Efficient Nonlinear Probabilistic Fracture Mechanics Analysis Based on Fast Monte-Carlo Algorithm

Gao-Wen YE
Century Research Center Corporation, Chiba, Japan
Genki YAGAYA, Shinobu YOSHIMURA, Min-Yan ZHANG
University of Tokyo, Tokyo, Japan

ABSTRACT

This paper is concerned with an improvement of the efficiency of the nonlinear Probabilistic Fracture Mechanics (PFM) analysis for the integrity study of structural components of nuclear power plants. The efficiency of the PFM analyses is improved by the following two methods: 1) the automatic subdivision of sampling space of initial cracks, and 2) the speedup of the Monte-Carlo algorithm. In this paper, the flow of the nonlinear PFM analyses is first described, and then the two methods are explained. Finally, the performance of the present methods is clearly demonstrated through a nonlinear PFM analysis of piping.

1 INTRODUCTION

The reliability analyses of nuclear structural components such as pressure vessels and piping, based on the probabilistic fracture mechanics (PFM), have become more and more important in the field of safety assessment and life extension of nuclear power plants (Harris *et al* 1981, Bloom 1984). Various PFM analyses have been performed so far, most of which are based on the linear elastic fracture mechanics (LEFM). However, as nuclear structural components are made of A533B class 1 or Type 304 stainless steels, their fracture process is essentially ductile, accompanied by stable crack growth under a large-scale yielding condition. For reliable and practical safety assessment , it is highly required to perform the PFM analyses based on the elastic-plastic fracture mechanics (EPFM).

Failure probabilities of pressure vessels or piping are usually calculated with the Monte-Carlo (MC) method which is very time-consuming. Although several speedup approaches such as Importance Sampling (Witt 1984) and Stratified Sampling (Harris *et al* 1981) techniques have been proposed and used so far, they are not always so satisfactory owing to their intrinsic features. In the Importance Sampling technique, it is difficult to find an optimum weight function. As for the Stratified Sampling technique, an optimum cell discretization and a proper sampling number that is chosen cell by cell are unknown *a priori*.

To solve the above problems, a novel PFM analysis code has been developed based on the EPFM database, which is used to evaluate nonlinear fracture mechanics parameters such as the J-integral and the crack opening displacement. In the present code, the efficiency of the analyses is improved by the following two methods: 1) the sampling space of initial cracks is automatically subdivided according to the probability density functions of initial crack sizes; 2) failure probabilities are calculated by a fast Monte-Carlo algorithm, named here MSMC-FSC algorithm (Modified Stratified sampling Monte-Carlo algorithm with Failure Samples Control). This algorithm is developed based on the two new speedup techniques suitable for the PFM analyses. The first technique, which is based on the SMC method, enables us to automatically cut off crack samples of smaller size, maintaining calculation

SMiRT 11 Transactions Vol. G (August 1991) Tokyo, Japan, © 1991

accuracy. The second technique is based on the relationship among the minimum number of failure samples, calculation accuracy as well as confidence coefficient. The number of crack samples of larger size is controlled by following this relationship. In the subsequent sections, the flow of the nonlinear PFM analysis is first described, and then the speedup methods are presented. Finally, the present PFM code is applied to the analyses of nuclear structural components.

2 FLOW OF NONLINEAR PFM ANALYSIS

A flow of the present nonlinear PFM analysis is given in Fig. 1.

At first, stratified sampling of initial crack geometry is performed after inputting the basic analysis data. All the cracks are assumed to be semi-elliptical surface cracks set in the circumferential direction of piping or pressure vessels. The configuration of each crack is defined by the two geometrical parameters: the nondimensional crack depth (a/t) and the crack aspect ratio (c/a). Both parameters are taken to be probabilistic variables. Then, the probability of non-detection of initial cracks during the pre-service inspection (PSI) is estimated. If a crack is not removed nor repaired, a simulation of fatigue crack growth is performed according to the crack growth criterion and the applied cyclic loads which occur during plant operation. At the same time, crack detection effects during the in-service inspection (ISI) is considered. For the purpose of simplicity, the applied stress considered in the simulation is assumed to be only cyclic axial stress. A crack extension amount is calculated both in the radial and the circumferential directions in case of surface cracks or only in the circumferential direction in case of througth-wall cracks. Finally the failure judgements of leak and unstable fracture are performed. After the simulation for all the cracks is completed, the failure probabilities of leak and unstable fracture are determined as the functions of plant operation time. The following criteria are used in the present analysis:

a) Crack growth rule:

$$da/dN = C(\triangle J)^m \tag{1}$$

b) Leak criterion:

$$L = l_0 \cdot c \cdot \delta \geq L_0 \tag{2}$$

c) Unstable fracture (tearing modulus criterion):

$$J \geq J_{Ic}, \qquad T \geq T_{mat} \tag{3a, b}$$

where C and m are the material constants (JWES 1984), l_0 a unit leakage per area. The J-integral (J) and the crack opening displacement (δ) are estimated according to the engineering approach using a 3-D EPFM database (Yagawa *et al* 1990):

$$J = J_e + J_p, \qquad \delta = \delta_e + \delta_p \tag{4a, b}$$

where J_e and δ_e are the elastic contributions, J_p and δ_p the plastic contributions.

3 AUTOMATIC SUBDIVISION OF SAMPLING SPACE

In the PFM analyses using the SMC method, the sampling space of initial cracks is divided into a number of small cells. It is easily imagined that computational accuracy as well as efficiency are strongly influenced due to the subdivision of sampling space and the number of samples taken from all cells. So far, optimum cell subdivision has been carried out by trial and error, consuming a lot of manpower and CPU time.

To solve this problem, we propose an automatic cell subdivison procedure. It is based on empirical knowledge on the PFM analyses.

For the purpose of simplicity, the basic principle of the present method is explained taking an one-dimensional example.

Let $f(\xi)$ be a density function of nondimensional crack depth a/t, which satisfies the following constraint:

$$\int_0^1 f(\xi)dx = 1 \tag{5}$$

Here we consider to divide the interval [0, 1] into n sub-intervals $[a_{i-1}, a_i]$ $(i = 1 \sim n)$, which satisfy the following relations:

$$\int_{a_0}^{a_1} f(\xi)d\xi = p_1 \tag{6a}$$

$$\int_{a_1}^{a_2} f(\xi)d\xi = p_2 = qp_1 \tag{6b}$$

$$\cdots$$

$$\int_{a_{i-1}}^{a_i} f(\xi)d\xi = p_i = qp_{i-1} \tag{6c}$$

$$\cdots$$

$$\int_{a_{n-2}}^{a_{n-1}} f(\xi)d\xi = p_{n-1} = qp_{n-2} \tag{6d}$$

$$\int_{a_{n-1}}^{a_n} f(\xi)d\xi = p_n = qp_{n-1} \tag{6e}$$

$p_i(i = 1 \sim n)$ is a geometric series, a quotient of two consecutive terms of which is q.

In Eq. (6), it is apparent that $a_0 = 0$ and $a_n = 1$. So, the separation points $a_k(k = 1 \sim n-1)$ can be determined by solving Eq. (6) subsequently from Eq.(6a) through Eq.(6d). In the above method of subdivision, the number of samples taken from each sub-interval, Ni, is chosen to be about $50/q \sim 100/q$ to avoid abrupt change of probability curves. According to experience of practical PFM analyses, it is recommended to take $p_1 = 0.8 \sim 0.9$ and $q = 0.01 \sim 0.1$.

For example, in the case of $f(\xi) = 32e^{-32\xi}$, the following separation points can be obtained easily under the condition that $p_1 = 0.9$, $q = 0.1$, $n = 13$:

$a_0 = 0$, $a_1 = 0.072$, $a_2 = 0.144$, $\cdots$, $a_{12} = 0.863$, $a_{13} = 1$.

In addition the number of samples in each sub-interval may be chosen to be abont 500.

4 MSMC-FSC ALGORITHM

In the PFM analyses based on the SMC method, the failure probability up to time t, $P_f(t)$, is calculated as follows:

$$P_f(t) = \sum_{i=1}^{m} \frac{N_i(t)}{N_i} P_i \tag{7}$$

where m denotes the total number of cells, N_i the total number of samples taken from the i-th cell, $N_i(t)$ the number of failure samples in the i-th cell up to time t, and P_i the probability with which a crack sample is taken from the i-th cell, respectively. An example of subdivided space is shown in Fig. 2. Herein, what is important in the sampling process of cracks is the treatment of uncertain area of whether to be failure or not, which is schematically bounded by two broken lines in the figure. For efficient and accurate PFM analyses, numerous samples have to be taken from this uncertain area. However, since such uncertain area is not known *a priori*, many cells and samples are required over a whole sampling space to assure computational accuracy. The method described in the previous section is capable of automatic cell subdivision of sampling space. In addition, appropriate control of numbers of samples is indispensable. For this purpose, the MSMC-FSC algorithm is proposed in the present paper. This algorithm is based on the following two approaches.

(a) MODIFIED SMC (MSMC) APPROACH

A semi-elliptical surface crack of initial size (a_{max}, c_{max}), designated S_M here, grows under various loading histories such as cyclic loads or earthquake loads. If this crack does not reach any failure, i.e. neither leak nor break within an operation life, a crack of initial size (a_{k0}, c_{k0}), designated S_0, which satisfies the conditions of both $a_{k0} \leq a_{max}$ and $c_{k0} \leq c_{max}$, namely $S_0 \leq S_M$, should not reach any failure within the same operation life. Of course, it is assumed here that all cracks are subjected to the same loading histories. The efficiency of the SMC method is improved by cutting off many calculations of smaller cracks which satisfy $S_0 \leq S_M$.

To apply this approach to the PFM analyses, the following issues are also taken into account:

1) Calculation is performed from cells of larger cracks to those of smaller cracks.

2) The maximum crack among the cracks which do not reach any failure is prepared for each cell, and continuously renewed during the calculation.

This process is schematically shown in Fig. 3. In this example, the MSMC analysis is performed as follows: Suppose that a crack of initial size S_I is the maximum crack among the cracks which do not reach any failure, i.e. $S_I = S_M$. The calculation of a crack of initial size S_R, such that $S_R < S_M$, can be skipped. The calculation of a crack of S_J, such that $S_J \not< S_M$, has to be performed. However if the crack of S_J dose not reach any failure, the calculation of a crack of S_Q such that $S_Q < S_J$ can be skipped in spite of $S_Q \not< S_M$, *i.e.* $a_{Q0} < a_M$, $c_{Q0} > c_M$.

Thus, an array of the maximum cracks among the cracks which do not reach any failure for each cell gives us the boundary between failure and no failure areas as schematically shown in Fig. 3 by broken line.

(b) Failure Samples Control (FSC) Approach

The MSMC approach is used to skip calculations of smaller crack samples. To obtain high efficiency, it is also necessary to control the number of failure samples of larger cracks. We propose a new control technique based on the following relationship among the computational accuracy, the number of failure samples and the confidence coefficient for the MC method:

$$M \leq \frac{C_\alpha^2 (1-P)(1+E)}{E^2} \tag{8}$$

where M is the number of failure samples among N samples, P the true probability of the event to be simulated, E the relative error of P, C_α a coefficient related to the confidence limit of estimation, respectively. The right-hand side of Eq. (8) gives us the upper limit of the number of failure samples M for the prescribed relative error E and confidence coefficient C_α. Table 1 gives the upper limits of M for various E and C_α. It should be noted here that, relative error of the order of $50\%(E = 0.5)$ to $100\%(E = 1)$ is not so significant since the probability to be calculated in the PFM analyses is extremely small. Computational efficiency can be improved by controlling the number of M with Eq. (8).

This algorithm combining the FSC approach with the MSMC method is called the MSMC with Failure Samples Control (MSMC-FSC) algorithm (Ye *et al* 1991).

5 RESULTS AND DISCUSSIONS

In order to investigate computational accuracy and efficiency of the aforementioned methods, a PFM analysis is carried out on a nuclear piping model under 200 cyclic loads with $\sigma_{max} = 50MPa$. The computer used is an EWS, NEWS/NWS-3460.

Fig. 4 shows the subdivided sampling space of initial cracks (52 cells) by the automatic subdivision procedure described in section 3. The failure probabilities obtained by this subdivision are almost the same as those by very fine subdivision (120 cells) obtained by trial and error. The failure curves of leak and break obtained by the present method are much smoother than those by the conventional method. Fig. 5 shows the time histories of failure probabilities within 40 year operation. Fig.6 shows the boundary line between failure area and no failure area in the sampling space obtained by MSMC-FSC method.

Table 2 gives CPU time and failure probabilities after 40 year operation obtained by the SMC, the MSMC and the MSMC-FSC methods. In the table, the number (m) denotes the control number of failure samples mentioned previously. The table clearly shows that both SMC and MSMC methods give us the same leak and break probabilities, and the MSMC-FSC algorithm gives almost the same leak and break probabilities as the SMC method. As for CPU time, the MSMC method takes only 2.48 minutes, and the MSMC-FSC methods with $m = 18$ and $m = 54$ take only 0.78 minutes and 1.72 minutes, respectively, while the CPU time of the conventional SMC method is 141.82 minutes. It is clearly demonstrated that the MSMC-FSC algorithm realizes excellent speedup of 80 to 180 times compared with the SMC method.

6 CONCLUSIONS

This paper proposed two methods to improve the effciency of nonlinear PFM analysis. The automatic subdivision method can save a lot of manpower and CPU time to find appropriate cell subdivision. The MSMC-FSC algorithm can increase computation speed of the SMC method up to 80 $\sim$ 180 times.

REFERENCES

Harris, D.O., Lim, E.Y. and Dedhia, D.D. (1981). Probability of Pipe Fracture in the Primary Coolant Loop of a PWR Plant, Vol.5: Probabilistic Fracture Mechanics Analysis, NUREG/CR-2188.

Bloom, J.M. (1984). Probabilistic Fracture Mechanics — a State-of-the-Art Review, ASME PVP-92, pp. 1-19.

Witt, F.J. (1984). Development and Applications of Probabilistic Fracture Mechnics for Critical Nuclear Reactor Components, ASME PVP-92, pp. 55-69.

Creep-Fatigue Crack Propagation Behaviors of Structural Materials for LMFBR, Report of FCC Subcommittee, JWES-AE-8403, Japan Welding Engineering Society, 1984.

Yagawa, G., Ye, G-W., Kitajima, Y. and Yoshimura, S. (1990) 3-D Fully Plastic Solutions and Related Probabilistic Fracture Mechanics Analysis, Proceedings of the Fifth International Conference on Numerical Methods in Fracture Mechanics, ed. by Luxmoore, A.R. and Owen, D.R.J., Freiburg, FRG., pp. 515-531.

Yagawa,G., Ye, G-W., Yoshimura, S. (1990). Probabilistic Fracture Analysis Based on Three Dimensional J-Integral Database, Proceedings of First KSME/JSME Joint Conference on Fracture and Strength, Seoul, Korea, pp. 696-701.

Ye, G-W., Yagawa, G. and Yoshimura, S. (1991). A Fast Monte-Carlo Algorithm and Its Application to Probabilistic Fracture Mechanics, Submitted to ICES'91, to Be Held in Melbourne, Australia, Aug. 11-16.

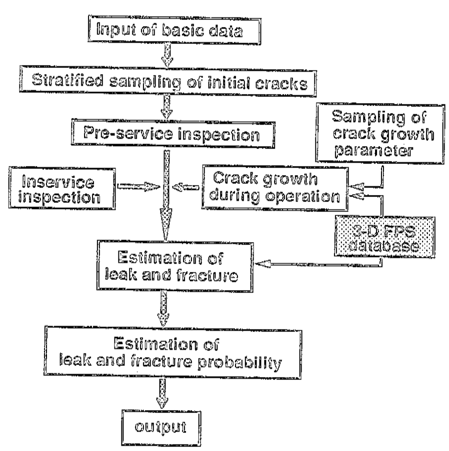

Fig. 1 Flow of PFM analysis based on 3-D EPFM database

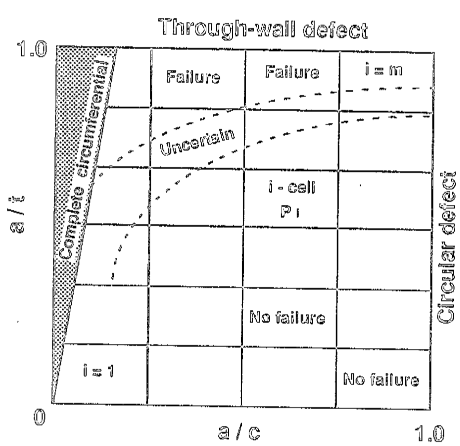

Fig. 2 Stratified sampling space of initial cracks

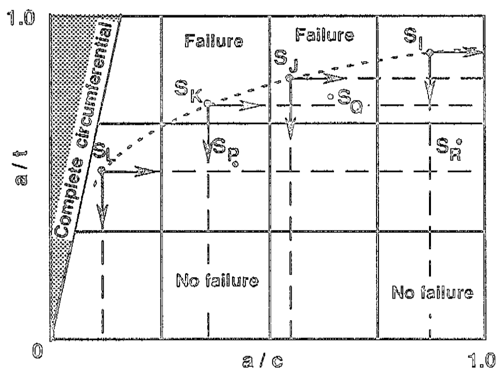

Fig. 3 Schematic illustration of MSMC approach

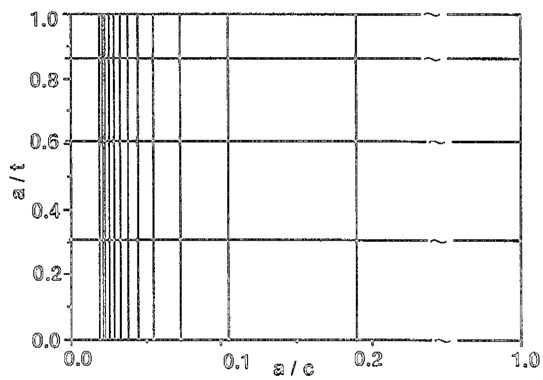

Fig. 4 Sampling space divided by automatic procedure

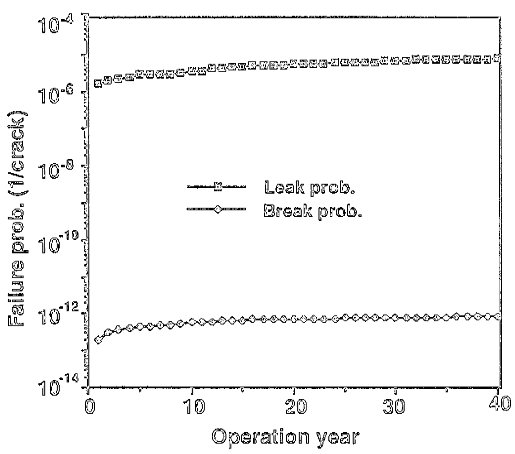

Fig. 5 Time histories of failure probabilites within 40 year operation

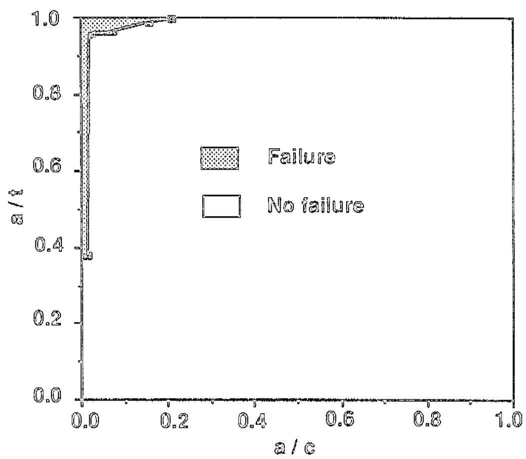

Fig. 6 Boundary line between failure area and no failure area obtained by MSMC-FSC method

Table 1 Relationship among E, C_α and m

E \ m ($C\alpha$)	1.96	2.58	3.00
0.05	38800	67230	90900
0.10	1613	2796	3780
0.50	23	40	54
1.00	8	13	18
5.00	1	2	2

Table 2 Comparison of probabilities and CPU time among SMC, MSMC and MSMC-FSC methods

	SMC	MSMC	MSMS-FSC	
			m = 18	m = 54
Leak prob.	7.82e-6	7.82e-6	7.65e-6	7.69e-6
Break prob.	8.57e-13	8.57e-13	6.81e-13	8.50e-13
CPU time (min)	141.82	2.48	0.78	1.72
Speedup	1	57.2	181.8	82.5