sigma)^{i_a+1}] \right. \quad (26)$$

$$\left. \prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} (\hat{b}_{i_b} l_1 + l_2 \hat{b}_i) \binom{M-s-i}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2)} \right\}$$

where \hat{b}_j is the expected number of slots that a new backlogged station is backlogged in the interval of length j slots given that the station becomes a new backlogged station in this interval.

Or secondly, a newly backlogged station stops the idle period, in which case

we have,

$$BK_{s,\sigma}^{s+l} = \sum_{i=1}^{M-s-1} \frac{\binom{M-i-1}{s}}{\binom{M-1}{s}} [1-(1-\sigma)^i] \prod_{j=1}^{i-1} (1-\sigma)^j \left\{ s \times i + \sum_{l=l_1+l_2} \sum_{E_{l,i}, a=1}^{l_1} \prod [1-(1-\sigma)^{i_a+1}] \right. \quad (27)$$

$$\left. \prod_{b=l_1+1}^i (1-\sigma)^{i_b+1} (\hat{b}_{i_b} l_1 + l_2 \hat{b}_i) \binom{M-s-i-1}{l_2} [1-(1-\sigma)^{i+1}]^{l_2} (1-\sigma)^{(i+1)(M-s-i-l_2-1)} \right\}$$

For cases in which there are no backlogged stations in the system at time t_e , we start counting the backlogged stations from t_e^* , the time instant that the number of backlogged stations in the system is not zero. The equations are similar except that the lower limit of the first summation starts at $i = 0$ since the system may start a busy period immediately. Therefore the B_i^l can be expressed as

$$B_{s \neq 0}^l = \sum_{l=1}^{M-1} BK_{s,o}^{s+l} + \sum_{l=0}^{M-1} BK_{s,n}^{s+l} \quad (28)$$

$$B_0^l = \sum_{s=1}^{M-1} \pi_s^* \left[\sum_{l=1}^{M-1} \overline{BK}_{s,o}^{s+l} + \sum_{l=0}^{M-1} \overline{BK}_{s,n}^{s+l} \right] \quad (29)$$

3.4.2. COMPUTATION OF B_i^B

Let $Z(j, m, l)$ represent the expected number of backlogged stations in the system over all the slots in the period between the instant that the system is in the state (j, m, l) to the end of the busy period. Then, we also have a set of linear equations for the Z 's.

$$\begin{aligned} Z(j, m, l) = l + \sum_{l'=\max\{l-1,0\}} [K_{m,l'}^{1,l'} + Z(j+1, l')] P \{(m, l) \rightarrow (1, l')\} \\ + \sum_{l'=\max\{l-1,0\}} Z(j+1, m+1, l') P \{(m, l) \rightarrow (m+1, l')\} \end{aligned} \quad (30)$$

The Z 's are defined at the moment when the token is released such that the number of time slots between two consecutive events will be either one or $(T-1+m)$; the latter case implies that the next station is active and it will hold the token until one time slot after its own transmission.

$K_{m,l}^{1,l'}$ represents the sum of backlogged stations over all the slots in the period we just described above but does not include the l backlogged stations which already existed

at

the current m^{th} slot.

$$\begin{aligned}
 K_{m,l}^{1,l'} = \frac{l(l-1)(T+1-m)}{\Delta} & \left[\binom{M-l-1}{l'-l+1} [1-(1-\sigma)^{T+2-m}]^{l'+1-l} (1-\sigma)^{(T+2-m)(M-l'-2)} \right. \\
 & (1-\sigma)^2 (l'-l+1) \hat{b}_{T+1-m} + \binom{M-l-1}{l'-l} [1-(1-\sigma)^{T+2-m}]^{l'-l} (1-\sigma)^{(T+2-m)(M-l'-1)} \\
 & \left. [1-(1-\sigma)^2] [(l'-l) \hat{b}_{T+1-m} + \hat{b}_2] \right] \\
 + \frac{(\Delta-l) \sigma l (T+1-m)}{\Delta} & \left[\binom{M-l-2}{l'-l} [1-(1-\sigma)^{T+2-m}]^{l'-l} (1-\sigma)^{(T+2-m)(M-l'-2)} \right. \\
 & (1-\sigma)^2 (l'-l) \hat{b}_{T+1-m} + \binom{M-l-2}{l'-l-2} [1-(1-\sigma)^{T+2-m}]^{l'-l-1} (1-\sigma)^{(T+2-m)(M-l'-1)} \\
 & \left. [1-(1-\sigma)^2] [(l'-l-1) \hat{b}_{T+1-m} + \hat{b}_2] \right]
 \end{aligned} \tag{31}$$

$$\text{where } \Delta = \begin{cases} M-1 & \text{if } m=1 \\ M-2 & \text{if } m \neq 1 \end{cases}, \quad (M-1) \geq l' \geq \max\{l-1, 0\}$$

The equation in matrix form is

$$Z = A + Z B$$

Then

$$Z = A (I - B)^{-1} \tag{32}$$

where B is the transition matrix developed in 3.3.2,

and

$$A = \begin{bmatrix} a(1,0) \\ a(1,1) \\ \vdots \\ a(M-1) \\ a(2,0) \\ \vdots \\ a(2,M-2) \\ \vdots \\ \vdots \\ a(T,0) \\ \vdots \\ \vdots \\ a(T,M-1) \end{bmatrix} \quad (33)$$

where

$$a(i,j) = l + \sum_{k=\max\{j-1,0\}}^{M-1} K_{i,j}^{1,k} P\{(i,j) \rightarrow (1,k)\} \quad (34)$$

Therefore we can obtain B_i^B as $Z(1,i)$. Finally, from Little's formula, the average packet delay DELAY is given by

$$DELAY = \frac{N}{S}$$

4. NUMERICAL RESULTS AND DISCUSSION

From (24) (32) we know that the calculation of the analytical solution involves a matrix with dimension $T(M-1) \times T(M-1)$ such that a small network is chosen as an example to show the accuracy of the analytical solution in order to avoid the computation difficulties. The analytical solution is verified by the simulation results in Figure 5 and 6. The arrival rate in Figure 5 represents the probability that an idle station will generate a packet in a slot of time. According to the assumption that a backlogged station will not generate any packet until the packet in the buffer is transmitted successfully such that the

maximum access delay can be calculated by $D_{\max} = (M-1)(T+1)$ and D_{\max} is 60 slots in this case. We can see that $D=60$ is the asymptote in Figure 5, at this moment all the stations in the network are busy at all time.

Figure 7 compares the performances of the new protocol with the conventional token bus protocol by a network with 20 stations and $T=20$. We can see that the new protocol performs better than the old one does in any case. The relative improvement which is calculated by $\frac{|D_o - D_n|}{D_o} \%$ is shown in Figure 8. From Figure 8 we can see that the optimal operating point is at $\sigma = 0.015$, at this point we can achieve 35% improvement and the system throughput is about 0.6.

An important notice from Figure 9 is that a smaller packet size network is more suitable than a system with larger packet size to implement the new protocol. In Figure 9, for a $M=20$ network, $T=20$ obtains better improvement than the improvement that $T=40$ can obtain. This result can be translated to that the new protocol will be a good choice for the system which use higher transmission speed medium because the transmission time of a packet will be shorter if a higher transmission speed medium is used. Figure 10 shows that a network will obtain better improvement if the number of stations in the network increases. This can be explained by the token passing time since at the same throughput a fewer population network has fewer number of token passing between two active stations such that the relative improvement is not significant as a network with larger number of stations.

By investigating Figure 8, 9, 10 we can see that the new protocol obtain the optimal improvement when the system throughput is in the range of 0.6 and 0.7, even though the

σ 's are different for different systems. And the throughput can be calculated roughly by $throughput = \sigma \times M \times T$ such that we can decide the optimal operating point for the new protocol by controlling the σ 's for each station. The new double bus protocol is proved to perform better than the conventional single bus network, especially, for a high speed local area network with larger number of stations.

In section 3, we assume that the transmission time of a packet in slots is bigger than the number of stations in the network. This assumption simplifies the analysis significantly, however, if the transmission capacity increases the transmission time of a packet decreases then, this assumption is no longer valid. The results, refer to Figure 11 12 13, show that the conclusions made above is still hold. In such environment, type I token ring protocol can only obtain throughput of 0.2 for a network $M=49$ $T=10$ based upon the simple model developed in [11]. A remarkable improvement is achieved by the new C-token protocol.

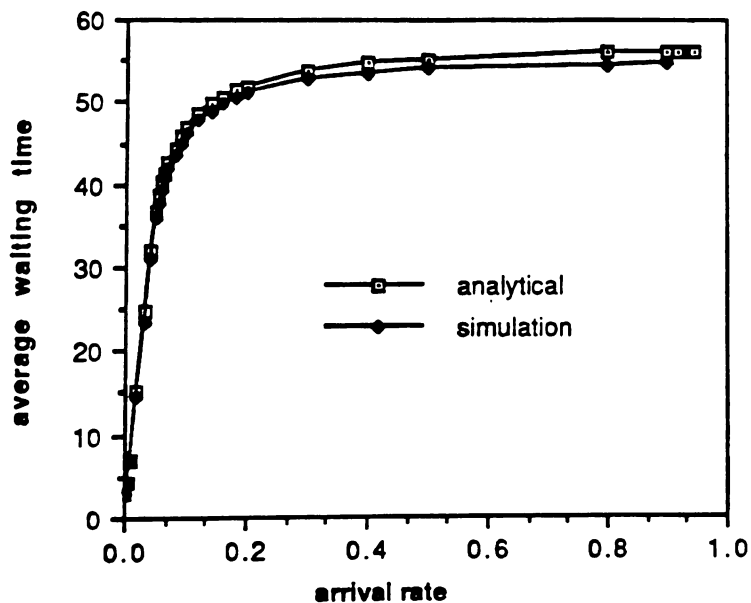


Figure 5. Analytical and simulation comparison for $M=6$ $T=10$.

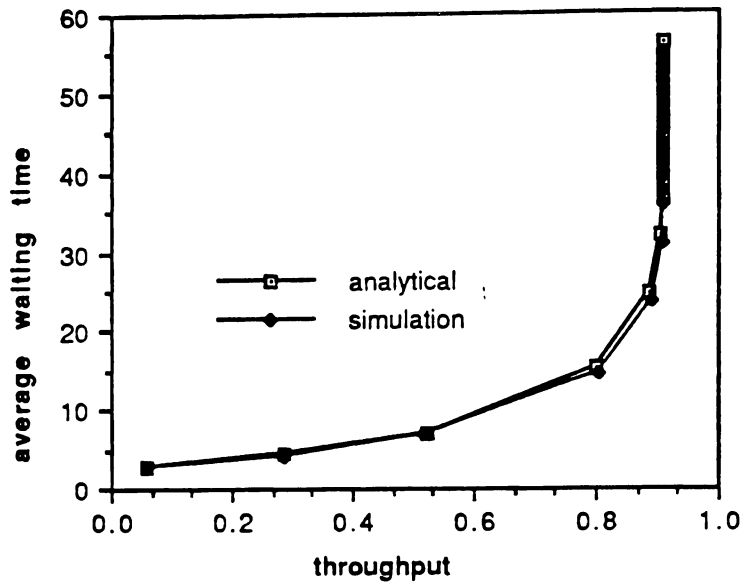


Figure 6. Analytical and simulation comparison for $M=6$ $T=10$.

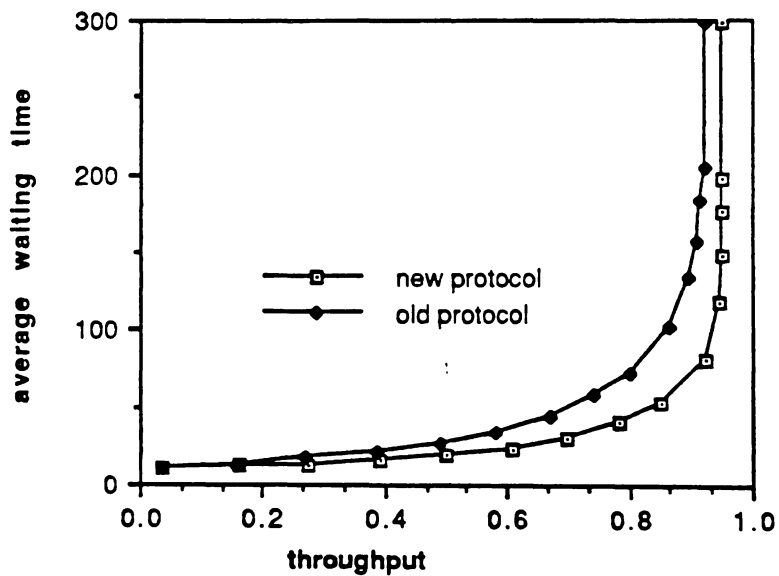


Figure 7. Comparison on the average waiting time for $M=20$ $T=20$.

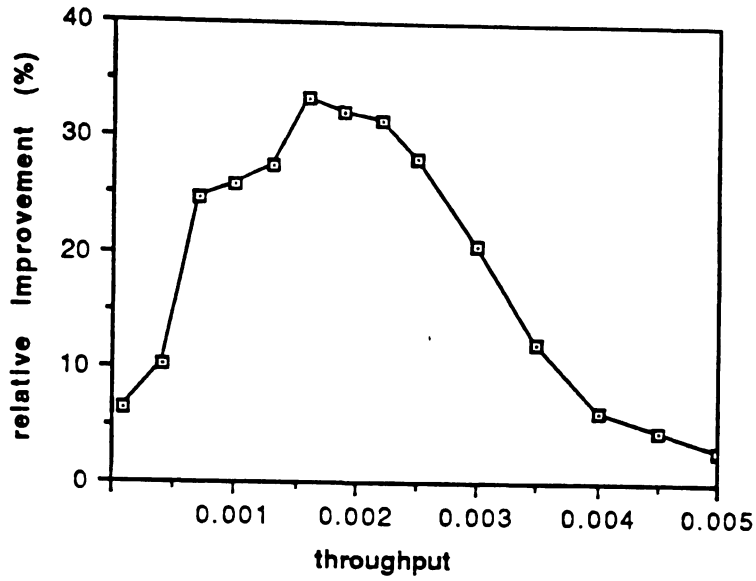


Figure 8. Relative improvement on average waiting time for $M=20$ $T=20$.

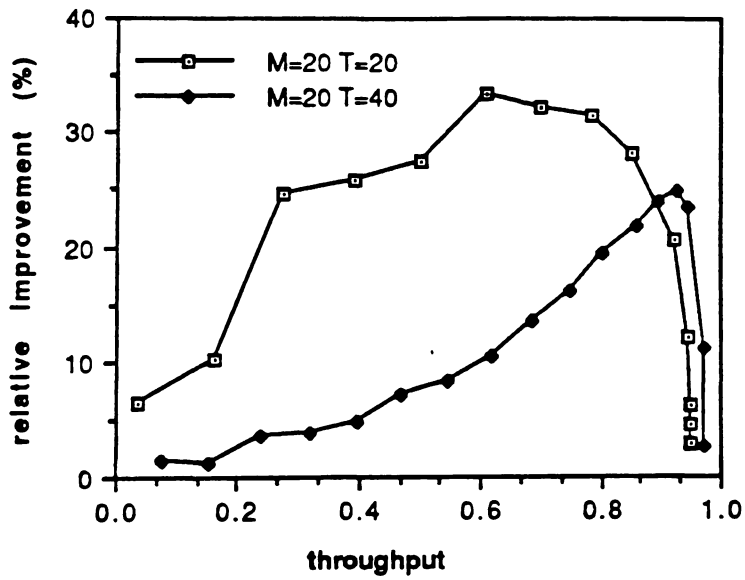


Figure 9. Relative improvement on average waiting time for different packet size.

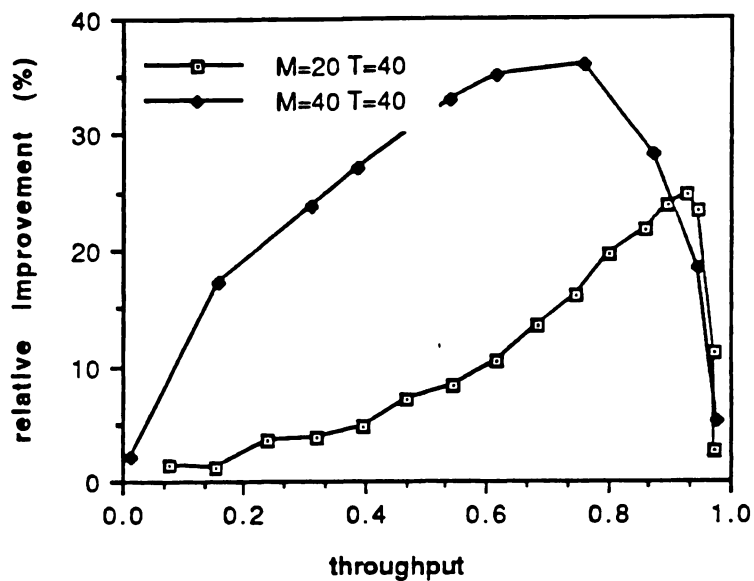


Figure 10. Relative improvement on average waiting time for different number of stations.

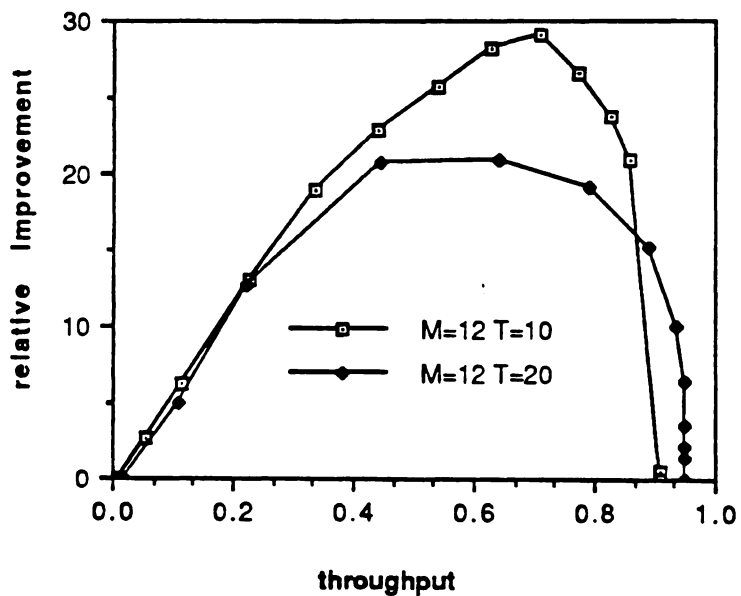


Figure 11. Relative improvement on average waiting time for different packet size, $a < 1$.

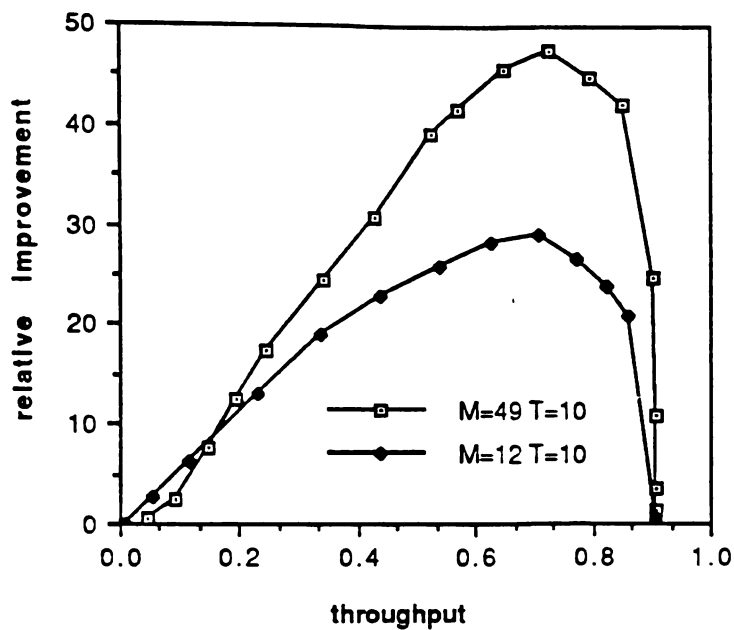


Figure 12. Relative improvement on average waiting time for different number of stations, $a < 1$

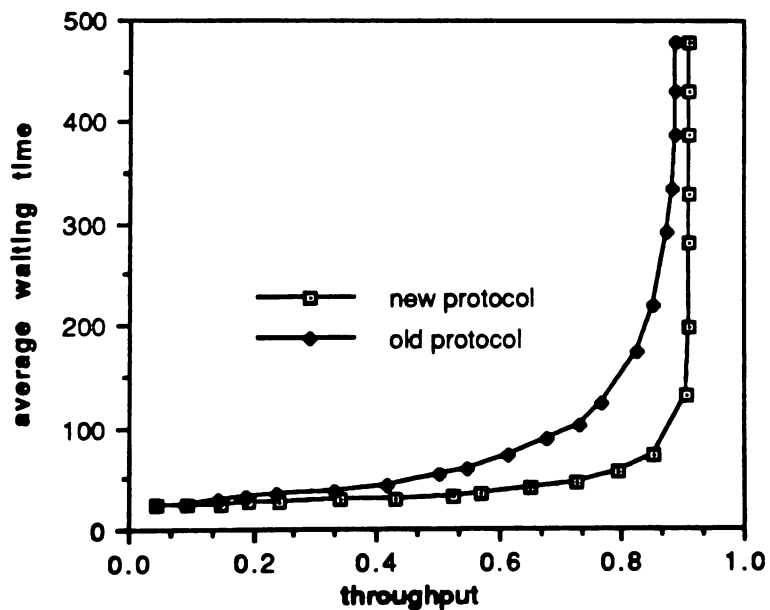


Figure 13. Comparison on the average waiting time for $M=49 T=10$, $a < 1$.

APPENDIX A

Computational algorithm for $\Pi_{l \neq 0}$ and $P_j^*(k)$

$$\begin{aligned}
\Pi_l &= \sum_{E_l k=1}^l \prod [(1-(1-\sigma)^{(T-i_k) \bmod M})] \prod_{k'=l+1}^{M-1} (1-\sigma)^{(T-i_{k'}) \bmod M} \\
&= \sum_{E_l k=1}^l \prod [(1-(1-\sigma)^{(T-i_k) \bmod M})] (1-\sigma)^{\sum_{k'=l+1}^{M-1} (T-i_{k'}) \bmod M} \\
&= (1-\sigma)^{\frac{M(M-1)}{2}} \sum_{E_l k=1}^l \prod [(1-\sigma)^{-(T-i_k) \bmod M} - 1]
\end{aligned} \tag{A.1}$$

Since,

$$\begin{aligned}
&\sum_{k'=l+1}^{M-1} (T-i_{k'}) \bmod M \\
&= \sum_{k'=1}^{M-1} (T-i_{k'}) \bmod M - \sum_{k=1}^l (T-i_k) \bmod M \\
&= \frac{M(M-1)}{2} - \sum_{k=1}^l (T-i_k) \bmod M
\end{aligned} \tag{A.2}$$

And,

$$\begin{aligned}
&\sum_{E_l k=1}^l \prod [(1-\sigma)^{-(T-i_k) \bmod M} - 1] \\
&= \sum_{i_1=1}^{M-1} [(1-\sigma)^{-(T-i_1) \bmod M} - 1] \cdot \sum_{i_2=i_1+1}^{M-1} [(1-\sigma)^{-(T-i_2) \bmod M} - 1] \\
&\quad \dots \sum_{i_j=i_{j-1}+1}^{M-1} [(1-\sigma)^{-(T-i_j) \bmod M} - 1]
\end{aligned} \tag{A.3}$$

Define:

$$\begin{aligned}
 P_j(k) = & \sum_{i_1=M-j-k+1}^{M-j} [(1-\sigma)^{-(T-i_1) \bmod M-1}] \cdot \sum_{i_2=i_1+1}^{M-j+1} [(1-\sigma)^{-(T-i_2) \bmod M-1}] \\
 & \dots \sum_{i_j=i_{j-1}+1}^{M-1} [(1-\sigma)^{-(T-i_j) \bmod M-1}]
 \end{aligned} \tag{A.4}$$

For fixed j , $j \leq M-1$, we can design the recursive relation to obtain $P_j(k)$'s:

$$P_0(k)=1, \text{ for } 1 \leq k \leq M-1,$$

$$P_j(1)=[(1-\sigma)^{-(T-M+j) \bmod M-1}]P_{j-1}(1), \tag{A.5}$$

$$P_j(k)=[(1-\sigma)^{-(T-M+j+k-1) \bmod M-1}]P_{j-1}(k)+P_j(k-1), \text{ for } 2 \leq k \leq M-j.$$

From the Π_l can be obtained by

$$\Pi_l = (1-\sigma)^{\frac{M(M-1)}{2}} P_l(M-l), \text{ for } 1 \leq l \leq M-1. \tag{A.6}$$

A similar algorithm is developed for the computation of

$$\sum_{E_{l_1, i}} \prod_{k=1}^{l_1} [1-(1-\sigma)^{i_k+1}] \prod_{k=l_1+1}^i (1-\sigma)^{i_k+1}$$

where

$$\begin{aligned} E_{l_1, i} &= \{i_1, i_2, \dots, i_{l_1}\} \quad \text{and} \quad i_k, k=1, \dots, l_1 \in \{1, 2, \dots, i\} \\ &= \sum_{E_{l_1, i}} (1-\sigma)^{i-i_1+1} \prod_{k=1}^{i_1} [1-(1-\sigma)^{i_k+1}] \\ &= (1-\sigma)^{\frac{i(i+3)}{2}} \sum_{E_{l_1, 1}} \prod_{k=1}^{l_1} [(1-\sigma)^{-(i_k+1)} - 1] \end{aligned} \quad (\text{A.7})$$

Again we rewrite as follows;

$$\begin{aligned} &\sum_{E_{l_1, i}} \prod_{k=1}^{l_1} [(1-\sigma)^{-(i_k+1)} - 1] \\ &= \sum_{i_1=1}^{i-l_1+1} [(1-\sigma)^{-(i_1+1)} - 1] \cdot \sum_{i_2=i_1+1}^{i-l_1+2} \dots \sum_{i_{l_1}=i_{l_1-1}+1}^i [(1-\sigma)^{-(i_{l_1}+1)} - 1] \end{aligned} \quad (\text{A.8})$$

Define :

$$P_{l_1}^*(k) = \sum_{i_1=i-l_1-k+2}^{i-l_1+1} \dots \sum_{i_{l_1}=i_{l_1-1}+1}^i \quad (\text{A.9})$$

The following recursive relations can be used to obtain $P_{l_1}^*(i-l_1)$ with a complexity of $O(i^2)$.

$$\begin{aligned} P_0^*(k) &= 1 \\ P_{l_1}^*(1) &= [(1-\sigma)^{-(i-l_1+2)} - 1] P_{l_1-1}^*(1) \\ P_{l_1}^*(k) &= [(1-\sigma)^{-(i-l_1-k+3)} - 1] P_{l_1-1}^*(k) + P_{l_1}^*(k-1) \end{aligned} \quad (\text{A.10})$$

APPENDIX B

Computation of the elements of the transition matrix B; $P\{(l_1, k) \rightarrow (l_2, m)\}$.

Define,

$$\Delta = \begin{cases} M-1 & l_1=1 \\ M-2 & l_1 \neq 1 \end{cases}$$

and,

$$\beta_1 \equiv 1 - (1-\sigma)^{T+2-m}$$

$$\beta_2 \equiv (1-\sigma)^{T+2-m}$$

PROPOSITION B.1. - For the case, $l_2 = l_1 + 1$.

$$\begin{aligned} & P\{(l_1, k) \rightarrow (l_1 + 1, m)\} \\ &= \binom{\Delta - k}{\Delta} \binom{\Delta - k - 1}{m - k} \sigma^{m-k} (1-\sigma)^{\Delta - m} \end{aligned} \quad (\text{B.1})$$

B.1 gives all the transitions for a station passing the token to an idle station.

PROPOSITION B.2. - For the case, $l_2 = 1$.

$$\begin{aligned} & P\{(l_1, k) \rightarrow (1, m)\} \\ &= \binom{k}{\Delta} \left\{ \binom{\Delta - k}{l - k + 1} \beta_1^{l-k+1} \beta_2^{\Delta-l-1} (1-\sigma)^2 + \binom{\Delta - k}{l - k} \beta_1^{l-k} \beta_2^{\Delta-l} [1 - (1-\sigma)^2] \right\} \end{aligned} \quad (\text{B.2})$$

PROOF. We assume that station 1 is currently occupying the data channel such that the station releases the token at l_1^{th} slot is numbered by $(l_1 \bmod M)$. For simplicity we call them stations l_1 and l_2 .

B.1 represents the case that station l_1 releases the token to the next station, l_2 , which is idle. If $(l_1 \bmod M) = 1$, station l_1 is occupying the data channel, station l_2 must be one

of the remaining $M-1-k$ idle stations (probability $\frac{M-k-1}{M-1}$). $m-k$ stations out of $M-2-k$ idle stations will generate new packets during this time slot given station l_2 is still idle (probability is shown in the remaining part of B.1).

If station l_1 is not the station that is transmitting , station l_2 may be one of the idle stations with probability $\frac{M-2-k}{M-2}$. The remainder of the analysis is the same as the first case.

B.2 represents the cases that station l_2 is waiting for permission to transmit. Therefore station l_2 will hold the token until one time slot after its own transmission which is $T+2-m$ slots later.

There are two possibilities for station l_2 needing to be examined; The first part in B.2 shows that station l_2 has already backlogged at the moment when station l_1 is releasing the token; this probability is given by $(\frac{k}{\Delta})$. Then in order to satisfy the given transition $l+1-k$ stations out of $\Delta-k$ stations in the system have to become backlogged in this period; this probability can be obtained as $(\binom{\Delta-k}{l-k+1})\beta_1^{l+1-k}\beta_2^{\Delta-l-1}$. Since the station who is currently occupying the data channel will become idle $T-m$ slots after the current slot, the probability for this station becoming a backlogged station at the next token releasing time, $T-m+2$ slots after the current slot, is $(1-\sigma)^2$, on the other hand, the probability of the station not becoming a backlogged station is $1-(1-\sigma)^2$.

The second part in B.2 shows that station l_2 is not one of the backlogged stations at the first token observed point such that it has to generate a new packet in this slot; the probability of satisfying this given transition is $\frac{\Delta-k}{\Delta}\sigma$. The rest of the probability for this case is the same as discussed

above.

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