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THERMAL STRESSES IN A SQUARE REGION WITH AN ELLIPTIC HOLE UNDER A HEAT GENERATION

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SUMMARY

The problem of determining the thermal stresses in a square region with an elliptic hole under heat generation occurs in nuclear engineering. For such problems, the use of the finite-difference or finite-element technique gives rise to a large number of linear simultaneous equations, and then the solution is very time-consuming.

In this paper, therefore, the analysis is developed by means of the point-matching technique in elliptic coordinates, as an extension of the five elementary function's method in plane thermoelasticity of multiply-connected region. Furthermore, we have tried to solve the same kind of problem by use of the complex variable method.

Numerical examples carried out are solutions of the problems of steady state thermal stresses in a square cylinder with an elliptic hole under constant heat generation.

1. INTRODUCTION

In nuclear reactor structures, the components are well insulated, but the heat is generated in the walls from irradiations-chiefly from gamma rays and neutrons, and the distribution of the temperature, which is steady-state except for starting or stopping, in the structure of reactor causes thermal stresses.

The thermal stress distributions in finite regions under the internal heat generation and perforated with a single circular or polygonal hole or an array of circular holes have been studied quite extensively from the theoretical viewpoints by authors[1],[2],[3],[4],[5]. Polygonal regions with an elliptic hole have also been used in the construction of nuclear reactor, but surprisingly few investigators have examined the thermal stress distributions around an elliptic hole in polygonal regions, even though analytical solutions for elliptic regions with a confocal elliptic hole and without any heat generation were obtained by Sekiya[6] by using the conformal mapping. So it is the object of this article to study the theoretical thermal stress distributions in polygonal regions with an elliptic hole under the internal heat generation.

For such problems, the application of the elliptic coordinates has not been published so far, and the use of the finite-difference or finite-element technique leads to large matrix systems if the region has even a moderately complicated shape and the solution of these systems can be very laborious and much time-consuming.

In this paper, therefore, the solutions for the temperature and thermal stress functions are obtained in the forms of the infinite series expressed by the elliptic coordinates and the unknown constants involved in these functions are determined so as to satisfy the boundary conditions using the point-matching technique, as an extension of the five elementary function's method in plane thermoelasticity of multiply-connected regions[1],[8].

Numerical examples are carried out for the problems of the steady thermal stresses in square regions with an elliptic hole under a constant heat generation and the thermal stress distributions around elliptic holes are examined.

2. ANALYSIS

Let us cosider a square region with a single elliptic hole under the steady state of temperature distribution with a constant internal heat generation as shown in Fig.1 and assume that the region is free from external forces and thermally insulated at the outer boundary with an internal convective boundary.

If the region is made from an isotropic linear elastic material, then its behavior under the influence of in-plane nonuniform temperature distribution which produces infinitesimal displacements is governed by the equation[7]

$$\Delta\Delta\chi = -\kappa\Delta\tau \qquad (1)$$

where

$$\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}$$

χ=Airy stress function

τ=temperature change from the reference state

$\kappa=\varepsilon E$ for plane stress problem, $\kappa=\varepsilon E/(1-\mu)$ for plane strain problem

E=Young's modulus

μ=Poisson's ratio

ε=coefficient of linear thermal expansion

The mean stresses in two dimensions may be expressed in terms of χ by the equation

$$\sigma_{ij}=(\delta_{ij}\Delta-\partial_i\partial_j)\chi\ , \qquad (i,j=1,2) \tag{2}$$

where

δ_{ij}=Kronecker delta, ∂_i=partial differentiation with respect to i

In discussing thermal stresses around an elliptic hole, it is advantageous to use the elliptic coordinates. The elliptic coordinates (α,β) are defined by

$$z=c\ \sinh\ \zeta, \quad z=x_1+ix_2, \quad \zeta=\alpha+i\beta \tag{3}$$

which gives

$$x_1=c\ \sinh\alpha\ \cos\beta, \quad x_2=c\ \cosh\alpha\ \sin\beta \tag{4}$$

or

$$\alpha=\ln(x_1+\sqrt{x_1^2+c^2\cos^2\beta}/c\ \cos\beta)$$
$$\beta=\tan^{-1}\sqrt{\frac{(x_1^2+x_2^2+c^2)-\sqrt{(x_1^2+x_2^2-c^2)^2+4c^2x_1^2}}{-(x_1^2+x_2^2-c^2)-\sqrt{(x_1^2+x_2^2-c^2)^2+4c^2x_1^2}}} \tag{5}$$

The coordinate α is constant and equal to α_1 on the ellipse. If the semiaxes are given by a and b as shown in Fig.1, c and α_1 can be found from

$$a=c\ \sinh\alpha_1, \quad b=c\ \cosh\alpha_1, \quad c=\ b^2-a^2 \tag{6}$$

The stresses $\sigma_{\alpha\alpha}$, $\sigma_{\beta\beta}$ and $\sigma_{\alpha\beta}$ associated with the elliptic coordinates are then transformed from Cartesian coordinates into elliptic coordinates, and one obtains

$$\sigma_{\alpha\alpha}=h^2\frac{\partial^2\chi}{\partial\beta^2}-h\frac{\partial h}{\partial\alpha}\frac{\partial\chi}{\partial\alpha}+h\frac{\partial h}{\partial\beta}\frac{\partial\chi}{\partial\beta}$$
$$\sigma_{\beta\beta}=h^2\frac{\partial^2\chi}{\partial\alpha^2}+h\frac{\partial h}{\partial\alpha}\frac{\partial\chi}{\partial\alpha}-h\frac{\partial h}{\partial\beta}\frac{\partial\chi}{\partial\beta} \tag{7}$$
$$\sigma_{\alpha\beta}=-h^2\frac{\partial^2\chi}{\partial\alpha\partial\beta}-h\frac{\partial h}{\partial\beta}\frac{\partial\chi}{\partial\alpha}-h\frac{\partial h}{\partial\alpha}\frac{\partial\chi}{\partial\beta}$$

where

$$h^{-2}=c^2(\cosh2\alpha+\cos2\beta)/2 \tag{8}$$

The thermal stress function χ for the doubly-connected region with zero surface traction on the boundaries may be expresented in terms of the system of elementary functions χ_τ, χ_{11}, χ_{21} and χ_{31} as follows[1],[8]:

$$\chi=\chi_\tau+\sum_{h=1}^{3}C_{h1}\chi_{h1} \tag{9}$$

where C_{h1} are constants which may be determined from the conditions that the displacements and rotation should be single-valued. Now the above elementary stress functions should satisfy the following equations in the elliptic coordinates:

[I] Fundamental differential equations in the region

$$h^2\Delta^* h^2\Delta^* \chi_\tau = -\kappa h^2\Delta^* \tau \qquad (10)$$

$$h^2\Delta^* h^2\Delta^* \chi_{h1} = 0 \qquad (11)$$

where

$$\Delta^* = \frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\beta^2}$$

[II] Boundary conditions at any point on the boundary (m=1,0)

$$(\chi_\tau)_{P_m} = (\frac{\partial}{\partial n}\chi_\tau)_{P_m} \qquad (12)$$

$$(\chi_{11},\ \chi_{21},\ \chi_{31})_{P_m} = (c\ \sinh\alpha\ \cos\beta,\ c\ \cosh\alpha\ \sin\beta,\ 1)\delta_{1m}$$

$$(\frac{\partial}{\partial n}\chi_{11},\ \frac{\partial}{\partial n}\chi_{21},\ \frac{\partial}{\partial n}\chi_{31})_{P_m} = (c\ \cosh\alpha\ \cos\beta,\ c\ \sinh\alpha\ \sin\beta,\ 0)\ _{1m} \qquad (13)$$

where m=1 and m=0 correspond to the elliptic and square boundary, respectively, and n denotes the outward normal to the boundary.

[III] Michell's conditions: The Michell's conditions in curvilinear coordinates, which have not been published so far, are derived as follows:

$$\oint_{\alpha=\alpha_1} \frac{\partial}{\partial\alpha}\{h^2\Delta^*(\chi_\tau + \sum_{h=1}^{3} C_{h1}\chi_{h1}) + \kappa\tau\}d\beta = 0$$

$$\oint_{\alpha=\alpha_1} (\cosh\alpha\ \sin\beta\frac{\partial}{\partial\alpha} - \sinh\alpha\ \cos\beta\frac{\partial}{\partial\beta})\{h^2\Delta^*(\chi_\tau + \sum_{h=1}^{3} C_{h1}\chi_{h1}) + \kappa\tau\}d\beta = 0 \qquad (14)$$

$$\oint_{\alpha\ \alpha_1} (\sinh\alpha\ \cos\beta\frac{\partial}{\partial\alpha} + \cosh\alpha\ \sin\beta\frac{\partial}{\partial\beta})\{h^2\Delta^*(\chi_\tau + \sum_{h=1}^{3} C_{h1}\chi_{h1}) + \kappa\tau\}d\beta = 0$$

Now the steady heat conduction equation with a constant heat generation is given as follows:

$$h^2\Delta^*\tau = -q_0/\lambda \qquad (15)$$

where

q_0=heat generation per unit volume per unit time

λ=thermal conductivity

Boundary conditions for the temperature are

$$\tau = 0 \text{ on the elliptic hole}$$

$$\frac{\partial\tau}{\partial n} = 0 \text{ on the square boundary} \qquad (16)$$

A classical method of solution for the boundary value problem, involving eq.(15), is to reduce eq.(15) to Laplace's equation by the introduction of a new variable τ_p such that

$$\tau = \tau_p + \tau_c \qquad (17)$$

and so choose that

$$\Delta^*\tau_c = 0 \qquad (18)$$

The general plane harmonic temperature distribution in elliptic coordinates is expressed in the following series

$$\Phi = \bar{A}_0 + \bar{B}_0\alpha + \sum_{n=1}^{\infty}\{(\bar{A}_n e^{n\alpha} + \bar{B}_n e^{-n\alpha})\cos n\beta + (\bar{C}_n e^{n\alpha} + \bar{D}_n e^{-n\alpha})\sin n\beta\} \qquad (19)$$

Considering the symmetry of the region about x_1 and x_2-axes, the solution for τ_c is given by

$$\tau_c=\overline{A}_0+\overline{B}_0\alpha+\sum_{n=1}^{\infty}(\overline{A}_{2n}e^{2n}+\overline{B}_{2n}e^{-2n})\cos 2n\beta \tag{20}$$

where $\overline{A}_0$, $\overline{B}_0$, $\overline{A}_{2n}$ and $\overline{B}_{2n}$ are unknown constants. On the other hand, as a particular solution τ_p is given by

$$\tau_p=-q_0c^2(\cosh 2\alpha-\cos 2\beta)/8\lambda, \tag{21}$$

the temperature function τ may be expresses as follows:

$$\tau_p=-q_0c^2(\cosh 2\alpha-\cos 2\beta)/8\lambda+\overline{A}_0+\overline{B}_0\alpha+\sum_{n=1}^{\infty}(\overline{A}_{2n}e^{2n\alpha}+\overline{B}_{2n}e^{-2n\alpha})\cos 2n\beta \tag{22}$$

From the condition that the outer square boundary is thermally insulated, under the steady-state condition the amount of heat generation in the region must carry away by the inner elliptic boundary, and gives the condition

$$\lambda\oint\frac{\partial\tau}{\partial n}ds=(4-\pi ab)q_0 \tag{23}$$

Substituting eq.(22) into eq.(23), one obtains

$$\overline{B}_0=2q_0/\lambda\pi \tag{24}$$

Next we will consider the elementary stress functions. Substituting eq.(]5) into eq.(10), one obtains

$$h^2\Delta^*h^2\Delta^*\chi_\tau=\kappa q_0/\lambda \tag{25}$$

A particular solution of this equation is given by

$$\chi_{\tau p}=\kappa q_0c^4(\cosh 4\alpha+\cos 4\beta)/512\lambda \tag{26}$$

Defining a new stress function $\chi_{\tau c}$ by the relation

$$\chi_\tau=\chi_{\tau p}+\chi_{\tau c} \tag{27}$$

and substituting eq.(27) into eq.(25) gives finally the relation

$$\Delta^*h^2\Delta^*\chi_{\tau c}=0 \tag{28}$$

The general plane biharmonic equation in elliptic coordinates is expressed in the following series

$$\begin{aligned}\Psi=&A_0+B_0\alpha+C_0(\cosh 2\alpha-\cos 2\beta)+D_0\{\alpha(\cosh 2\alpha-\cos 2\beta)-\sinh 2\alpha\}+K_0\sin 2\beta\\ &+N_0\sinh 2\alpha+A_1\cosh\alpha\cos\beta+B_1\sinh\alpha\cos\beta+M_1\alpha\sinh\alpha\cos\beta+C_1(e^{3\alpha}\cos\beta-e^{\alpha}\cos 3\beta)\\ &+D_1(e^{-3\alpha}\cos\beta-e^{-\alpha}\cos 3\beta)+E_1\sinh\alpha\sin\beta+F_1\cosh\alpha\sin\beta+L_1\alpha\cosh\alpha\sin\beta\\ &+G_1(e^{3\alpha}\sin\beta-e^{\alpha}\sin 3\beta)+H_1(e^{-3\alpha}\sin\beta-e^{-\alpha}\sin 3\beta)\\ &+\sum_{n=2}^{\infty}\{A_ne^{n\alpha}\cos n\beta+B_ne^{-n\alpha}\cos n\beta+C_n(e^{(n+2)\alpha}\cos n\beta-e^{n\alpha}\cos(n+2)\beta)\\ &\quad+D_n(e^{-(n+2)\alpha}\cos n\beta-e^{-n\alpha}\cos(n+2)\beta)\}\\ &+\sum_{n=2}^{\infty}\{E_ne^{n\alpha}\sin n\beta+F_ne^{-n\alpha}\sin n\beta+G_n(e^{(n+2)\alpha}\sin n\beta-e^{n\alpha}\sin(n+2)\beta)\\ &\quad+H_n(e^{-(n+2)\alpha}\sin n\beta-e^{-n\alpha}\sin(n+2)\beta)\}\end{aligned} \tag{29}$$

Therefore, considering the symmetry of the region about both x_1 and x_2-axes, the general solution of $\chi_{\tau c}$ is given by

$$\chi_{\tau c}=A_{00}+B_{00}\alpha+C_{00}(\cosh 2\alpha-\cos 2\beta)+D_{00}(\alpha\cosh 2\alpha-\alpha\cos 2\beta-\sinh 2\alpha)$$
$$+\sum_{n=1}^{\infty}[A_{2n0}e^{2n\alpha}\cos 2n\beta+B_{2n0}e^{-2n\alpha}\cos 2n\beta$$
$$+C_{2n0}\{e^{(2n+2)\alpha}\cos 2n\beta-e^{2n\alpha}\cos(2n+2)\beta\}$$
$$+D_{2n0}\{e^{-(2n+2)\alpha}\cos 2n\beta-e^{-2n\alpha}\cos(2n+2)\beta\}] \tag{30}$$

Therefore χ_τ may be given as follows:

$$\chi_\tau=\kappa q_0 c^4(\cosh 4\alpha+\cos 4\beta)/512\lambda+A_{00}+B_{00}\alpha+C_{00}(\cosh 2\alpha-\cos 2\beta)$$
$$+D_{00}(\alpha\cosh 2\alpha-\alpha\cos 2\beta-\sinh 2\alpha)+\sum_{n=1}^{\infty}[A_{2n0}e^{2n\alpha}\cos 2n\beta+B_{2n0}e^{-2n\alpha}\cos 2n\beta$$
$$+C_{2n0}\{e^{(2n+2)\alpha}\cos 2n\beta-e^{2n\alpha}\cos(2n+2)\beta\}$$
$$+D_{2n0}\{e^{-(2n+2)\alpha}\cos 2n\beta-e^{-2n\alpha}\cos(2n+2)\beta\}] \tag{31}$$

Similarly as before, considering the symmetry of the region, the general solution for χ_{h1} formed by terms which satisfy the biharmonic equations in elliptic coordinates are respectively given as follows:

$$\chi_{11}=A_{01}+B_{01}\alpha+C_{01}(\cosh 2\alpha-\cos 2\beta)+D_{01}(\alpha\cosh 2\alpha-\alpha\cos 2\beta-\sinh 2\alpha)$$
$$+A_{11}\cosh\alpha\cos\alpha+B_{11}\alpha\sinh\alpha\cos\beta+C_{11}(e^{3\alpha}\cos\beta-e^{\alpha}\cos 3\beta)$$
$$+D_{11}(e^{-3\alpha}\cos\beta-e^{-\alpha}\cos 3\beta)+\sum_{n=1}^{\infty}[A_{n1}e^{n\alpha}\cos n\beta+B_{n1}e^{-n\alpha}\cos n\beta$$
$$+C_{n1}\{e^{(n+2)\alpha}\cos n\beta-e^{n\alpha}\cos(n+2)\beta\}+D_{n1}\{e^{-(n+2)\alpha}\cos n\beta-e^{-n\alpha}\cos(n+2)\beta\}] \tag{32}$$

$$\chi_{21}=E_{12}\sinh\alpha\sin\beta+F_{12}\alpha\cosh\alpha\sin\beta+G_{12}(e^{3\alpha}\sin\beta-e^{\alpha}\sin 3\beta)$$
$$+H_{12}(e^{-3\alpha}\sin\beta-e^{-\alpha}\sin 3\beta)+\sum_{n=1}^{\infty}[E_{n2}e^{n\alpha}\sin n\beta+F_{n2}e^{-n\alpha}\sin n\beta$$
$$+G_{n2}\{e^{(n+2)\alpha}\sin n\beta-e^{n\alpha}\sin(n+2)\beta\}+H_{n2}\{e^{-(n+2)\alpha}\sin n\beta-e^{-n\alpha}\sin(n+2)\beta\}] \tag{33}$$

$$\chi_{31}=A_{03}+B_{03}\alpha+C_{03}(\cosh 2\alpha-\cos 2\beta)+D_{03}(\alpha\cosh 2\alpha-\alpha\cos 2\beta-\sinh 2\alpha)$$
$$+\sum_{n=1}^{\infty}[A_{2n3}e^{2n\alpha}\cos 2n\beta+B_{2n3}e^{-2n\alpha}\cos 2n\beta$$
$$+C_{2n3}\{e^{(2n+2)\alpha}\cos 2n\beta-e^{2n\alpha}\cos(2n+2)\beta\}$$
$$+D_{2n3}\{e^{-(2n+2)\alpha}\cos 2n\beta-e^{-2n\alpha}\cos(2n+2)\beta\}] \tag{34}$$

Where $A_{00}, B_{00},\cdots; A_{01}, B_{01},\cdots; A_{02}, B_{02},\cdots; A_{03}, B_{03},\cdots;$ involved in eqs.(31), (32), (33), (34) are unknown constants to be determined from the boundary conditions.

Substituting τ, χ_τ and χ_{h1} given by eqs.(22), (31), (32), (33), (34) into eqs.(14), and taking into account of the next relations

$$\frac{1}{\cosh 2\alpha+\cos 2\beta}=\operatorname{cosech} 2\alpha\{1+2\sum_{k=1}^{\infty}(-1)^{-2k\alpha}\cos 2k\beta\}$$
$$\frac{1}{(\cosh 2\alpha+\cos 2\beta)^2}=\operatorname{cosech}^2 2\alpha\{\coth 2\alpha+2\sum_{k=1}^{\infty}(-1)^k(k+\coth 2\alpha)^{-2k\alpha}\cos 2k\beta\} \tag{35}$$

one obtains

$$C_{11}=0,\quad C_{21}=0,\quad C_{31}=-(8D_0+c^2\kappa\bar{B}_0)/8D_{03} \tag{36}$$

Hence, from eq.(9), the Airy stress function considered herein becomes

$$\chi=\chi_\tau+C_{31}\chi_{31} \tag{37}$$

It is obvious from eqs.(31), (34) and (36) that the constants $\overline{A}_0$, $\overline{A}_{2n}$ and $\overline{B}_{2n}$ in the temperature function (22) do not appear in the stress function (37). Therefore, concerning to the thermal stresses, the constants in the temperature function are of less important except for the constant $\overline{B}_0$ which can be determined from the equilibrium condition of the heat as eq.(24).

The unknown constants in the temperature function τ and the stress functions X_τ and X_{31} are now determined by satisfying the respective boundary conditions. As these functions have been derived in order to satisfy the requirements of symmetry of the region about both x_1 and x_2-axes, it is only necessary to consider henceforce the conditions of one quadrant of the region.

For this purpose, as these functions are expressed in the forms of infinite series, so the conditional equations to get the unknown constants become infinite. However, since the outer boundaries ($x_1=\pm1$, $x_2=\pm1$) are not coordinate lines for the solutions of the elliptic coorsinates, so exact solutions cannot be found. To obtain approximate solutions, we use the method of the point-matching to satisfy the boundary conditions. That is, if we replace $\sum^{\infty}$ in eq.(22), (31) and (34) by $\sum^{N}$ approximately, the temperature and the stress functions contain 2N+1 and 4(N+1) unknown constants, respectively. The values of the unknown constants remaining in the truncated series would then have to be evaluated by satisfying the boundary conditions of selected finite sets of boundary points around the perimeter of the elliptic hole and the outer edge of the quadrant.

3. Numerical Examples

In order to avoid a large computer program, it is decided to retain terms in the finite series expansion of the temperature function up to and including N=20 and of the stress functions up to and including N=15. Then, the values of 41 and 64 unknown constants in the temperature and the stress functions, respectively, are determined by means of the boundary points least squares method.

Representative analytical results for several shapes of elliptic hole are shown in Figs.2 to 8.

Given data are as follows:

(A) elliptic hole of a=0.495, b=0.505
(B) elliptic hole of a=0.500, b=0.550
(C) elliptic hole of a=0.500, b=0.600
(D) elliptic hole of a=0.500, b=0.650

In Fig.2 and 3, to estimate the accuracy of the present work, the results for the case of the elliptic hole of a=0.495, b=0.505, which may be considered to be a circle of radious a=0.5 approximately, are checked by the previous work for a circular hole [1] by changing β into equivalent angle θ in polar coordinates by the equation

$$\theta=\tan^{-1}\left(\frac{b}{a}\tan\beta\right) \qquad (38)$$

From the comparison between both results, it is seen that the present method satisfactory accuracy.

Fig.4 shows the variations of the stress components $\sigma_{\beta\beta}$ around the elliptic holes and Figs.5 to 8 represent the isothermals of the temperature distributions for several shapes of the elliptic holes.

As a whole, the reselts which are obtained in this paper are justified by the computation to the boundary conditions of zero traction on the both boundaries. We see that the values of σ_{11} and σ_{12} on the outer boundary $x_1=1$, and those of $\sigma_{\alpha\alpha}$ and $\sigma_{\alpha\beta}$ on the inner boundary are nearly equal to zero. Moreover, for the temperature function we examined the analytical results by the sheet analogue method of electrical resistance paper.

According to our calculations, in the point-matching technique, the accuracy does not depend on the increase in the number of points on the boundaries. That is, if we plot the results, we cannot distinguish the difference in the distribution diagrams by increasing the number of points, because all the results fall on the same single curve except for the domain around the corner of the polygon. However, the convergency of the elliptic coordinates is not so good as the convergency of the polar coordinates.

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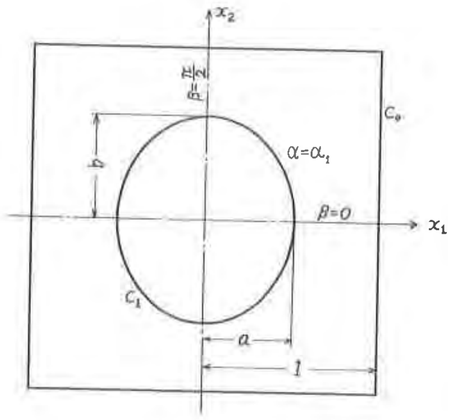

Fig.1 Notation for a single elliptic hole in a square region

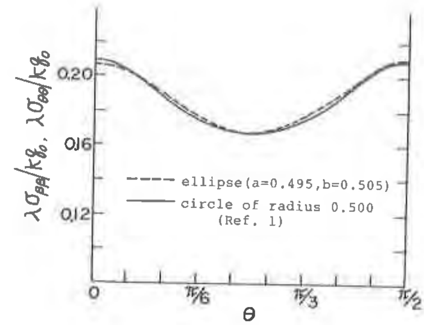

Fig.2 Tangential stresses around an elliptic hole and a circular hole

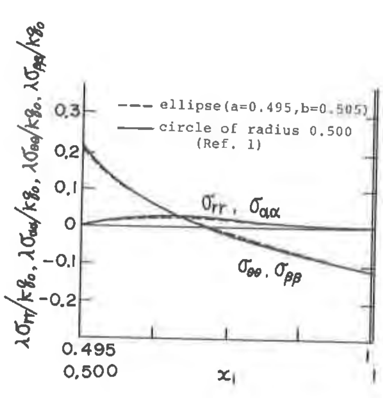

Fig.3 Principal stresses on the x_1-axis

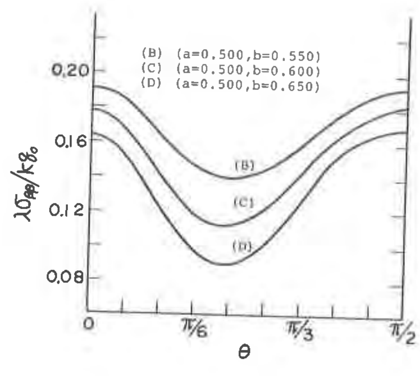

Fig.4 Variations of the tangential stresses around elliptic holes

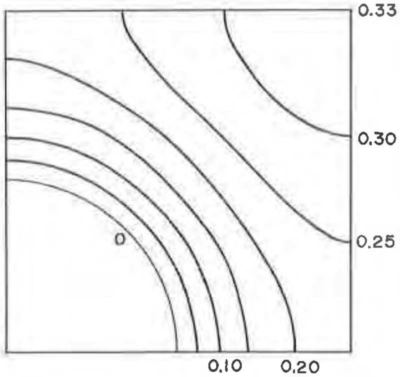

Fig.5 Isothemals for the elliptic hole (a=0.495, b=0.505)

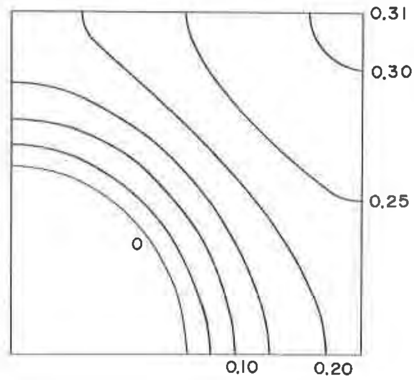

Fig.6 Isothermals for the elliptic hole (a=500, b=550)

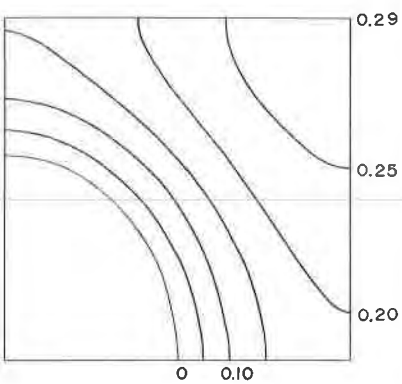

Fig.7 Isothermals for the elliptic hole (a=0.500, b=0.600)

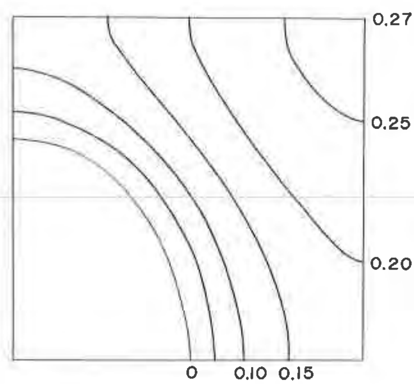

Fig.8 Isothermals for the elliptic hole (a=0.500, b=0.650)