Transactions of the **14th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 14), Lyon, France, August 17-22, 1997

K20/6

Substructuring in large-scale pseudo-dynamic tests

Pinto A.V., Pegon P.
European Commission - Joint Research Centre - ISIS, Italy

ABSTRACT. The distinctive features of the pseudo-dynamic test method as implemented at the largest European reaction-wall facility are described. Software aspects are considered. Particular attention is devoted to a coupled numerical-experimental substructuring technique allowing realistic earthquake testing of very large structures. Mathematical and implementation details of this testing technique with substructuring both for synchronous and asynchronous input motion are given. Selected test results illustrate the advantages of the presented features.

INTRODUCTION

A research programme on Bridges under the European Commission's programme on Human Capital and Mobility [1], outlined in [2], included the pseudo-dynamic (PSD) testing of six bridges in the ELSA Laboratory using substructuring techniques, which is the first large scale testing campaign successfully performed with such a technique.

The PSD test method is a hybrid method combining the numerical integration of the equations of motion for complex structures condensed on a reduced number of degrees-of-freedom (d.o.f.), with the experimental measurement of the restoring forces resulting from this motion. Despite the potential of the PSD technique, direct testing of very large civil engineering structures like bridges would require, on top of the size problem, the control of too many d.o.f., thus exceeding the experimental capabilities. It is however possible to extend the PSD range of application to such cases, at least when the behaviour of a part of the structure can be modelled computationally, by introducing a substructuring technique [3]. This technique takes advantage of the hybrid character of the PSD method in combining the numerical modelling of a part of the structure, the modelled structure, with the physical testing of the remaining structural part, the tested structure.

This paper presents the pseudo-dynamic testing method with substructuring by recalling its mathematical foundations and by highlighting the specific aspects of its implementation in ELSA. The presentation first concentrates on the particular case of synchronous motion [4], that is a base acceleration loading which is uniform in space. Subsequently, the more general case of asynchronous base motion is dealt with [5]. The interest and the flexibility of the implementation is then illustrated through some results of the tests conducted on large-scale four-span irregular bridges [6].

SUBSTRUCTURING IN CASE OF SYNCHRONOUS MOTION

The α-Operator Splitting scheme

Consider the following system of semi-discrete differential equations

$$\boldsymbol{Ma} + \boldsymbol{Cv} + \boldsymbol{r}(\boldsymbol{d}) = \boldsymbol{f} \tag{1}$$

which describe the motion of a structure with m d.o.f.s, where $\boldsymbol{a}$, $\boldsymbol{v}$ and $\boldsymbol{d}$ represent the acceleration, the velocity and the displacement vectors, $\boldsymbol{r}$ and $\boldsymbol{f}$ the structural internal and the external force vectors and $\boldsymbol{M}$ and $\boldsymbol{C}$ the mass and damping matrices. In the case of synchronous seismic loading, $\boldsymbol{a}$, $\boldsymbol{v}$ and $\boldsymbol{d}$ represent the motion of the structure in a reference frame which is relative to the uniform ground motion. The seismic action is taken into account by means of an inertial contribution to the external force vector $\boldsymbol{f}$

$$\boldsymbol{f} = -\boldsymbol{MI}\,{}^{\text{base}}a \tag{2}$$

where ${}^{\text{base}}a$ is the intensity of the base acceleration and $\boldsymbol{I}$ a vector which account for the direction of earthquake loading at the level of each d.o.f. To solve the system of equations given by Eq.(1), a numerical step-by-step integration technique is adopted: in this work it is the so-called α method implemented by means of an Operator-Splitting technique. According to the α method [8], the displacement and velocity vectors at step $(n+1)$ can be written in terms of both the acceleration vector and the previous step values

$$\begin{aligned} \boldsymbol{d}^{n+1} &= \tilde{\boldsymbol{d}}^{n+1} + \Delta t^2 \beta \boldsymbol{a}^{n+1} \qquad & \tilde{\boldsymbol{d}}^{n+1} &= \boldsymbol{d}^n + \Delta t \boldsymbol{v}^n + \frac{\Delta t^2}{2}(1-2\beta)\boldsymbol{a}^n \\ \boldsymbol{v}^{n+1} &= \tilde{\boldsymbol{v}}^{n+1} + \Delta t \gamma \boldsymbol{a}^{n+1} & \tilde{\boldsymbol{v}}^{n+1} &= \boldsymbol{v}^n + \Delta t(1-\gamma)\boldsymbol{a}^n \end{aligned} \tag{3}$$

$$\beta = (1-\alpha)^2/4 \qquad \gamma = (1-2\alpha)/2 \qquad \alpha \in [0,-1/3] \tag{4}$$

and then introduced into the following time discrete system of equilibrium equations

$$\boldsymbol{Ma}^{n+1} + (1+\alpha)\boldsymbol{Cv}^{n+1} - \alpha\boldsymbol{Cv}^n + (1+\alpha)\boldsymbol{r}^{n+1} - \alpha\boldsymbol{r}^n = (1+\alpha)\boldsymbol{f}^{n+1} - \alpha\boldsymbol{f}^n \tag{5}$$

This scheme is implicit since $\boldsymbol{d}^{n+1}$ depends on $\boldsymbol{a}^{n+1}$ related to $\boldsymbol{r}^{n+1}$ which is a function of $\boldsymbol{d}^{n+1}$, and then it implies an iterative procedure. It is however possible to implement the α method without iterating by using an Operator Splitting (OS) method [9]. This method maintains the potential stability of the scheme, remaining implicit for the elastic part of the response, but does not require any iteration, being explicit for the non-linear part of the response. The OS method is based on the following approximation of the restoring force $\boldsymbol{r}^{n+1}$

$$\boldsymbol{r}^{n+1}(\boldsymbol{d}^{n+1}) \cong \boldsymbol{K}^I\boldsymbol{d}^{n+1} + (\tilde{\boldsymbol{r}}^{n+1}(\tilde{\boldsymbol{d}}^{n+1}) - \boldsymbol{K}^I\tilde{\boldsymbol{d}}^{n+1}) \tag{6}$$

where $\boldsymbol{K}^I$ is a stiffness matrix, generally chosen as close as possible to the elastic one $\boldsymbol{K}^E$ and in any case, for stability reason, higher or equal to the current tangent stiffness $\boldsymbol{K}^T(\boldsymbol{d})$ of the structure. Note that the digital PSD experimental set-up clearly offers all what is needed for an accurate measurement of the elastic characteristics of the structure to be tested or of its current stiffness at the beginning of any test.

PSD testing using the α-Operator Splitting scheme

All useful quantities being known at time t^n, the step-wise operations for reaching the time $t^{n+1} = t^n + \Delta t$ are: i) Compute (*prediction* phase) $\tilde{\boldsymbol{d}}^{n+1}$ and $\tilde{\boldsymbol{v}}^{n+1}$ according to Eq.(3), ii) Apply (*control* phase) the displacement $\tilde{\boldsymbol{d}}^{n+1}$ to the tested structure in order to get (*measuring* phase) the restoring force $\tilde{\boldsymbol{r}}^{n+1}$, iii) Solve for $\boldsymbol{a}^{n+1}$ the system of linear equations

$$\hat{\boldsymbol{M}} \cdot \boldsymbol{a}^{n+1} = \hat{\boldsymbol{f}}^{n+1+\alpha} \tag{7}$$

where the pseudo mass matrix $\hat{\boldsymbol{M}}$ and the pseudo force vector $\hat{\boldsymbol{f}}^{n+1+\alpha}$ are given by

$$\hat{\boldsymbol{M}} = \boldsymbol{M} + \gamma\Delta t(1+\alpha)\boldsymbol{C} + \beta\Delta t^2(1+\alpha)\boldsymbol{K}^I \tag{8}$$

$$\hat{f}^{n+1+\alpha} = (1+\alpha)f^{n+1} - \alpha f^{n} + \alpha\tilde{r}^{n} - (1+\alpha)\tilde{r}^{n+1} + \alpha C\cdot\tilde{v}^{n} - (1+\alpha)C\cdot\tilde{v}^{n+1} + \alpha(\gamma\Delta t C + \beta\Delta t^{2}K^{I})\cdot a^{n} \qquad (9)$$

and iv) compute (*correction* phase) d^{n+1} and v^{n+1} according to Eq.(3). Note that the computation, and possibly the factorization of $\hat{M}$, which usually does not depend on the time, may be performed during the initialization phase of the algorithm, before entering the time stepping loop. Note also that the verification of the α-shifted system of equilibrium equations Eq.(5) strongly influences the definition of the pseudo force vector $\hat{f}^{n+1+\alpha}$ in Eq.(9) and explains the introduction of the α-shifted time index $n+1+\alpha$ instead of $n+1$.

Substructuring and condensation

Among the m d.o.f.s of Eq.(1), m_S d.o.f.s belong to a modelled structure which is described using a finite element model having, for instance, an elastic behaviour, whereas m_T d.o.f.s belong to the structure which is effectively tested. Note that $m_S + m_T > m$ since some d.o.f.s are common to the modelled structure and the tested structure (connecting d.o.f.s). To be more specific, each of the m d.o.f.s is an element of one of the following collections: m_{SS} d.o.f.s internal to the modelled structure (index i, j, etc.), m_{ST} d.o.f.s common to the tested structure and the modelled structure (index δ, θ, etc.), m_{TT} d.o.f.s internal to the tested structure (index I, J, etc.), with $m = m_{SS} + m_{ST} + m_{TT}$, $m_S = m_{SS} + m_{ST}$ and $m_T = m_{TT} + m_{ST}$. Distinguishing between the quantities coming from the modelled structure (S) and the tested structure (T), Eq.(7) may be rewritten as

$$\begin{bmatrix} {}^{S}\hat{M}_{ij} & {}^{S}\hat{M}_{i\theta} & 0 \\ {}^{S}\hat{M}_{\delta j} & {}^{S}\hat{M}_{\delta\theta} + {}^{T}\hat{M}_{\delta\theta} & {}^{T}\hat{M}_{\delta J} \\ 0 & {}^{T}\hat{M}_{I\theta} & {}^{T}\hat{M}_{IJ} \end{bmatrix} \bullet \begin{bmatrix} a_j^{n+1} \\ a_\theta^{n+1} \\ a_J^{n+1} \end{bmatrix} = \begin{bmatrix} {}^{S}\hat{f}_i^{n+1+\alpha} \\ {}^{S}\hat{f}_\delta^{n+1+\alpha} + {}^{T}\hat{f}_\delta^{n+1+\alpha} \\ {}^{T}\hat{f}_I^{n+1+\alpha} \end{bmatrix} \qquad (10)$$

Condensing the unknowns a_j, the controlling equation becomes

$$\begin{bmatrix} {}^{T}\hat{M}_{\delta\theta} + ({}^{S}\hat{M}_{\delta\theta} - {}^{S}\hat{M}_{\delta j}\,{}^{S}\hat{M}_{ji}^{-1}\,{}^{S}\hat{M}_{i\theta}) & {}^{T}\hat{M}_{\delta J} \\ {}^{T}\hat{M}_{I\theta} & {}^{T}\hat{M}_{IJ} \end{bmatrix} \bullet \begin{bmatrix} a_\theta^{n+1} \\ a_J^{n+1} \end{bmatrix} = \begin{bmatrix} ({}^{S}\hat{f}_\delta^{n+1+\alpha} - {}^{S}\hat{M}_{\delta j}\,{}^{S}\hat{M}_{ji}^{-1}\,{}^{S}\hat{f}_i^{n+1+\alpha}) + {}^{T}\hat{f}_\delta^{n+1+\alpha} \\ {}^{T}\hat{f}_I^{n+1+\alpha} \end{bmatrix} \qquad (11)$$

and the modelled structure is governed by

$${}^{S}\hat{M}_{ij}a_j^{n+1} = {}^{S}\hat{f}_i^{n+1+\alpha} - {}^{S}\hat{M}_{i\theta}a_\theta^{n+1} \qquad (12)$$

Implementation

The PSD testing algorithm with substructuring is hold by two processes running in parallel on two different hardware connected through the network: one process is responsible for the tested structure and runs in the PC (master PSD computer) and the other, responsible for the modelled structure, runs in a remote workstation. This choice is in line with the decentralized architecture of the system implemented in ELSA [1] and allows to run experiment with substructuring only marginally changing the PSD program of the master PSD computer, as shown hereafter. Since the flux of informations exchanged by the two processes is very limited (only the connecting d.o.f.s are involved), this exchange can be managed relying on a low level library, working on an heterogeneous network, such as the Berkeley Sockets. Consider an existing PSD/α-OS algorithm, for which the generalized mass matrix corresponding to the tested structure alone is already known

$$ {}^{T}\hat{M} = \begin{bmatrix} {}^{T}\hat{M}_{\delta\theta} & {}^{T}\hat{M}_{\delta J} \\ {}^{T}\hat{M}_{I\theta} & {}^{T}\hat{M}_{IJ} \end{bmatrix} \qquad (13) $$

In order to take into account the modelled structure, this matrix should be modified. According to Eq.(11), only the term ${}^{T}\hat{M}_{\alpha\beta}$ is modified by an additional generalized mass term. This term is supplied by the process handling the modelled structure through a static condensation. Note that this condensation only requires the knowledge of the m_{ST} d.o.f.s which are in connection with the tested structure. During the time loop, at the end of the measuring phase of the test, the generalized effective force ${}^{T}\hat{f}^{n+1+\alpha}$ should be also modified. According to Eq.(11), only the term ${}^{T}\hat{f}_{\theta}^{n+1+\alpha}$ is modified by an following additional term. This term is supplied by the process holding the modelled structure, and may be computed during the measuring phase of the process holding the tested structure. The solution of Eq.(11) being performed by the tested structure process, the value of the acceleration at the connecting d.o.f.s has to be supplied to the modelled structure process, which in turn is able to compute the solution of Eq.(12). Note that during the test, the information exchange between the tested structure and the modelled structure is computationally well balanced: the tested structure needs the additional terms for the pseudo force vector but, symmetrically, the modelled structure requires the acceleration at the connecting d.o.f.s to complement Eq.(12).

SUBSTRUCTURING IN CASE OF ASYNCHRONOUS MOTION

PSD-Testing with asynchronous motion

From strong motion arrays installed in seismic areas it is clearly demonstrated that the soil motion of close surface points is not synchronous; even relatively close points can experience significant relative displacements, due mainly to reflection and refraction of seismic waves through underlying soil layers with different mechanical characteristics. Recent numerical studies on the responses of R/C concrete bridges, conventional and isolated ones, conclude that the conventional design under synchronous input provides a global upper bound of the response and consequently leads to a conservative design. However, the damage pattern in the bridge tends to be altered with respect to the one resulting from the synchronous motion. It is therefore required to assess the effects of this asynchronous input motion which can be partially accomplished by shaking table and/or PSD tests.

For the PSD tests with asynchronous input motion, special attention must be devoted to the mathematical and implementation aspects, which is the object of this section. The same type of partitioning of the d.o.f.s as for the case of synchronous motion is introduced. As it will be shown, the PSD testing with substructuring for asynchronous motion is not a trivial extension of the case with synchronous motion. However, at the practical implementation level, the process responsible for the tested structure is only very marginally affected by the introduction of the asynchronous motion. The additional complexity is mainly treated by the process responsible for the modelled structure, which can be easily modified.

Relative and absolute motion

The Eq.(1) may describe a *relative* or an *absolute* motion.

- The description with a *relative motion* is the most widely used in earthquake engineering. The base of the structure of interest is considered to be subjected to a uniform (ground motion) base acceleration field. The basic principle of the dynamics is expressed in a reference frame which follows this ground motion. The motion of the structure is originated by the inertial forces considered as being part of the external force vector $\boldsymbol{f}$. The relative motion description is quite natural since it expresses directly the contribution of the structure response to the overall motion.

- The description with an *absolute motion* is scarcely used, only when the other approach is impossible. This is the case of an asynchronous ground motion where the base acceleration field changes spatially from point to point. The motion of the structure is now originated by the motion of some of its internal points. In consequence, provision should be made while performing the discretization of the structure not to eliminate the ground connecting d.o.f.s and to subject them to the convenient base acceleration ${}^{\text{base}}\boldsymbol{a}$ (Dirichlet boundary condition).

Clearly, in the case of synchronous motion, both descriptions lead to the same results in term of intensive variables (internal stress state for instance). It is however interesting to precise how to maintain this equivalence at the numerical level. In order to clarify this point, consider the simple spring/mass system of Fig. 1. The relative motion of the mass is given by considering only *one* d.o.f. according to

$$Ma + Kd = -M\,{}^{\text{base}}a \tag{14}$$

while the same motion relies on *two* d.o.f.s in the absolute reference frame

$$\begin{aligned} Ma_2 + K(d_2 - d_1) &= 0 \\ a_1 &= {}^{\text{base}}a \end{aligned} \tag{15}$$

In order to maintain $a = a_2 - a_1$ at the *numerical level*, the Dirichlet condition in Eq.(15) should be treated in the α method in the same way as the inertial external force in Eq.(14), that is by ensuring, as in Eq.(7) and Eq.(9)

$$a_1^{n+1} = {}^{\text{base}}a^{n+1+\alpha} = (1+\alpha)\,{}^{\text{base}}a^{n+1} - \alpha\,{}^{\text{base}}a^{n} \tag{16}$$

instead of the obvious but wrong solution $a_1^{n+1} = {}^{\text{base}}a^{n+1}$.

Substructuring and condensation for asynchronous motion

Since its base is always fixed to the floor, the structure in the laboratory can only be tested in a reference frame *relative* to the earthquake motion. Then, in order to realize a meaningful test on a structure (tested and modelled structures) undergoing an asynchronous motion, only physically unconnected parts of the tested structure can be submitted to different base accelerations. This condition is easily verified for bridges: the tested structure consists of a set of different piers which do not interact one with the others, apart through the modelled deck. Each pier can be tested in a different relative reference frame, and this is the task of the modelled structure to make the convenient synchronization between the various relative frames.It is assumed for convenience that the m_{ST} connecting d.o.f.s belong to m_{ST} unconnected parts of the tested structure, each of which subjected to a different input motion ${}^{\text{base}}a_\theta$. Note that the behaviour of these d.o.f.s will be described by two *different* motions whether they are considered as being part of the tested structure (relative motion) or of the modelled structure (absolute motion). To clarify this point it is interesting to write Eq.(1) independently for the tested structure and the modelled structure, and then to focus on the connecting d.o.f.s.

- Eq.(1) for the tested structure is written according to

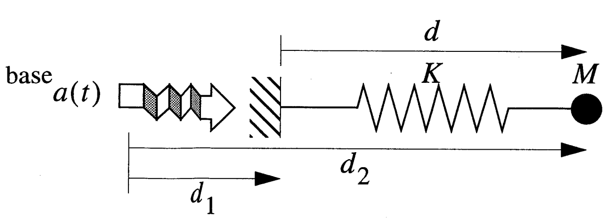

Figure 1 - Simple spring/mass system

$$\begin{bmatrix} {}^{T}\hat{M}_{\delta\theta} & {}^{T}\hat{M}_{\delta J} \\ {}^{T}\hat{M}_{I\theta} & {}^{T}\hat{M}_{IJ} \end{bmatrix} \bullet \begin{bmatrix} {}^{T}a_{\theta}^{n+1} \\ {}^{T}a_{J}^{n+1} \end{bmatrix} = \begin{bmatrix} {}^{T}\hat{f}_{\delta}^{n+1+\alpha} \\ {}^{T}\hat{f}_{I}^{n+1+\alpha} \end{bmatrix} \tag{17}$$

Since the vector ${}^{T}\boldsymbol{a}$ represents the *relative* motion of each part of the tested structure, the external force vector ${}^{T}\boldsymbol{f}$ should account for the inertial forces, and is obtained through the following generalization of Eq.(2)

$${}^{T}\boldsymbol{f} = -\boldsymbol{M}\left(\sum_{\delta} \boldsymbol{I}_{\delta}\, {}^{\text{base}}a_{\delta}\right) = -{}^{T}\tilde{\boldsymbol{M}} \cdot {}^{\text{base}}\boldsymbol{a} \tag{18}$$

where ${}^{T}\tilde{\boldsymbol{M}}$ is a rectangular $m_S \mathrm{x} m_{ST}$ matrix depending only on the mass matrix ${}^{T}\boldsymbol{M}$ of the tested structure and on the direction of the various earthquake loading.

- Eq.(1) for the modelled structure takes the form

$$\begin{bmatrix} {}^{S}\hat{M}_{ij} & {}^{S}\hat{M}_{i\theta} \\ {}^{S}\hat{M}_{\delta j} & {}^{S}\hat{M}_{\delta\theta} \end{bmatrix} \bullet \begin{bmatrix} {}^{S}a_{j}^{n+1} \\ {}^{S}a_{\theta}^{n+1} \end{bmatrix} = \begin{bmatrix} {}^{S}\hat{f}_{i}^{n+1+\alpha} \\ {}^{S}\hat{f}_{\delta}^{n+1+\alpha} \end{bmatrix} \tag{19}$$

Since some parts of the modelled structure, connected directly with the ground (e.g. the ends of a bridge deck), can already experience different ground accelerations, it is convenient to assume that the vector ${}^{S}\boldsymbol{a}$ represents the *absolute* acceleration of the modelled structure. In this case the external force vector is ${}^{S}\boldsymbol{f} = \boldsymbol{0}$

The ground connected d.o.f.s should be subjected to the appropriate base acceleration, by means of the α-shifted Dirichlet boundary conditions Eq.(16).

- The connection between the tested structure and the modelled structure is simply realized by relating the absolute and the relative motion of each d.o.f. to the associated base acceleration ${}^{S}a_{\theta} = {}^{T}a_{\theta} + {}^{\text{base}}a_{\theta}$, whose discrete counterpart, according to Eq.(16), is

$${}^{S}a_{\theta}^{n+1} = {}^{T}a_{\theta}^{n+1} + {}^{\text{base}}a_{\theta}^{n+1+\alpha} \tag{20}$$

By eliminating the m_{ST} absolute accelerations following Eq.(20), it possible to derive the system of equations defining at time level $n+1$ the motion of the whole structure

$$\begin{bmatrix} {}^{S}\hat{M}_{ij} & {}^{S}\hat{M}_{i\theta} & 0 \\ {}^{S}\hat{M}_{\delta j} & {}^{T}\hat{M}_{\delta\theta} + {}^{S}\hat{M}_{\delta\theta} & {}^{T}\hat{M}_{\delta J} \\ 0 & {}^{T}\hat{M}_{I\theta} & {}^{T}\hat{M}_{IJ} \end{bmatrix} \bullet \begin{bmatrix} {}^{S}a_{j}^{n+1} \\ {}^{T}a_{\theta}^{n+1} \\ {}^{T}a_{J}^{n+1} \end{bmatrix} = \begin{bmatrix} {}^{S}\hat{f}_{i}^{n+1-\alpha} - {}^{S}\hat{M}_{i\theta}\, {}^{\text{base}}a_{\theta}^{n+1+\alpha} \\ {}^{T}\hat{f}_{\delta}^{n+1+\alpha} + {}^{S}\hat{f}_{\delta}^{n+1+\alpha} - {}^{S}\hat{M}_{\delta\theta}\, {}^{\text{base}}a_{\theta}^{n+1+\alpha} \\ {}^{T}\hat{f}_{I}^{n+1+\alpha} \end{bmatrix} \tag{21}$$

If the unknowns ${}^{S}a_{j}$ of Eq.(21) are now condensed again, this system is split in two parts

$$\begin{bmatrix} {}^{T}\hat{M}_{\delta\theta} + ({}^{S}\hat{M}_{\delta\theta} - {}^{S}\hat{M}_{\delta j}\, {}^{S}\hat{M}_{ji}^{-1}\, {}^{S}\hat{M}_{i\theta}) & {}^{T}\hat{M}_{\delta J} \\ {}^{T}\hat{M}_{I\theta} & {}^{T}\hat{M}_{IJ} \end{bmatrix} \bullet \begin{bmatrix} a_{\theta}^{n+1} \\ a_{J}^{n+1} \end{bmatrix} =$$

$$\begin{bmatrix} {}^{T}\hat{f}_{\delta}^{n+1+\alpha} + [\, {}^{S}\hat{f}_{\delta}^{n+1+\alpha} - {}^{S}\hat{M}_{\delta\theta}\, {}^{\text{base}}a_{\theta}^{n+1+\alpha} - {}^{S}\hat{M}_{\delta j}\, {}^{S}\hat{M}_{ji}^{-1} (\, {}^{S}\hat{f}_{i}^{n+1+\alpha} - {}^{S}\hat{M}_{i\theta}\, {}^{\text{base}}a_{\theta}^{n+1+\alpha})] \\ {}^{T}\hat{f}_{I}^{n+1+\alpha} \end{bmatrix} \tag{22}$$

$${}^{S}\hat{M}_{ij}\, {}^{S}a_{j}^{n+1} = {}^{S}\hat{f}_{i}^{n+1+\alpha} - {}^{S}\hat{M}_{i\theta}({}^{T}a_{\theta}^{n+1} + {}^{\text{base}}a_{\theta}^{n+1+\alpha}) \tag{23}$$

where Eq.(22) is now the system over d.o.f.s to be solved by the tested structure process and Eq.(23) the system that can be solved afterwards and elsewhere, in particular by a concurrent process dedicated to the modelled structure.

Implementation

Considering again an existing PSD/α-OS algorithm, the same modification as before has to be performed for the generalized mass matrix ${}^T\hat{M}$ of the tested structure. During the time loop, at the end of the measuring phase of the test, the pseudo force $\hat{f}^{n+1+\alpha}$ still has to be modified. As can be seen by comparing Eq.(11) to (22), this modification is however more complex than in the synchronous case. However, this term is supplied by the process running the modelled structure so that the process responsible for the tested structure remains unchanged. Still as before, the solution of Eq.(22) being performed by the tested structure process, the value of the acceleration at the connecting d.o.f.s has to be supplied to the modelled structure process. Note that, in contrast with the case with synchronous motion, this value is added to the base acceleration in order to obtain the absolute acceleration according to Eq.(23). However, this modification is again of the responsibility of the process handling the modelled structure and in particular does not affect the process responsible for the tested structure.

At the implementation level, the differences between the synchronous and asynchronous motion situations are the following. i) The PSD process handling the tested structure is marginally affected: the *only* difference lies in the derivation of the loading vector ${}^T\boldsymbol{f}$ (Eq.(18) instead of Eq.(2)) which does no longer depend on only one accelerogram. ii) The modelled structure process is more deeply changed: first, being in the absolute space, the loading vector ${}^S\boldsymbol{f}$ is null and, second, the passage between each relative frames and the absolute frame affects the derivation of the pseudo force vector which is sent to the tested structure process and the acceleration vector which is received from the tested structure process.

ILLUSTRATIVE RESULTS

A recent research programme on bridges included the pseudo-dynamic testing of six large-scale bridge models in the ELSA Laboratory using substructuring techniques. The first part of the experimental research was concerned with the testing of four reinforced concrete bridges [6]. The test campaign in ELSA was completed with two special tests, namely: a bridge with isolation/dissipation devices and another bridge with asynchronous input motion.

The seismic tests in the ELSA laboratory were carried out using the pseudo-dynamic testing method with substructuring: the bridge piers have been constructed and physically tested while the continuous bridge-deck behaviour was simulated on-line. A synchronous input motion was assumed in most of the tests. However, a test with asynchronous input motion was carried out, which required the mathematical and implementation procedures developed in the previous section.

Two illustrative results are given in this paper: one concerns the test with synchronous motion and the other the one with asynchronous motion. The tested bridges, shown schematically in Figure 2, were constructed at a 2.5 reduced scale. The pier were tested physically and the deck simulated on-line. Time histories of the piers top displacement are given in the figure, including, for the asynchronous motion, the absolute and relative displacements. A detailed analysis of the test results from the engineering point of view can be found elsewhere [7].

REFERENCES

1. Donea, J., Magonette, G., Negro, P., Pegon, P., Pinto, A.V. and Verzeletti, G. 1994. Pseudo-dynamic testing of large-scale models of civil engineering structures at the ELSA reaction wall.
Proc. of the 10th European Conference on Earthquake Engineering, Vienna.

2. Calvi, G.M. & Pinto, P.E. 1994. Seismic design of bridges: Experimental and analytical research.
Proc. of the 10th European Conference on Earthquake Engineering, Vienna.

3. Dermitzakis, S.N. & Mahin, S.A. 1985. Development of substructuring techniques for on-line computer controlled seismic performance testing.
Report No. UCB/EERC-85/04, Berkeley, California.

4. Buchet, P. & Pegon, P. 1994. PSD Testing with Substructuring - Implementation and Use.
JRC Special Publication, No. I.94.25, 21 pages, Ispra, Italy, 1994.

5. Pegon, P. 1996. PSD Testing with Substructuring - the case of asynchronous motion.
JRC Special Publication, No. I.96.42, 17 pages, Ispra, Italy.

6. Pinto, A.V., Verzeletti, G., Pegon, P., Magonette, G., Negro, P. and Guedes, J. 1996. Pseudo-Dynamic Testing of Large-Scale R/C Bridges.
EUR report No. 16378 EN, 109 pages, JRC.

7. Pinto, A.V. (Editor). 1997. Pseudodynamic and shaking table tests on RC bridges.
ECOEST/PREC8 Concerted Action Report (EC-HCM), LNEC, Lisbon.

8. H.M. Hilber, T.J.R. Hughes and R.L. Taylor. 1997. Improved Numerical Dissipation for Time Integration Algorithms in Structural Dynamics.
Earthquake Eng. Struct. Dyn. **5**, 283-292.

9. D. Combescure and P. Pegon. 1994. Alpha-Operator Splitting Time Integration Technique for Pseudodynamic Testing. Error Propagation Analysis.
JRC Special Publication No. I.94.65, JRC, Ispra, Italy.

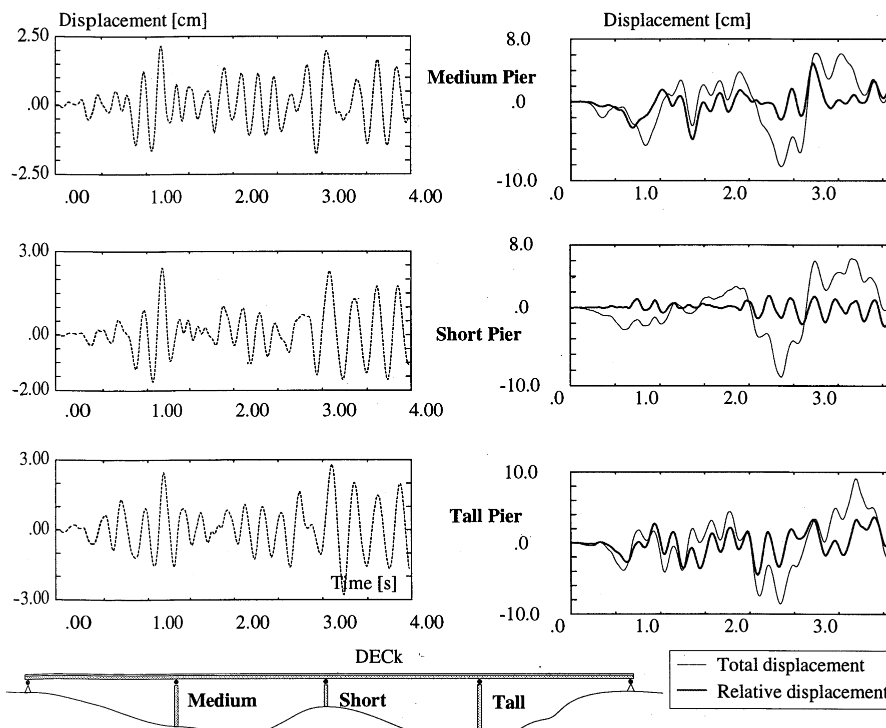

Figure 2 - Time histories of top piers displacement for the design earthquake from: Synchronous input motion (*Left*) and Asynchronous input motions (*Right*)