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Calculation of the Fluid Induced Forces Acting on a Pipe by Means of the Surfaces Integral and its Application to a Blowdown Test in the HDR

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Abstract

This contribution describes a method to calculate hydrodynamic forces acting on pipes by applying the surface integral method. These fluid forces were calculated based on measured pressure time histories from a piping system during blowdown with subsequent closing of a check-valve. A prediction of the structural response of the piping system was done with these forces and compared to measured displacements.

1. Introduction

One aim of blowdown investigations at the HDR was to study the dynamic behavior of a pipe as a result of a simulated break with subsequent rapid closure of a feedwater check valve. Predictions of the structural behavior based on calculated fluid dynamics yielded significant differences in the calculated displacements and stresses. To investigate the reason of these differences it was looked for a method to calculate the forces acting on the pipe only via pressures. Using measured pressures it should be possible to exclude false calculated fluid dynamics. A method described herein was developed to calculate fluid induced forces by means of the surface integral.

2. Usual methods to calculate fluid forces

Given the transient fluid response, there are two possible approaches to determine the fluid forces acting on a control volume of a pipe. One approach involves the use of the linear momentum equation from fluid mechanics:

$$\vec{F} = -\iint p\,\vec{n}\,dA - \iint \varrho\,\vec{u}\,(\vec{u}\cdot\vec{n})\,dA - \frac{\partial}{\partial t}\iiint \varrho\,\vec{u}\,dV \qquad (1)$$

The other approach is the integral of the stresses acting on the wetted surface of the pipe:

$$\vec{F} = \iint \vec{t}\,dS \qquad (2)$$

where $\vec{F}$ is the force acting on the pipe, p is the fluid pressure, $\vec{u}$ is the fluid velocity, ϱ is the fluid density, $\vec{t}$ is the stress-tensor at the wetted surface, $\vec{n}$ is an outward unit vector normal to the control surface, dA is an area element between two control volumes, dS is an area element of the wetted surface and dV is a volume element.

In most cases eq. (1) is used to evaluate the forces. To use this equation it is necessary to know the velocity and its time derivative in the momentum and acceleration terms. In many applications of this equation the acceleration term is only neglected or estimated. To prove these estimations and to calculate the forces only with measured (or calculated) pressures an evaluation was made by the use of eq. (2). To use this equation shear stress between fluid and pipe will be neglected. So it is possible to write:

$$\vec{F} = \iint p \, \vec{n} \; dS \qquad (3)$$

Only the pressure on the wetted surface produces a force on a pipe section. It can be seen, that a straight pipe section will not be affected by a force. The next chapter shows the development of eq. (3) to calculate the force acting on a pipe elbow.

3. Calculation of the force on a pipe bend by means of the surface integral

As shown before it is possible to calculate the force by integrating the pressure acting on the wetted surface by eq. (3) when the friction of the fluid at the walls is neglected. Given the pressure distribution along the bend it is possible to integrate eq. (3) to receive the force at any time-step. If measured pressure should be used, which is only known at some points at the pipe, it is not possible to use eq. (3) directly. In the following it will be described how it is possible to evaluate eq. (3) with local measured (or calculated) pressure time histories.

As a further neglect we assume, that the pressure waves travel in the direction of the pipe axis also in the bends and the waves change their shape only slightly. With this assumptions a real pressure distribution can be interpreted as a sum of many small pressure jumps which travel with the velocity of sound through the pipe. Fore one pressure jump it is possible to evaluate eq. (3).

At a fixed time the situation at a bend is shown in fig. 1, the pressure jump has reached the angle β. To calculate the surface integral it is necessary to split the force into two vectors:

$$F_x = \iint p \, \vec{n} \, \vec{x} \; dS$$
$$F_y = \iint p \, \vec{n} \, \vec{y} \; dS \qquad (4)$$

The products of the normal vector with the coordinate vectors are:

$$\vec{n} \, \vec{x} = \sin\psi \quad \cos\varphi$$
$$\vec{n} \, \vec{y} = \sin\psi \quad \sin\varphi \qquad (5)$$

The surface element dS can be written as:

$$dS = R\,d\psi\,(r + R\sin\psi)\,d\varphi \tag{6}$$

The pressure is only a function of :

$$p = \begin{cases} p_o & \text{for } 0 < \varphi < \beta \\ p_1 & \text{for } \beta < \varphi < \alpha \end{cases} \tag{7}$$

The integration limits are:

$$\varphi = (0, \alpha) \qquad \psi = (0,\ 2\pi). \tag{8}$$

With these declarations the force components are calculated as:

$$F_x = \pi R^2\,(p_o \sin\beta + p_1 \sin\alpha - p_1 \sin\beta) \qquad 0 \le \beta \le \alpha$$
$$F_y = \pi R^2\,[p_o\,(1-\cos\beta) + p_1\,(\cos\beta - \cos\alpha)] \tag{9}$$

The factor πR^2 is equal to the cross section A of the bend. The angle β can be described with the velocity of sound a as a function of time: $\beta = at/r$. The pressure jump is $\Delta p = p_1 - p_o$. Then eq. (9) gives:

$$F_x = A\,[p_1 \sin\alpha - \Delta p \sin(at/r)] \qquad 0 < t < r\alpha/a$$
$$F_y = A\,[p_1\,(1-\cos\alpha) - \Delta p\,(1-\cos(at/r))] \tag{10}$$

To use the force for further calculation it must act on discreet points of the pipe. This is shown in fig. 2 where the forces F_1 and F_2 and the moment must have the same reaction on the bend as F_x and F_y. With simple static the results are:

$$F_1 = A\,[p_1 - \Delta p\,(1-\cos(at/r) + \sin(at/r)\cos\alpha / \sin\alpha)]$$
$$F_2 = A\,[p_1 - \Delta p \sin(at/r) / \sin\alpha] \tag{11}$$
$$M = A\,r\,\Delta p\,[\sin(at/r)\,(1-\cos\alpha)/\sin\alpha - 1 + \cos(at/r)]$$

Until now we can not use eq. (11) for pressure time histories because the calculated forces are not proportional to any pressure at a fixed point. Fig. 3 (full lines) shows the time dependent forces and moment while the pressure jump travels through the bend.

The two forces are equal to Ap_1 before the pressure jump reach the bend and they are equal to Ap_o after it leaves the bend.

It is not necessary to load the pipe system with the exact forces to calculate dynamic reactions because the structure does not reply to the high frequences of forces. This is the reason why it is possible to make the forces proportional to the pressure at fixed points as fig. 3 shows (dashed lines). These amended forces and moment have the same impuls on the bend as the original ones. They are proportional to the pressures measurable at points near the middle of the bend as fig. 4 shows. The angle γ is a function of the bend angle α:

$$\gamma = \tan(\alpha/2) - \alpha/2 \tag{12}$$

and the forces are:

$$F_1 = A\ p_a$$
$$F_2 = A\ p_b \tag{13}$$
$$M = A\ r\ (p_a - p_b).$$

Neither the travel direction of the pressure jump nor the velocity of sound appear in the solution so it can be used for real pressure distribution.

4. Application of the force calculation using measured pressures

The method of force calculation described above has been used to determine forces from measured pressures which were employed to recalculate the dynamic behavior of an HDR pipe during a blowdown with subsequent valve closing.

Fig. 5 shows the pipe, its length is approx. 18 m and the nominal diameter is 350 mm. Instead of the pressures p_a and p_b at the bends (see fig. 4) the pressures were measured in the near of the bends. Nevertheless it is possible to get the corresponding pressures by an interpolation.

Fig. 6 shows the measured and calculated displacement. Both agree well in their amplitude and frequency. The structural dynamic calculation was done with the code ASKA /2/.

A further investigation indicated that the vibrating pipe produced repercussions on the enclosed fluid. Differential pressures between two elbows can be measured in the pipe which are derived from the movement of the pipe and which are proportional to the accelerations occuring at the pipe. As forces were calculated with the measured pressures, one obtains from these passive differential pressures active forces which do not occur in reality. These artificial active forces produce higher loads for the structur calculation, which are expressed as higher vibration amplitudes. Moreover, an enforced phase shift of the calculated vibration results, which produces major deviations between the measurement and the calculation than expected from the difference between the measured and calculated eigenfrequencies of the pipe, see fig. 6.

Calculating the fluid dynamics, this repercussion does not arise, of course, and the over-estimation of the vibration amplitudes and the phase shift do not occur. Thus the force method can be regarded as being suitable and reliable.

References

/ 1 / MEIER, D.; HUNGER, H. "Untersuchungen über die Reaktion und Beanspruchung einer Rohrleitung mit Speisewasserrückschlagventil bei Kühlmittelverluststörfällen", Kernforschungszentrum Karlsruhe, Projekt HDR-Sicherheitsprogramm, Technischer Fachbericht Nr. 43-84.

/ 2 / PLATZ, G. "Nachrechnungen zu den SRV 350-Versuchen V60.4.1 und V60.5 mit ASKA für verschiedene Kraftmethoden", SDF Stuttgart, Nov. 83

/ 3 / MOSBY, W.R.; DOOLEY, W.T. "Comparison of NUPIPE II and SAP IV displacement and acceleration response predictions for German standard problem 4A", EGG-EA-5566, December 1981

/ 4 / WATKINS, J.C. "R5FORCE: a program to compute fluid induced forces using hydrodynamic output from the RELAP5 code, EGG-EA-5920, October 1982

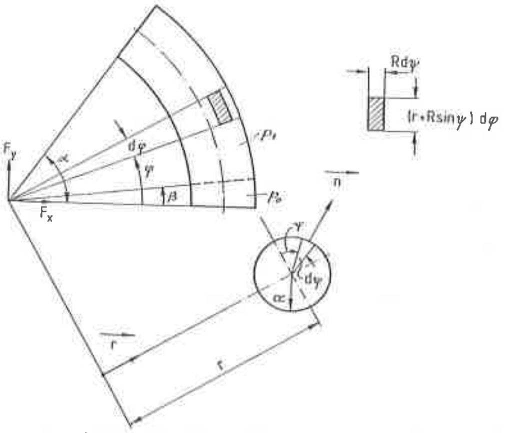

Fig. 1 Relations at the elbow to use the surface integral

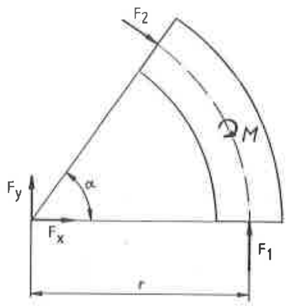

Fig. 2 Forces at the elbow

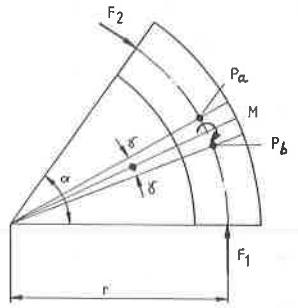

Fig. 4 Forces at the elbow with proportional pressures

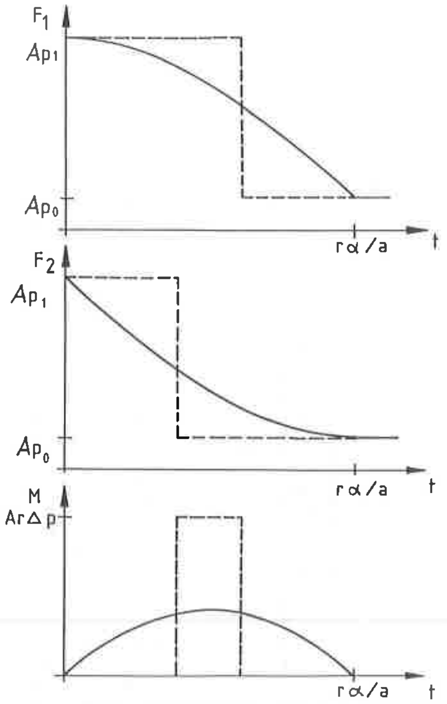

Fig. 3 Time dependent forces and moment during a pressure jump travels through the bend

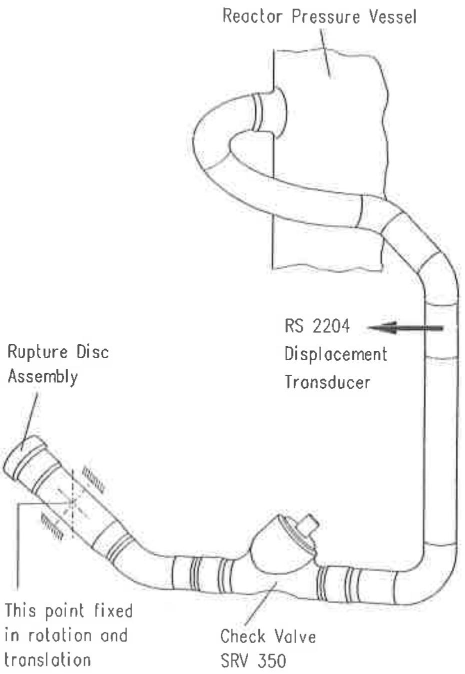

Fig. 5 HDR blowdown pipe

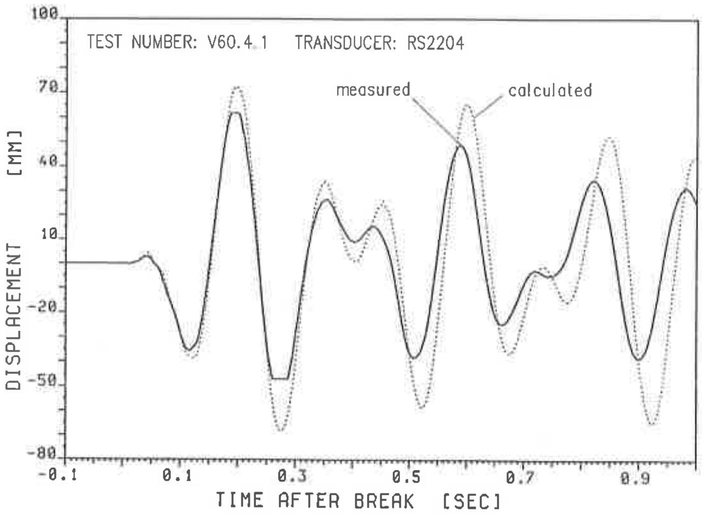

Fig. 6 Comparison of measured and calculated displacement