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Biaxial Loading Effects on the Growth of Cracks

M.W. Brown, K.J. Miller
Dept. of Mechanical Engineering, University of Sheffield, Mappin Street, Sheffield S1 3JD, U.K.
T.J. Walker
U.S. Nuclear Regulatory Commission, Office of Nuclear Regulatory Research, MS 1130-SS, Washington, D.C. 20555, U.S.A.

Summary

Fatigue crack growth under different biaxial stress states is considered for both small scale yielding and high bulk stress conditions. Analytical and elastic finite element results are compared favourably alongside experimental results on a AISI 316 stainless steel at both room and elevated temperatures. Differences in crack growth rates are compared against different crack tip cyclic plastic zone sizes for various degrees of mixed mode loading, thereby overcoming the limitations of the Paris Law and LEFM.

The usefulness of the approach is indicated for studies in the behaviour of materials subjected to thermal shock. Where steep temperature gradients are introduced due to rapid thermal transients, high strains are produced which propagate fatigue cracks under cyclic conditions. Since stress gradients are generally associated with thermal shock situations, the cracks grow through a plastically deformed region near the surface into an elastic region. A unified approach to fatigue behaviour, encompassing both linear elastic and elastic-plastic fracture mechanics, will enable analysis of thermal shock situations.

The approach to crack propagation developed here shows that cyclic growth rates are a function of a severe strain zone size in which local stresses exceed the tensile strength, i.e. monotonic instability. The effects of stress biaxiality and mixed mode loading are included in the analysis, which may be extended to general yielding situations.

1. Introduction

Linear elastic fracture mechanics (LEFM) has been successfully used to analyse fatigue crack growth of mode I cracks. However the well established Paris law needs to be extended to include not only higher stress levels where the strict small scale yielding (SSY) conditions are no longer valid, but also biaxial stress field effects and the influence of mixed mode loading. A more general equation describing fatigue crack growth should enable improved design methods to be devised and assist our understanding of the fatigue behaviour of components which contain notches, complex stress fields and high strain gradients.

This paper summarizes the progress in a multidisciplinary collaborative project concerned with the creep-fatigue behaviour of 316 stainless steel. A number of projects have been initiated to study creep damage accumulation, fatigue and creep crack growth and their interaction, but this short paper will concentrate on the fatigue crack propagation aspects. Elastic-plastic finite element studies of cracks in biaxial stress fields have been made alongside experimental work, to relate the applied stress fields to crack extension. Thermal shock is discussed as just one example to which these studies are applicable.

2. Elastic-Plastic Fracture Mechanics

The results of elastic-plastic analyses by finite element methods (FEM) have been published [1][2] for inclined cracks in biaxially loaded plates. The local stress strain field at the crack tip may be characterised in terms of the size and shape of plastic zone, crack tip opening and shear displacements, the J integral, energy release rate and the G^{Δ} criterion for co-planar or non-coplanar crack extension. One approach to the high strain fatigue crack growth problem is based on crack tip plasticity [3], since this is clearly related to fatigue endurance through the Coffin-Manson equation [4] as well as to models based on crack opening displacement.

Typical plastic zones obtained in the finite element analyses are illustrated in Fig. 1. These show the conventional plastic ears emanating from the crack tip, but when the mode II component of the load is large, one ear becomes dominant ahead of the crack, making shear mode growth feasible. It is also found [2] that the non-singular stress parallel to the crack has a large effect on the size of plastic zone. This may be characterised in terms of the T stress, where

$$T = \sigma_L - \sigma_N \tag{1}$$

and σ_N and σ_L are the applied stress components normal and parallel to the crack, respectively. Large negative values of T increase crack tip plasticity, together with the crack opening displacement (COD) and J integral. The COD is largest for pure mode I cracks, so that one would expect inclined cracks to follow a path that tends to maximise the mode I components.

3. Fatigue Crack Growth Results

Studies of mode I fatigue cracks in biaxially stressed cruciform specimens have shown a distinct effect of the T stress on propagation rate. At low stress levels LEFM conditions are applicable to long cracks, and the range of stress intensity factor ΔK was found to characterise both threshold and growth rate satisfactorily in materials with a reasonably high yield stress (i.e. SSY conditions) [5]. But as the applied stress ranges were raised, a biaxiality effect was observed, as shown in Fig. 2, similar to the trend observed in a number of different materials [6]. Tests on two heats of 316 stainless steel at 20 and 550°C demonstrated the same behaviour in load controlled tests, i.e. faster growth for a negative T stress.

Clearly as the crack tip plasticity is increased by the negative T stress, the mechanism for crack extension is enhanced to give deviations from the Paris law. This indicates that growth rate cannot be determined from ΔK alone. A similar result is also observed in low cycle biaxial fatigue tests [7], where two strain parameters are required to describe the dependence of crack growth on stress state. The apparently higher fatigue strength under shear loading for specimens in traditional low cycle fatigue tests is compatible with high growth rates in shear (Fig. 2), because the cyclic stress strain curve for biaxial loading shows that the torsional shear strain amplitude is seven times that for the uniaxial case with the same range of principal stress σ_N [8].

Results of mixed mode I and mode II loading are presented in Fig. 3, plotted in terms of cyclic plastic zone size, $r_p(\theta^*)$, measured from the crack tip in the direction θ^* that shows the greatest extent of plasticity. Since the stresses applied were not too high, crack tip plasticity could be estimated from singular crack tip stress components and the T stress with a von Mises yield criterion, assuming LEFM stress fields could adequately predict SSY behaviour [9]. The five tests presented in Fig. 3 were all at different stress biaxialities, Λ ($= \sigma_L/\sigma_N$) where $0.1 \geq \Lambda \geq -1$, and different initial crack inclinations to give $\infty \geq \Delta K_{II}/\Delta K_I \geq 0.8$. It can be seen that crack growth rate is approximately proportional to plastic zone size. It was also observed that inclined cracks branched to follow the direction of maximum tensile stress, eventually becoming pure mode I cracks as predicted above.

Tests at threshold levels showed that an initial period of shear mode crack growth preceded the formation of a mode I branch crack [10]. This shear mode growth could be characterised by the simple equation

$$da/dN = k\, r_p(\theta^*)/\gamma_f \qquad (2)$$

where the shear fracture ductility, γ_f, on the radial plane in the direction θ^* depends on crack tip stress fields, defined by $\Delta K_{II}/\Delta K_I$. The influence of the γ_f term was of secondary importance compared to cyclic plastic zone size, nevertheless it did enable a satisfactory correlation between thresholds for pure mode I and pure mode II situations. Once the crack branches to follow a mode I path, the value of ΔK_{II} becomes zero, so that the term γ_f in eq. (2) does not vary as crack growth proceeds, giving the concurrence of results in Fig. 3.

Although eq. (2) gave a satisfactory description of threshold behaviour for coplanar crack extension, the shearing displacement at the crack tip introduced fretting and frictional resistance between the fracture surfaces as the crack grew. This reduced the effective stress intensity at the crack tip, allowing the crack to arrest. By raising the cyclic stress amplitude to a sufficiently high level a mode I branch crack was formed, enabling continuous propagation as depicted in Fig. 3.

4. Determination of Plastic Zone Size

Plastic zone size may be determined by either elastic-plastic finite element analysis or from the LEFM stress field for SSY conditions. Both methods require numerical analysis [1][9], so it is advantageous to derive a simpler closed form solution that predicts the extent of plasticity for SSY and the intensification of the plastic strain around the crack tip where more general yielding is involved. The Dugdale-Barenblatt model [11] was extended in reference [5] to show the influence of T stress on mode I plastic zones for plane stress. Although the strip yield model cannot accurately represent the shape or orientation of plastic shear ears, particularly in strain hardening materials, it did provide good quantitative prediction for the SSY size of the zone, $r_p(\theta^*)$, defined above.

The strip yield model has also been used for mode II shear cracks. The mode I and II analyses may be combined to give the plastic zone size for mixed mode conditions, if loading is proportional, to give a fixed ratio of shear and tensile cohesive stresses within the yielding region. For plane strain conditions at the crack tip, the plastic zone r_p is given by

$$r_p = a[\sec(\pi\sigma_N/2\sigma_o) - 1] \tag{3}$$

where

$$2\sigma_o = \frac{4AY}{(1+\nu)} \{[1 - \left(\frac{\alpha T}{Y}\right)^2 (1 - A^2)]^{\frac{1}{2}} - \left|\frac{\alpha T}{Y}\right| A \} \tag{4}$$

In eq. (4),

$$A = (1+\nu)/2\sqrt{1 + \nu^2 - \nu + 3(\tau/\sigma_N)^2} \tag{5}$$

and τ is the mode II shear stress, Y is the yield stress, ν is Poisson's ratio and α is a constant equal to unity. Equations (3) to (5) show that the plastic zone is determined by the three applied stresses σ_N, τ and T.

For strain hardening materials, the strip yield plastic zone is replaced by the more diffuse shear ears in Fig. 1. These are also predicted by the SSY approach used in reference [9], although a correction factor of $(\pi/2)^2$ is required for plastic zone size to give agreement with eq. (3). Figure 4 shows that, for different biaxial stress states, putting α equal to $1/\sqrt{2}$ gives an excellent representation for plane strain SSY behaviour with strain hardening. Since the crack length was 2.54 mm, the SSY equations must break down where the lines diverge ($r_p \approx 0.1$ a), but eq. (3) is still valid for the higher stresses, as shown by the FEM results, plotted as points in Fig. 4. Although there is considerable scatter in the points on this figure, this reflects the accuracy of r_p values which is directly related to element size in Fig. 1. With this is mind eq. (3) gives satisfactory agreement with all the FEM data presented, in spite of the dependence on Λ in Fig. 4. However, if desired, improved correlation may be achieved by introducing suitable functions for α in eq. (4).

The mode I growth rates are plotted in Fig. 5 in terms of plastic zone size defined by eq. (3), normalised by the equibiaxial values for each stress level, $(da/dN)_1$ and r_{p1} respectively. To obtain the cyclic zone size, Y was replaced by 2Y and σ_N by $\Delta\sigma_N$. Since general yield was attained in the specimens at the highest stress levels, r_p becomes infinite, giving the dotted curve in Fig. 5 which clearly deviates from eq. (2). As crack growth rates are not infinite in low cycle fatigue, this unrealistic equation may be improved by replacing Y in eq. (4) with the ultimate strength [5], as proposed by Tomkins [12]. This gives a much simpler relationship,

$$\frac{da/dN}{(da/dN)_1} = (r_p/r_{p_1})^{1.5} \tag{6}$$

which applies to both room and elevated temperature for this material. This equation can also be used to represent the mixed mode data in Fig. 3.

5. Discussion

The results presented indicate that for both mode I and mixed mode I/II loading, crack propagation rate in biaxial stress fields is related to crack tip plasticity by eq. (6). Equation (4) shows that the presence of a T stress will increase the plasticity, accelerating crack growth in the high strain regime. The use of the tensile strength in eq. (4) has already found widespread applicability [12], and this suggests that the fatigue process zone is a region of unstable plastic flow ahead of the crack, which must be closely related to the cyclic strain hardening exponent and strength coefficient.

High strain crack growth may be observed in thermal shock tests where thermal transients in excess of 50°C lead to plastic deformation [13]. Figure 6 shows crack growth monitored in a plate sample experiencing a periodic thermal downshock of 280°C on one edge. It's rate decreases as the crack extends into regions of lower temperature gradients, which correspond to reduced stress. However the application of an additional mechanical stress, σ, normal to the crack, accelerates growth. Being a steady stress, this cannot affect the stress intensity factor range. But it does have an influence on T in eq. (4), reducing the stress state from equibiaxial (for pure thermal shock) towards the uniaxial condition. This will lead to increased growth rates (Fig. 2), as observed. This does not, however, account for the dramatic increase in growth at the higher stress, 38 and 76 MPa, where creep damage due to the mean stress applied leads to a change of mechanism and rapid creep crack growth.

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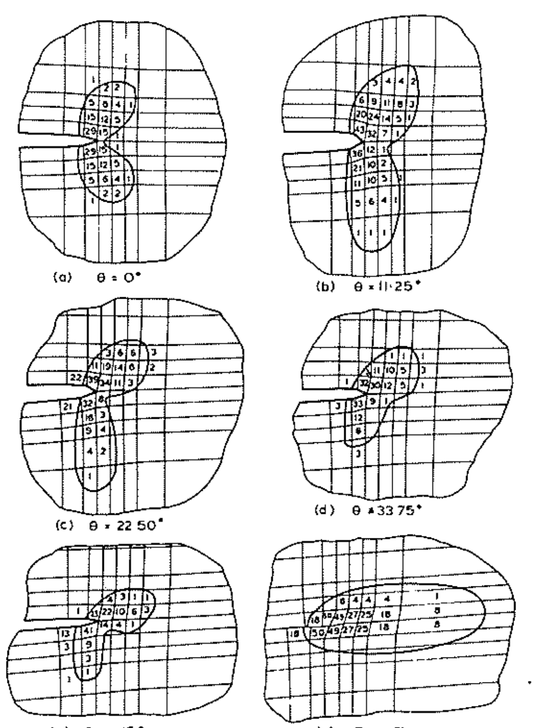
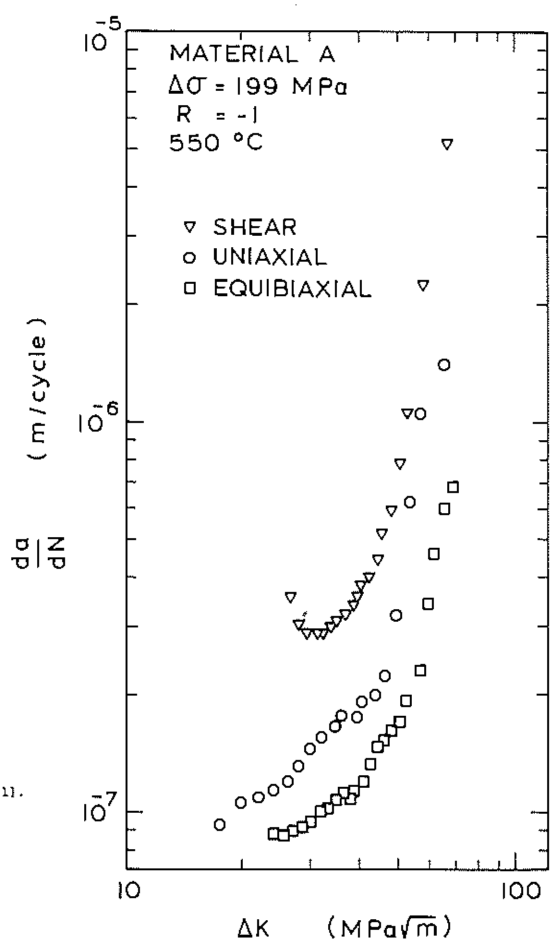
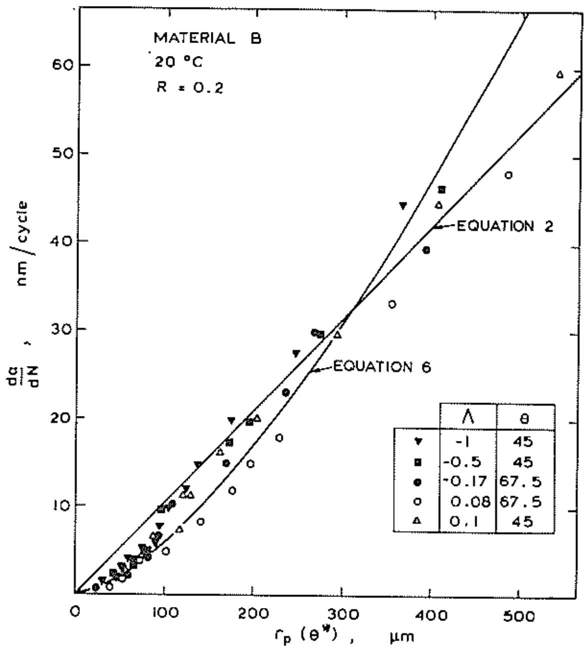


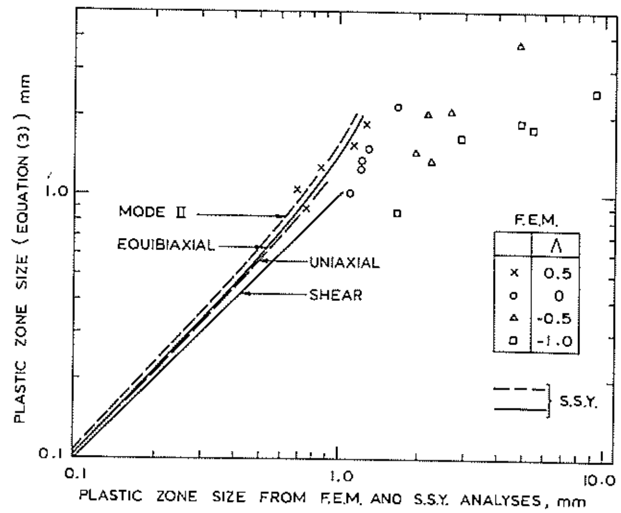
1. Plastic zones for different crack inclinations θ in uniaxial loading (a to e), and pure shear (f). The numbers give plastic strain x10^4 [1].



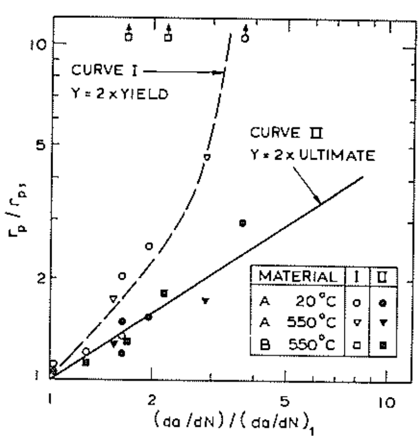
2. Influence of biaxial stress on fatigue crack growth in 316 stainless steel at 550°C [5].



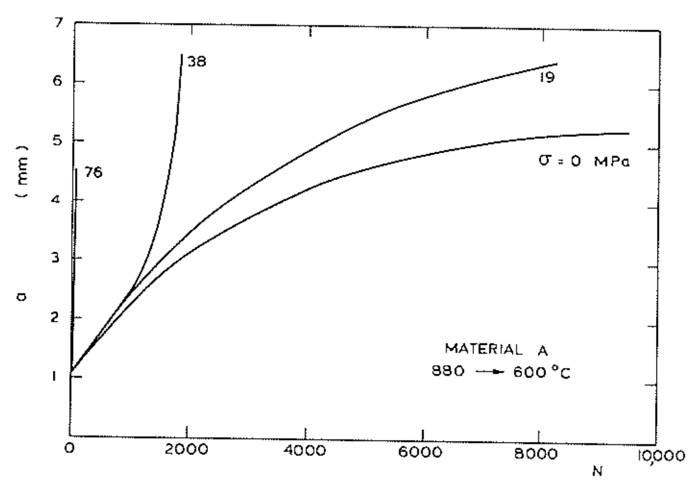
3. Fatigue crack propagation for mixed mode loading of inclined cracks (θ) in biaxially stressed cruciform specimens, as a function of maximum cyclic plastic zone size [9].



4. Plastic zone size from eq. (3) compared with SSY and FEM results for a crack length of 2.54 mm.



5. Mode I growth rates as a function of cyclic plastic zone size (curve I) and severe strain zone size (curve II), derived from eq. (3) and normalised to equibiaxial values.



6. Growth of a crack in a single edge notched plate due to thermal downshock of 880° to 600°C with a superposed tensile stress, σ [13].