Simple Numerical Techniques Using Boundary Element Method for the Determination of K_I in Fracture Mechanics

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ABSTRACT

In many countries, the electricity companies have stopped the construction of new nuclear power plants. Instead of new plants, there is a great effort to extend the lifetime of the existing power plants, taking from them as much as they can supply in terms of electricity. To extend the lifetime of the existing plants, it is important to evaluate, via Fracture Mechanics, the structural integrity of some components like pressure vessels and pipelines. To estimate the structural integrity of such nuclear components it is essential to well access the Stress Intensity Factor *K*. Among the Stress Intensity Factor Modes (I, II and III), Mode-I, with the K_I parameter, is the most important to know. K_I characterizes the stress field in the neighborhood of a crack tip when the crack is under tension. Nuclear pipes and vessels usually hold internal pressure and their walls are generally under tension. Knowledge of K_I is essential to establish if an existing crack is stable or not. For some simple crack configurations, analytical expressions for K_I are available in the literature and also in some handbooks. However, for cracks with more complex geometry configuration no handbook method exists for the determination of the Stress Intensity Factor. For complex geometries, the Stress Intensity Factor may be obtained by experiments; however, such procedure is expensive when compared to numerical approaches. The use of numerical methods for the determination of K_I seems to be an economic tool. Among other numerical methods, the use the BEM (Boundary Element Method) for the computation of K_I appears advantageous since it involves only meshing the boundaries of a structural component. In this paper, some simple techniques for obtaining K_I in planar structures using a classical (standard) formulation of the BEM without quarter-point elements are employed. The techniques include: a) displacement extrapolation, b) energy release rate, and c) *J*-integral. For the computation of the *J*-integral we propose an approach with fictitious internal elements surrounding the crack tip. On the fictitious internal elements, the terms $\partial u_i \,/\, \partial x$ necessary to compute the *J*-integral are approximately evaluated by finite differentiation of the Boundary Element (BE) fundamental solutions. To show the effectiveness of the proposed formulation an example is presented and compared with other formulations. Conclusions and suggestions are also given at the end of this article.

INTRODUCTION

In recent years, due to the great effort to extend lifetime of existing power plants and the growth of analytical procedures for predicting the structural behavior of cracked structures, the importance of Linear Elastic Fracture Mechanics (LEFM) for the analysis of structural integrity of nuclear components has increased considerably. Most nuclear components are made of ductile materials and when subjected to certain loads may experience significant plasticity in front of the crack tip. In such situation the structural integrity evaluation of such components containing cracks would require the precise use of Elastic-Plastic Fracture Mechanics (EPFM) parameters. However, numerical methods to apply the EPFM concepts are very expensive and time consuming [1]. The existence of simplified methods for integrity evaluation is of great relevance and despite the limitations concerning LEFM its fundamentals with appropriate improvements can be used for the evaluation of the integrity of a cracked structure. Even in the DFM (Ductile Fracture Method) with calibration functions for the prediction of crack behavior [1] the *K* parameter is used. Therefore, although, the LEFM theory seems more adequate to deal with fragile materials, in practical situations it can be used (with some consciousness of its limitations) in the evaluation of crack stability of nuclear components when the plastic or yielding zone in front of the crack tip is very limited.

The most important parameter in LEFM for the evaluation of structural integrity is the Stress Intensity Factor *K*. In LEFM, *K* describes the local stress and strain magnification at the crack tip. *K* depends on the material properties, loading and boundary conditions applied to the structure, and also to geometric parameters like size, shape, orientation, and depth of the crack. Most of the analytical solutions for determining *K*, available in fracture mechanics handbooks, are limited to idealized structures and simple crack geometric configurations. In most cases the domain is even taken as infinite, the material is homogeneous and isotropic, and the boundary conditions applied are relatively simple and set in the infinity.

For more realistic and complex crack geometries, it is difficult to find good values of K to be used in practical situations. Experimental techniques may be employed to estimate K, but such techniques are very time consuming and expensive. One advantage of using numerical methods in engineering analysis is that if it is proven to be reliable, then it can be employed in more general situations. Alturi and Nakagaki [2] suggest that numerical methods are mandatory to deal with practical situations of fractures in flawed structural system with cracks of finite size, arbitrary shape, complicated boundary conditions, and arbitrary material properties. With the help of efficient numerical algorithms for the determination of K one may study structures with cracks of different shape configuration and under diverse load and constraint conditions.

The Finite Element Method (FEM) has been successfully employed for the determination of the stress intensity factor [3,4], and more recently, due to its precision and simpler meshing scheme, the BEM has also received attention [3,5] from the engineering community. However, the main difficult in the precise numerical computation of the parameter K is the singularity that must be well represented at crack tip.

The presence of a crack in a structure reduces the fatigue and static strength of the structure because the stresses and strains are highly magnified at the crack tip [3]. The LEFM theory depicts this magnification as infinite in a sharp crack. In practical problems the material cannot resist such a high stress field and yielding and blunting at the tip will take place. Many numerical formulations for K try to simulate an infinite stress field at the crack tip and to do that they employ special elements like the quarter-point elements to model the crack tip. Several other strategies have also been proposed for the analysis of crack problems using the BEM. The strategies include representing the crack as a notch, symmetric crack modeling, use of special Green's functions, and flat crack modeling. More detail about those strategies can be seen in [3].

In real structures, inevitably, yielding and blunting will take place at the crack tip. Admitting the yielding region limited to a very small extent and, therefore, the region of unreal infinite stress considered a very small area, this paper tries to obtain the Stress Intensity Factor K using standard BEM. That is, this paper employs no special Boundary Element (BE) with quarter-point for obtaining stress singular field near the crack tip. We try to determine the K parameter using some simple procedures and standard BEM programs available in classical literature [6,7]. To compute the K parameter, see next section, three techniques will be employed; Displacement Extrapolation, Energy Release, and J-Integral for limited plastic zone. In this last technique the J-integral will be related to K. For the calculation of the J-integral we use an alternative and simple approach defining fictitious internal elements for the J-integral path S. In S the terms $\partial u_i / \partial x$ (derivatives of the displacements $u_i = (u, v)$ with respect to x) present in the J-integral equation are approximately evaluated by a finite difference method of the Boundary Element displacements $u_i = (u, v)$.

OBTAINING K_I WITH THREE DIFFERENT TECHNIQUES

In this section we describe three numerical techniques for obtaining the Stress Intensity Factor K limited to Mode-I in Fracture Mechanics and to two-dimensional (2D) structures. The attractiveness of the three techniques presented hereon is that they can be implemented in a standard BEM program for elastotastic problems without much extra programming.

Basically, the approaches for calculating the Stress Intensity Factor K_I may be divided into two categories: point matching and energy evaluation. The former technique calculates the Stress Intensity Factor K_I from stress or displacement fields near the crack using an extrapolation method. The energy techniques compute the energy release rate in the structure or the J-integral and subsequently relate this energy rate to K_I. Figure-1 shows the sketches of the three techniques that will be used in this work and will be explained in detail afterwards.

The three techniques that will be used in this paper include: Displacement Extrapolation (DET), Energy Release variation due to a virtual crack extension (ERT), and J-Integral evaluations (JIT), and are discussed lately. These techniques have also been applied with success in other numerical schemes [8].

We propose, in this paper, an approach for the calculation of the J-integral with the help of the finite difference evaluation of the terms $\partial u_i / \partial x$ in the internal points defining the path S for a simple calculation of the J-integral. For the application of these three techniques in a FEM schemes see Cavalcanti [9] and Owen and Fawkes [10], and for more details in the application of these three techniques with the BEM see Medeiros [4].

Displacement Extrapolation Technique (DET):

The easiest way to determinate K_I parameter is the Displacement Extrapolation Technique (here named DET). Although the goal is to determine K_I at the crack tip, that is, when $r = 0$, this technique avoids the stress singularities that appear when $r = 0$. In our formulation with a standard BEM program there is no singularity at the crack tip, as at that point the stresses are not reliable KI will be evaluated indirectly.

To obtain K_I with this technique, consider a cracked body as shown in Figure-1a, subject to pure Mode-I loading. On the crack plane $(\theta = 0)$, K_I is related to the stress in the y-axes (see Figure-1a) according to the following equation when $r \to 0$

$$K_I = \lim_{r \to 0}[\sigma_y \sqrt{2\pi r}\,] \tag{1}$$

With the knowledge of the stress σ_y at a certain point specified by r one could determine the value of K_I at that point (which means at that r). Before estimating K_I at the crack tip one can estimate more values of K_I in points without stress singularities, that is, for different values of $r > 0$ (away from the upset region). In this way one could extrapolate K_I for the case when $r = 0$ and obtain K_I at the crack tip, see Figure-2a.
As the crack tip is a point of stress singularity this technique works better if one avoids calculating K_I based on stress because in standard BEM the modal displacements can be inferred with a higher degree of precision than the stress values. Stresses are related to strain that are derived from the displacements and, therefore, less accurate. Instead of relating K_I to the stress field, which is less reliable, a better way to get more accurate values of K_I is to use the displacement field u and v as in Equation (2) below. A general expression for the determination of K_I, as a function of u and v along a radial line r emanating from the crack tip in the direction θ, can be given as

$$K_I \begin{Bmatrix} (2k-1)\cos(\theta/2) - \cos(3\theta/2) \\ (2k+1)sin(\theta/2) - sin(3\theta/2) \end{Bmatrix} = 4\mu\sqrt{\frac{2\pi}{r}} \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{2}$$

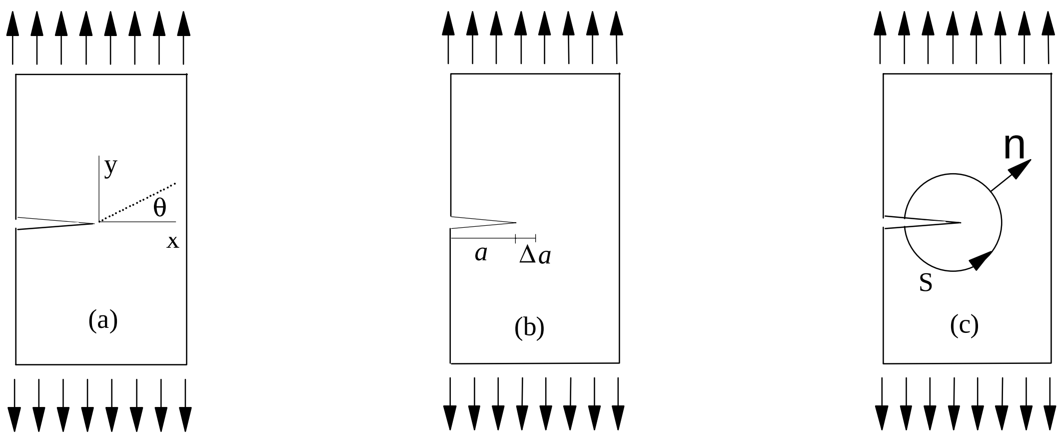

Figure-1 –Common numerical techniques used for the determination of the Stress Intensity Factor using
(a) extrapolation technique, (b) elemental crack advance giving the variation of the energy, and (c) J-integral.

In Equation (3), θ is the inclination angle of the radial line r along which K_I is to be calculated. θ is measured with respect to the x-axis according to Figure-1a. μ is the elastic shear modulus; u and v are the displacements and k is a function of plane stress or plane strain: $k = (3\text{-}\nu)/(1+\nu)$, for plane stress, and $k = 3\text{-}4\nu$, for plane strain [11].

For the point defining a certain values of r, u and v can be promptly read from a BEM code. Substituting the values of u or v in Equation (2) it is possible to plot K_I against the radial distance r. By discarding the results from points very close to the crack tip the value of K_I can be extrapolated to $r = 0$, see Figure-2a. It is very important to note that the DET is easier to use with the BEM than with the FEM since the points defining the radial line r along which K_I is to be calculated can be readily defined only specifying the coordinate of the internal points along the radial line r without any worry about internal mesh like in the FEM. When using the FEM one has to define an appropriate mesh so that the points (defining r) where the displacements u and v are to be read might correspond to FE nodes. A great disadvantage of DET is that it is highly dependent on the accuracy of the element in representing high stress gradient when approaching the upset region of stress in the vicinity of the crack. This method is also dependent of the number of points rejected in the vicinity of the crack tip.

Strain Energy Release Rate Technique (ERT):

The advantage of the Strain Energy Release Rate Technique (here named ERT) is that the crack tip stress field, which (as we discussed earlier) in LEFM is singular and corresponds to an unreal situation in practice, makes only a relatively small contribution to the total strain energy of the cracked body. In addition, the ERT can be applied to structures under linear or nonlinear regime. This technique seems very appropriate to be used with simple elements without any particular formulation to provide singular stress field, that is; without quarter-point [5]. The ERT is very easy to use with standard boundary element formulation [6,7], since only a refined mesh near the crack tip is able to capture the high stress gradient effect on the energy.

The energy release rate, named G (or G_I for Mode-I,) is related to the Stress Intensification Factor K_I according to the following equation

$$K = \left(\frac{8 \mu G}{1+k} \right)^{1/2} \tag{3}$$

where: μ is the shear modulus, k is a function of plane strain ($k = 3\text{-}4\nu$) or plane stress ($k = (3\text{-}\nu)/(1+\nu)$). G can be calculated from the rate of change in the global potential energy U with respect to a crack growth Δa as represented in Equation (4). In practice, G can be evaluated considering the variation of the global potential energy ΔU when the crack growths from an extension a to $a+\Delta a$. The gray part of Figure-2b represents the variation of the energy ΔU due to the crack growth Δa.

$$G = -\frac{\Delta U}{\Delta a} = -\frac{U_2 - U_1}{a_2 - a_1} \tag{4}$$

where U_1 and $U_2 = U_1 + \Delta U$ are the strain energies associated with the crack lengths a_1 and $a_2 = a_1 + \Delta a$, respectively. This approximation is only valid if the difference between the two crack lengths is very small. With G_I (for Mode-I) calculated from Equation (4), the stress intensity factor K_I can be obtained according to relationship expressed in Equation (3). The disadvantage of such procedure is that at least two solutions must be considered in order to obtain the difference between potential energies of the structure, respectively, when it has a crack of length a and $a+\Delta a$.

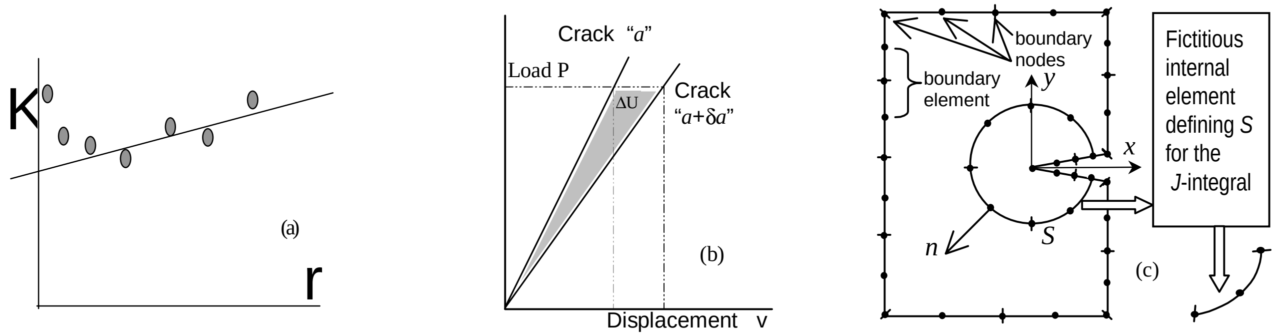

Figure-2 – (a) Stress Intensity Factor evaluation by extrapolation. (b) Energy variation due to a crack extension growth from "a" to "$a + \delta a$" under constant load P. (c) Definition of path S around the crack tip for determination of J-integral

J-Integral Technique (JIT):

This is another technique (here named JIT) that uses the concepts of energy conservation and, therefore, is less conditioned to the singular stress field next to the crack tip. Although the meaning of the J-integral equation is somewhat elusive, it can be seen as a balance between the internal strain energy U and the external energy [11] represented by the work done by the applied traction t_i on the volume enclosed by the closed path S. The J-integral is evaluated along the path S surrounding the crack tip. The initial and the end points defining the path S must be located on the surface of the crack sizes, see Figure-1c. The equation defining the J-integral is known as

$$J = \int_S \left(U dy - t_i \frac{\partial u_i}{\partial x} ds \right) \tag{5}$$

In this equation, S is an arbitrary contour path around the tip of the crack defined from one surface of the crack to the other surface of the crack in the x-y plane; S can be defined by the outward normal n; U is the strain energy density; t_i are the components of the traction vector acting on the contour S; u_i is the displacement vector (in 2D problems $u_1 = u$ and $u_2 = v$); dy is an infinitesimal element along the y-direction and ds is the differential distance on the path S. For plane stress problems, considering the expressions for the strain energy density U and for the traction vector, $t_i = \sigma_{ij} n_j$, also dividing S in P pieces (S $= \sum S_i$) for the numerical determination of the J-integral, Equation (5) can be rewritten in matrix form as

$$J = \sum_{i=1}^{P} \int_{S_i} \left\{ \frac{1}{2} \begin{bmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{xy} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix} dy - \begin{bmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{xy} \end{bmatrix} \begin{bmatrix} n_x & 0 \\ 0 & n_y \\ n_y & n_x \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{Bmatrix} \right\} d\Gamma \tag{6}$$

Rice [11] showed that the J-integral is path independent. For a closed path not containing the crack, $J = 0$. The J-integral can be directly related to the Stress Intensity Factor K (for Mode-I, K_I) by the following equation where, as before, μ is the shear modulus, the constant $k = (3-\nu)/(1+\nu)$, for plane stress, and $k = 3-4\nu$, for plane strain.

$$K = \left(\frac{8\mu J}{1+k}\right)^{1/2} \tag{7}$$

To perform the numerical integration of Equation (6), we need all the quantities involved in that equation at the integration points (here Gauss points) along the path S, including the derivatives $\partial u/\partial x$ and $\partial v/\partial x$ - see Equation (6), all the other quantities are promptly read from a standard BEM program. Note that in the BEM formulation the path S is easily defined throughout a set of internal points around the crack tip and not attached to a mesh as in the FEM, see Figure 2c.

THE BEM FOR THE DETERMINATION OF K_I

The Boundary Element Method has many positive attributes that make it advantageous for the use in engineering applications [6,7]. A widely recognized advantage of the BEM is that it reduces by one the dimensionality of the problem. 3D problems are reduced to 2D integrals and 2D problems are reduced to line integrals. For problems with no body force, the BEM involves only the meshing of the boundaries.

Discretizing only the boundaries with the BEM also leads to a system of equations with a smaller number of degrees of freedom. This is particularly advantageous for numerical fracture mechanics because fewer degrees of freedom imply greater numerical stability in the solution process. In the BEM the domain of the continuum does not have to be discretized. The calculation of the structural response at all the internal nodes, as is done in the FEM, can thus be avoided. The calculation of displacements and tractions are also more accurate with the BEM because the analytical solution (i.e. Green's function represented by the fundamental solutions) of the governing differential equation is part of the numerical solution while in the FEM, for instance, an approximate displacement or traction field has to be assumed.

Another characteristic of the fracture mechanics problem that can take advantage of the BEM is the problem of crack propagation. In the algorithms treating this problem, generally, a continuous updating of the mesh is required so that the crack propagation path can be pursued. With the BEM the update process of the crack propagation becomes less laborious compared with other methods that involve discretizing the entire continuum. Geometric modeling problems of mesh generation and mesh update, associated with evolving cracks are, therefore, tremendously reduced with the BEM. The main disadvantage of the BEM is that its formulation leads to a nonsymmetric, fully populated system of equations.

The fundamental Equation of the BEM is the Somigliana's identity [6,7] expressed in the following equation and valid for any internal or boundary point.

$$u_k^i + \int_\Gamma p_{kj}^* u_j \, d\Gamma = \int_\Gamma u_{kj}^* p_j \, d\Gamma + \int_\Omega u_{kj}^* b_j \, d\Omega \tag{8}$$

In this equation, the terms with star u^*_{kj} and p^*_{kj} are the fundamental solutions or kernels. For two-dimensional problems in elastostatics, the mathematical expressions for the fundamental solutions u^*_{kj} and p^*_{kj} may be found in standard BEM textbooks [6,7] and are expressed in Equations (9) and (10)

$$u_{kj}^* = \frac{1}{8\pi\,(1-\nu)\,\mu}\left[(3-4\nu)\delta_{kj}\ln\left(\frac{1}{R}\right) + R,_k\, R,_j\right] \tag{9}$$

$$p_{kj}^* = -\frac{1}{4\pi\,(1-\nu)\,R}\left[\frac{\partial R}{\partial n}\left[(1-2\nu)\delta_{kj} + 2R,_k\, R,_j\right] + (1-2\nu)\left(n_k\, R,_j - n_j R,_k\right)\right] \tag{10}$$

Equations (9) and (10) were obtained for plane strain [6,7] but they are also valid for plane stress substituting the Poisson's coefficient ν for $\bar{\nu} = \nu \div (\nu + 1)$. In Equations (9) and (10) n is the unit vector normal to the outer boundary of the body, μ is the shear modulus and R is the distance from between the load point and the field point [6,7]. The discretization of Equation (8) can be done using a variety of polynomials, of different orders, such as linear, quadratic, cubic, and so on. In this work, we used standard quadratic boundary elements which means that the geometry, displacements, and tractions acting on the boundary of the body are discretized with quadratic polynomials, therefore, written in terms of three nodal quantities. After application of the boundary element discretization, the boundary integral Equation (8) may be rewritten (for the case of zero body forces) in discrete form as

$$[F]\{U\} = [G]\{P\} \tag{11}$$

where $[F]$ and $[G]$ are the boundary element matrices. $\{U\}$ and $\{P\}$ are the vectors of nodal displacements and tractions, respectively. After applying the given boundary conditions of the problem, Equation (11) is manipulated, taking all known values (specified boundary conditions) to the right hand side and all the unknown values to the left hand side and, accordingly, Equation (11) can be rewritten as in Equation (12). At the points where nodal displacements are defined (as boundary conditions) the tractions are unknowns and at the points where the tractions are given (as boundary conditions) the displacements are unknowns. After the manipulation, the right hand side of Equation (11) results in a vector of known quantities, $\{b\}$, and the left hand side results in a matrix of coefficients $[A]$ times the unknowns, as shown in Equation (12).

$$[A]\{V\}=\{b\} \tag{12}$$

where $\{V\}$ is the vector containing both unknown displacements and unknown tractions at the boundary nodes. After solving Equation (12), the complete values of tractions and displacements at any boundary point are known. Using again the discretized form of Equation (8), the displacements u_i at any point (internal or at the boundary) can be calculated. With the displacements the strains can be calculated, and with the strains, making use of Hook's law, the stresses are also available at any point.

Examining Equation (6), for the numerical calculation of the *J*-integral, except for the derivatives $\partial u/\partial x$ and $\partial v/\partial x$ all the other term are obtained at any internal point *i*. The path *S* for the *J*-integral is defined by a set of internal points *i* chosen around the crack tip as indicated in Figure 2c. Through each set of three internal points one fictitious internal element (with quadratic shape functions) is defined so that the *J*-integral can be determined numerically.

For the calculation of the derivatives $\partial u/\partial x$ and $\partial v/\partial x$ at internal point *i* defining the path *S*, we use a finite difference strategy computing the terms as

$$\frac{\partial u}{\partial x} \approx \frac{u_2 - u_1}{\Delta x} \quad \text{and} \quad \frac{\partial v}{\partial x} \approx \frac{v_2 - v_1}{\Delta x} \tag{13}$$

The path *S* is divided in *P* sub paths (*P* elements) each corresponding to a fictitious element through three internal points around the crack tip – see Figure-2c. Therefore, the *J*-integral, in Equation (6), can be promptly calculated. It means that all the quantities involved in the three techniques (DET, ERT and JIT) described before are well know.

APLICATION TO A CLASSICAL CRACK CONFIGURATION

To show the effectiveness of the three techniques DET, ERT and JIT implemented using a standard BEM program listed in reference [7] and also to compare the precision among these techniques for the calculation of the Stress Intensification Factor, a classical example corresponding to a panel with a central crack and under uniform traction is considered. The geometry of the plate and the crack, the modulus of elasticity and the Poisson's ratio are also given in Figure-3a. Half of the panel and the corresponding crack were discretized with Boundary Element. Two meshes were used. The first mesh has 29 quadratic BEs in total being 25 for the panel external boundaries and 4 for the crack surfaces. In the other mesh, the number of BE for the external boundaries was maintained as 25 and 8 BEs were used for the crack surfaces. In the line of symmetry special boundary conditions were imposed. Figure 3b shows the Boundary Element discretization including the approximation of the *S* path for the introduction of fictitious internal elements through the internal points around the crack tip for the *J*-integral technique (JIT). To see the influence of the ligament, different ratios of a/W were considered.

The results obtained using the three techniques (DET, ERT and JIT) in a BEM framework are presented in Table-1. The analytical solution of this problem can be found in any classical text of fracture mechanics [11] and is also listed in Table-1.It can be seen that the technique that presented the worst result was the strain energy release rate technique (ERT) followed by the displacement extrapolation technique (DET). The best result was achieved with the *J*-integral technique (JIT) with fictitious internal elements (K_I is determined from the relationship between *K* and the *J*-integral represented in Equation (7)). The accuracy in the calculation of the *J*-integral is due to the fictitious elements passing through internal points defining the path *S* around the crack tip.

The value of K_I was also determined from a FEM code. Table-2 shows the results from the FEM and the BEM. Both methods used quadratic interpolation functions but the formulation with the FEM also employed the quarter-point approach to simulate the singularity next to the crack tip. The BEM with the *J*-integral technique (JIT) and no quarter-points used 37 external elements and the FEM with quarter-point elements next to the crack tip used 42 elements in total. In this case, observing Table-2, one can see that one result from the FEM formulation was better than the results from the standard BEM for the ratio of a/W=0.1. In this case, the FEM percentage error (compared with the analytical solution) was estimated to be 0.058% against 1.27% from the BEM with standard elements. In Table-2, both methods made use of the *J*-integral technique for obtaining K_I. However, the results from the BEM showed frequent small errors (less than 1.8%) for all ratios of a/W while with the FEM the errors ran from 0.058% to 5.478%. Therefore, the simple numerical techniques (specially the JIT) using standard Boundary Element Method for the determination of K_I presented in this paper seems to be efficients, given small errors in the results and moreover, very simple to be implemented in standard BEM codes.

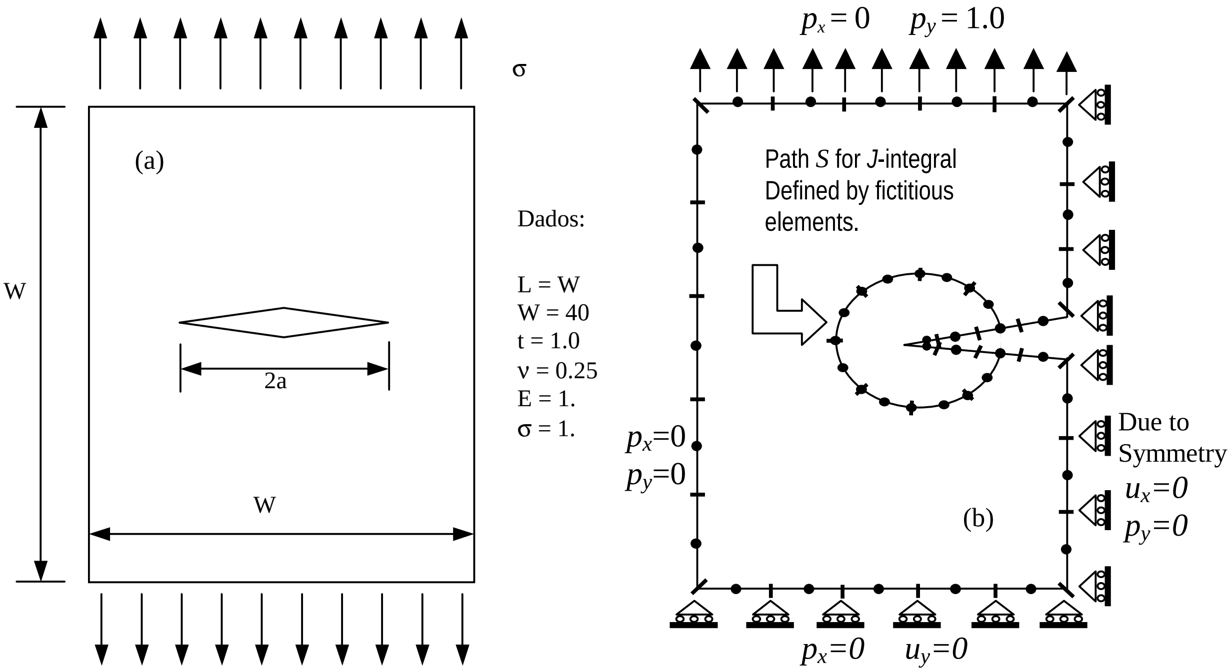

Figure – 3 - (a) Plate with a central crack “2a” and (b) the corresponding boundary element modeling

Table – 1 – Comparison of different methods for obtaining the Stress Intensity Factor K_I

Crack with	a/W	K_0 ($\sigma\sqrt{\pi a}$)	K_I (EDT)	ε (%)	K_I (ERT)	ε (%)	K_I (JIT)	ε (%)
4 elements at crack surfaces	0,05	3,142	3,208	2,11	2,963	5,70	3,201	1,87
	0,10	6,283	6,401	1,87	6,167	1,85	6,370	1,39
	0,15	9,425	9,556	1,39	9,183	2,57	9,528	1,09
	0,20	12,566	12,814	1,98	12,156	3,26	12,713	1,17
	0,25	15,708	16,076	2,34	15,041	4,25	16,002	1,87
8 elements at crack surfaces	0,05	3,142	3,207	2,06	3,321	5,69	3,199	1,80
	0,10	6,283	6,379	1,53	6,171	1,78	6,363	1,27
	0,15	9,425	9,318	1,14	9,210	2,28	9,342	0,88
	0,20	12,566	12,761	1,55	12,228	2,69	12,689	0,98
	0,25	15,708	15,416	1,86	15,201	3,23	15,493	1,37

Table - 5.5 – BEM with 37 standard BE and the FEM with quarter-point and 42 FE

a/W	K_{ANAL}	K_{MEC} (JIT) - no quarter-point	ε (%)	K_{MEF} (JIT) - with quarter-point	ε (%)
0,05	3,142	3,199	1,80	2,969	5,478
0,10	6,283	6,363	1,27	6,280	0,058
0,15	9,425	9,342	0,88	9,517	0,981
0,20	12,566	12,689	0,98	12,876	2,461
0,25	15,708	15,493	1,37	16,358	4,136

CONCLUSIONS AND SUGGESTIONS

For the calculation of the stress intensity factor K_I in 2D structures, this paper used three different techniques implemented in a simple Boundary Element code. The use of boundary elements allowed the use of an easy mesh discretizing the structure and also the crack. The three techniques carried out were the displacement extrapolation, the strain energy release rate, and the *J*-integral, this last one calculated with the help of fictitious internal elements defined along internal points around the crack tip.

All these techniques were made feasible with the use of a standard Boundary Element formulation available in classical BEM books [6,7]. In this paper the quarter-point boundary elements to produce stress singularity at the tip of the crack were not used. It is important to conclude, that even with this simple Boundary Element formulation the technique using *J*-integral gave very accurate results when compared to the analytical solution of classical problems available in the literature. Comparing the BEM formulation with other numerical methods, like the FEM with quarter-point revealed that the BEM formulation with the *J*-integral is more numerically stable. The numerical results presented in this paper illustrate the use of BEM in fracture mechanics. In the example, the BEM error estimation for the calculation of K_I - when using the *J*-integral technique - was always very small (ranging from 0.88% to 1,87% for different a/W ratios). The same is not true for the FEM. The accuracy in calculating the *J*-integral is due the employment of fictitious internal elements passing through internal points defining the path *S* around the crack tip. In the internal points the calculation of the derivative terms $\partial u/\partial x$ and $\partial v/\partial x$ present in the *J*-integral equation were obtained with finite difference. It is believed that the procedures presented here could be easily implemented and could be a tool for the determination of K_I in fractured structures of varied shapes not commonly found in handbooks. Although the obtained results in the given example were very accurate, the authors believe that even better results could be obtained with the use of quarter-point boundary elements and also with the implicit differentiation of the fundamental solution of the BEM for the precise calculation of the derivatives $\partial u/\partial x$ and $\partial v/\partial x$ present in the *J*-integral equation.

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REFERENCES

1. Cruz, J. R. B., "Procedimento Analítico para Previsão do Comportamento Estrutural de Componentes Trincados". Tese de Doutorado, IPEN-SP, 1998 (in Portuguese).
2. Alturi,, N. S. and Nakagaki, M., "Computational Methods in the Mechanics of Fracture". ed. S. N. Alturi. North-Holland, Amsterdam, pp. 169-228, 1986.
3. Blandford, G. E.. "Two-dimensional Stress Intensity Factor Computations Using the Boundary Element Method". Int. J. Num. Meth. Engng., vol. 17, pp. 387-401, 1981.
4. Medeiros, J., M. S., "Obtenção do Fator de Intensidade de Tensão Através de Utilizando Várias Formulações com o Método dos Elementos de Contorno", Dissertação de M.Sc., Universidade de Brasília, 2000 (in Portuguese).
5. Barsoum, R. S., "On the Use of Isoparametric Finite Elements in Linear Fracture Mechanics", Int. J. Num. Meth. Engng., vol. 10, pp. 25-37, 1976
6. Banerjee, P. K., and Butterfield R., "Boundary Element Methods in Engineering Science". London: McGraw-Hill. 1981
7. Dominguez, J., and Brebbia, C. A., "Boundary Elements: An Introductory Course", Mc-Graw Hill,1989.
8. Anderson, T. L., "Fracture Mechanics - Fundamentals and Applications", CRC Press, 1991
9. Cavalcanti, M. V. S., "Obtenção do Fator de Intensidade de Tensão Através de Variadas Formulações Utilizando o Método dos Elementos Finitos", Dissertação de M.Sc., Universidade de Brasília, 1997 (in Portuguese).
10. Owen, D. R. J. and Fawkes, A. J., "Engineering Fracture Mechanics: Numerical Methods and Applications". Swansea, U.K., Pineridge Press Ltd. 1983.
11. Broek, D., "Elementary Engineering Fracture Mechanics", Nijhoff, 1985.