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DYNAMIC INSTABILITY OF CYLINDRICAL ELASTO-PLASTIC SHELL UNDER COMBINED STATIC LOAD AND DYNAMIC SHORT PULSE

J. Ivanova[1] and F. Pastrone[2]

[1]Institute of Mechanics and Biomechanics, Bulgarian Academy of Sciences, Acad. G. Bonchev str. 4, 1113 Sofia, Bulgaria
[2]University of Torino, Department of Mathematics, v. C. Alberto 10, I-10123 Torino, Italy

1. INTRODUCTION

Many thin walled structures like shells can be subjected to dangerous loading, as example: a dead loads q_o and an short-time dynamic pulse, namely a load q, applied dynamically in the axial direction, during an interval of time [0, τ]; we shall summarize this action with the notation ($q\tau$). The following conditions must be satisfied: $q_o << q_e$ and $q > q_e$, where q_e is the maximum value of the static load at which the shell loses its stability. When we apply dynamic pulse the shell does not react immediately as shown by experiments, thus process of buckling has a delay in comparison with the static case. This case of a "delay" causes the change of the load under which the same buckling of the middle shell surface takes place in comparison with the Euler static load q. As we will see further, for shells the critical dynamic load can surpasse the maximum static load q_e.

The problem arising here is: is it possible to predict theoretically the critical values of the amplitude q of the pulse together with the value of the static load q_o such that the shell can go back to the initial undeformed configuration, making oscillations around it? Is it possible to find the curve (q, q_o) dividing the zones of stability and instability of the shell? The method used follows Pogorelov's investigations [1], but will be extended to the problem of dynamic shell stability.

2. DYNAMIC INSTABILITY.

Firstly we have to expline the nature of the critical point of the dynamic loss of stability: which type of extremal point we have - a minimum, a maximum or an inflexion? Starting from the full energetic functional $W = \int(K - U - A)dt$ we shall try to find the character of the stationary point of W. As an example we take the clamped spherical cap under action of the pulse load $p > q_e$. The problem on the consideration is to investigate the deformations of the shell in time and to expline the physical meaning of the snap. Let us define the dynamic shell behaviour in the neighbourhood of the point of "dynamic" instability on some surface F which well approximates the behaviour of the shell. This surface is constructed in [1] and is isometric to the initial undeformed surface in the class of partially-regular surfaces. The surface is generated using the initial undeformed form and the miror images through the planes paralell to the shell base. To define the approximation we assume that surface F can be given by the aid of twice mirror images: the mirror image of the shell segment through the plane α closed to the shell edgeand the next mirror image through the plane β.

Introducing phase coordinates h(t), h′ where h and h′ are height and the velocity of mirror image of a shell cap, we can define W on F. If the planes α and β which determine the mirror bucling are not close to each other then we have:

$$U = \pi c E \delta^{5/2}(1/R_1 + 1/R_2)\ (2h)^{3/2} + (*)$$

where E is Young modulus, $R_{1,2}$ are main radii of curvatures in the centre of buckling, δ is the thickness of the shell, c=0.19. With (*) we note the analogical energy for the mirror buckling, corresponding to α, which is stationar and we can ignore it.

The work A produced by the external load p is:

$$A = 2pV + (*) \quad \text{where } V = \pi h^2\sqrt{R_1 R_2}$$

The kinetic energy of the shell motion is:

$$K = S\delta\gamma(2h')^2 \qquad \text{where } S = 2\pi h\sqrt{R_1 R_2}$$

The following Euler-Lagrange equation is obtained:

$$2hh' + h'^2 + 1.5h^{1/2} + 2c_2 h = 0$$

where c_1 and c_2 are positive constants. The first integral is:

$$h' = [\frac{c_3 - c_1 h'^{3/2} - c_1 h^2}{h}]^{1/2} \text{ where } c_3 \text{ is constant.}$$

In the neighbourhood of the critical point after which the shell jumps to the new stable configuration, h tends to 0, h tends to infinity, thus the velocity of the bending h has a jump. Physically the snap is accompanied by the impact. Mathematically, on the diagrams (h, h) and (W, h) before reaching the value h= 0, we have an inflexion point h_{cr}.

3. THE GEOMETRIC METHOD.

Let us consider a simply supported elasto-plastic cylindrical shell with radius R, length L, thickness h under a load conditions given in the introduction. The brief commentary on the isometric partially regular map Z of initial undeformed cylinder is given in [2]. If the curve (rib) γ is given by the formula:y(x, t) u(t)y(x),then the way to find the amplitude u(t) can be obtained separately from geometric characteristics of Z which depend only on coordinates. The curve is periodical with the number of the complite waves m. From the other hand the wave numbers n and m divide the surface Z into 4nm congruent region Q isometric to a rectangle. The height of Q is b = L/2m, the width is a = R/n. We have to find the energy of the post critical deformations in Q and multiply by 4nm in order to find energy on whole surface Z. Geometric characteristics of Z are:

$$k_1 = \mp \pi y''/2n \ ; \ k_2 = 0 \ ; \ \Delta k_2 = -1/R \ ; \ \Delta k_{1,2} = k_{1,2} - k_{1,2}$$

where with ()′′ we indicate the differentiation by x. For initial cylindric surface normal curvatures are $k_1 = 0$, $k_2 = 1/R$.

In the neighbourhood of the curve γ (γ divides the neighbourhood to two parts) the normal curvatures k_e and k_2 have equal values but differ by the sign, thus $k_e + k_i = 0$. The angle α between the plane of a rib γ and the tangent planes of Z along γ and the length of the arc of γ are:

$$\alpha = \pi(1 + y'^2)^{1/2}/2n \quad ; \; ds = (1 + y'^2)^{1/2}dx$$

The angle between the generator of initial surface and direction corresponding isometrically to it can be expressed by:

$$\cos\theta = 1/(1+y'^2)^{1/2} \; ; \text{ curvature of } \gamma \text{ is } k = y''/(1+y'^2)^{3/2}$$

The load-shortening of the part of the ideal cylinder is: $\Delta b = b - b' = \frac{\pi^2}{8n^2} \int_b y'^2\, dx$ where b' is the heigth of the already deformed part of the cylinder with a heigth b.

In [2] the following formula for the elasto-plastic post-critical deformation is obtained:

$$U = \frac{Eh^3}{24(1-\upsilon^2)} \int_F (k_{11}^2 + k_{22}^2 + 2\upsilon k_{11}k_{22})dF + Eh^{5/2}cJ_o \int_{s_\gamma} \alpha^{5/2}/\rho^{1/2}\, ds_\gamma$$

$$+\frac{Eh^3}{9} \int_{s_\gamma} \alpha[0.25(\Phi_s+3\Phi_t)(k_e+k_i-2k_n) - \Phi_s k_\gamma]ds_\gamma \tag{1}$$

where E, υ, Φ_s, Φ_t are Young, Poisson ratio and secant and tangent moduli from a stress-strain diagram of shell material, $k_{\alpha\beta}$ (α,β = 1,2) is a tensor of changing of curvatures atthe transformation from initial to Z surface $k_{\alpha\beta} = \Delta k_{\alpha\beta}$. J_o = 1.11, α is an angle between plan of γ and tangent plans of Z on $\gamma\rho$ is a radius of curvature of γ, k_e and k_i are normal curvatures of Z in direction perpendicular to γ; k_γ, k_n are normal curvatures of initial surface in respective direction;$c=[(\phi_s+\phi_t)/36]^{3/4}\phi_t^{1/4}J_o$. Introducing non-dimensional coordinates as follows:x = bx/2 y =$a(\lambda/2)^{1/2}$y and putting geometrical characteristics in (2), we obtain formula for the energy of the post-critical deformation:

$$U = 2nU_Q = \frac{E\pi^2h^3}{12(1-\upsilon^2)n} \left\{ \lambda \int_{-1}^{1} y''^2dx + \frac{2^{1/4}c3(1-\upsilon^2)\pi\lambda^{1/4}}{0.91} * \right.$$

$$\left. *\int_{-1}^{1} y''^{1/2}(1+2\lambda y'^2)dx + 2\lambda(\upsilon \frac{2(1-\upsilon^2)\phi_s}{3}) \right\}$$

We shall approximate function y(x), giving an asymmetric mode of deformation, by two parabolas with verteces on lines x = 1, -1 and by two straight lines smoothly passing into parabolas. Parameters characterizing y(x) are k =|y''| at x = 1,-1, l=y(0).
Using the approximation of y(x) by k and l we get:

$$U = \frac{E\pi^2h^3}{12(1-\upsilon^2)n} J^*(k, l, \lambda)$$

where

$$J^* = 2kl\lambda + c_1\lambda + c\,[2\lambda^{1/4}k^{1/2} + 4l^3\lambda^{5/4}/3k^{1/2}]$$

and $k = \dfrac{4l^3}{3(2l^3-1)}$,in [3] is shown that $l = 0.82$, $\lambda < 4/3$

Now introducing the parameter ω equal to $\omega = \lambda^{1/4}k^{1/2}$ and putting $s = l/k$ we have that:

$$U = 2nU_Q = \frac{E\pi^2h^3}{12(1-\upsilon^2)n}[\,C\omega^4 + B\omega^5 + D\omega\,] \tag{2}$$

where:

$C = 2s + c_1/k^2$; $B = 4s^3c/3$; $D = 2cs$; $\upsilon = 0.3$;

$c = (\phi_s + \phi_t)^{3/4}\phi_t^{1/4}J_o/36^{3/4}$; $c_1 = 2[\upsilon - 2\phi_s(1 - \upsilon^2)/3]$;

The work A done by the static load q_o [2] has the form:

$$A = \frac{E\pi^4h^3\omega^4 q_o}{3\,.\,3nk^2} \;;\; q_o = q_o Eh/R$$

At the very begining the shell buckling could be described by the formula:$w(x,y,t) = u(t)\sin 2\pi mx/L \sin ny/R$, where w is normal deflection of the shell. Let us find the function u(t). The shell motion can be determined from stationarity condition of the functional $J = \int(K - U + A)dt$. Now the equation of the shell motion can be obtained as Lagrange equation for J.

$$u'' - \mu^2 u = 0 \quad \text{where } \mu^2 = \frac{4\pi^2m^2}{L^2\gamma}(q - q_e)$$

and the following initial conditions have to be satisfied:
$u(0) = 0$, $u'(0) = d\lambda$. Then $u(t) = d\ \mathrm{sh}\ \mu t$, where

$$d = \frac{2}{\mu}\left\{\frac{q^2-q_e^2}{E\gamma}\right\}^{1/2}$$, γ is a density of shell material

Now we can find the deformation and kinetic energy as well as the work done by the external force q. At the moment of the stop of the action of q the total energy of the shell (kinetic energy and buckling deformation energy) has a form:

$$V = \frac{\pi hRL}{E}(q^2-q_e^2)(\mathrm{ch}^2\mu\tau - \frac{q_e}{q - q_e}\mathrm{sh}^2\mu\tau)$$

After stopping the action of the pulse q the shell continues to buckle under load q_o and if q_o is not too large then the buckling at some moment stops, the shell restores it initial form. On the contrary, if q_o is large then the buckling grows up. The critical value of q_o at which the shell could restore its initial form can be determined by the condition that at the moment of the stopping the action of the pulse the shell force is exactly equal to q_o. The other condition is connected to the fact that at the same moment of the stopping a pulse the energy V, produced by the pulse and the work A done by q_o have to be in a balance with the deformation energy U of the shell. Hence, according to the bouth conditions we have:

$$V + A(q_o, \omega) = U(\omega) \tag{3}$$

$$q(\omega) = q_o \tag{4}$$

After elimination of ω, we obtain a relation between q. q_o and τ. Putting the second equation in the first and satisfying the conditions for having the inflexion point of q as a function of ω we can find the critical point q_{cr} of dynamic instability of the shell. Let us suppose that the loss of stability has the same character as in the static case.In our consideration the critical value (maximum) for the shell under a static load is q_e =0.6E h/R, assuming that the begining no plastic zones.

The energy V, produced due to the action of the pulse $q\tau$ is:

$$V = 0.36\pi Eh^3\frac{L}{R}(k^2 - 1)(ch^2\mu\tau + \frac{1}{k-1}sh^2\mu\tau) \tag{5}$$

where $k = \frac{q}{q_e}$ and $\mu = 0.7\ [(k\text{-}1)E/\gamma R^2]^{1/2}$ then we have:

$$V(k,\ \tau) + A(q_o,\ \omega) = U(\omega) \tag{6}$$

The energy ballance together with the condition of the equality of a shell force to the value q_o can be rewritten:

$$k = \frac{1}{ch\mu\tau}\{[\ L(\ U - 0.25\omega^4\frac{q_o}{P}\ ) + \frac{sh^4\mu\tau}{4ch^2\mu\tau} + 1\ ]^{1/2} - \frac{sh^2\mu\tau}{2ch\mu\tau}\} \tag{7}$$

$$q_o = \frac{P}{\omega^3}\frac{dU}{d\omega} \tag{8}$$

where: $L = \frac{1}{4.32(1\text{-}\upsilon^2)}$; $P = \frac{3.3k}{12(1\text{-}\upsilon^2)\pi^2}$ $U = C\omega^4 + B\omega^5 + D\omega$

Putting (8) into (7) we obtain for k:

$$k = \frac{1}{ch\mu\tau}\{[\ L(U - 0.25\omega\frac{dU}{d\omega}\ ) + \frac{sh^4\mu\tau}{4ch^2\mu\tau} + 1]^{1/2} - \frac{sh^2\mu\tau}{2ch\mu\tau}\}$$

To find the boundary of the safety zone of combin loaded shell we have to eliminate ω from (8). It has already proved [2] that solution of system (7, 8) exists when $q_o < q_e$, thus for ω we have the interval $(\omega_e,\ w_i)$, where ω_i corresponds to minimal critical load q_i. Taking various values for ω the curve (k, q_o, τ) can be drown. This curve divides the stable and unstable zone. To find at which condition on the diagram (k,ω) has an inflexion point we put the first derivative of k equals to zero and obtain the following condition:

$$\frac{d^2U}{d\omega^2} - \frac{3}{\omega}\frac{dU}{d\omega} = 0 \tag{9}$$

From the other hand to obtain the minimal value of q_o (q_i) we derive (8) by ω and get the same condition (9). Now the second derivatives of k and q_o have to be zero. Taking into account (9) we can obtain that:

$$k'' = L(2\frac{d^2U}{d\omega^3} - \omega\frac{d^3U}{d\omega^3})k = 0 \quad \text{where } k = \frac{q}{q_e} > 1. \tag{10}$$

The last result confirms existance of inflexion point (k^{cr}, ω^{cr}). Now intervals for k and ω are (1, k^{cr}) and (ω_e, ω^{cr}). To find k_{cr} and ω_{cr} we have to solve an algebraic equation (9):

$$\omega^4 = \frac{3D}{5B}, \tag{11}$$

where $B = \frac{4s^3c}{3}$; $D = 2sc$; $c=[(\phi_s+\phi_t)/36]^{3/4}\phi_t^{1/4}J_o$

$s = l/k$, $l = 0.82$, then $k = 2.2$. Thus, $s= 0.37$ and $\omega_{cr} = 1.3$.

We note here that for the case of a loss of dynamic stability of a shell under combine loading the obtained value for q_{ocr} is the maximum one for q_o. For every $1< k < k_{cr}$(for every $q_e <q< q_{cr}$) and $q_{ocr}< q_o < q_e$ the shell can restore its initial form.

The final resultat for ω_{cr} does not depend on the material properties of the shell. The critical value for k depends on the material properties (ϕ_t, ϕ_s) and time τ of pulse. Variating ω in the interval, corresponding to the interval for q_o we obtain the safety curve (k, q_o, τ). The safety curve (q, q_o) is a strongly convex curve which gives both the stable and unstable zone.

The present method for solving the problem of dynamic loss of stability can be successfully applied to earthquake excited structures like shells. We could follow bellow written sheme:

(i) Express into a serie of short-time rectungle axial pressure pulses the earthquake excitation $q_{(1)}, q_{(2)},\ldots, q_{(n)}$.

(ii) Let on the shell act the continuously applied q_1^o and the short-time pulse, both of axial pressure, satisfying the initial conditions for the loading i.e. $q_1^o < q_e$ and $q_{(1)} > q_e$; ($q_{(i)}> q_e$, $i = 1,\ldots n$). Following the proposed model find a critical combination of the values of q^o and $q_{(1)}$. If q^o and $q_{(1)}$ are such that their values are less than the critical ones the shell restores its initial form but with some initial imperfactions on the shell surface due to the elasto-plastic behaviour.

(iii) Let on the shell act next pair q_2^o and $q_{(2)}$ (initial imperfactions have to be taken into account). Check values of (q_2^o, $q_{(2)}$) with the new critical ones. The process ends with a reaching the critical values of the combin loading.

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