The Instability of an Asymmetric Through-Wall Circumferential Crack in a Pipe Subject to Bending Deformation

Edwin Smith
Joint Manchester University/UMIST, Manchester, UK

ABSTRACT

This paper examines the fracture instability of a pipe, fabricated from a ductile material such as 304 stainless steel, in which a circumferential cross-section contains a through-wall crack, the pipe being subject to bending deformation. The crack is situated asymmetrically with regards to the bending axis, and the theoretical analysis, which is based on the tearing modulus procedure, demonstrates the extent to which crack asymmetry influences the fracture instability criterion.

INTRODUCTION

Intergranular stress corrosion cracking (IGSCC) of Type 304 stainless steel Boiling Water Reactor piping is a technological problem that has posed many interesting questions relevant to the stability of crack growth in a ductile material. From a safety viewpoint, a central question is whether unexpected loads, arising from for example a very severe earthquake, will convert a part-through circumferential stress corrosion crack into a through-wall crack which might then extend unstably around the pipe circumference and develop into a large crack, through which there might be a major loss-of-coolant, or worse still, lead to complete pipe severance, i.e. a guillotine rupture. Against this background, Tada, Paris and Gamble (1980), using the tearing modulus procedure (Paris et al, 1979), examined the stability of circumferential growth of a through-wall crack in a pipe subjected to bending deformation, the crack being symmetrically oriented with respect to the bending axis. They concluded that, because of the high crack growth resistance of 304 stainless steel, circumferential growth should be stable for pipe run lengths in a typical reactor system, a result that demonstrates the essential stability of cracks in 304 stainless steel pipes. Subsequent analyses have considered various aspects of this general problem, a major objective being to see what factors might erode the large safety margins demonstrated via the Tada-Paris-Gamble analysis. Thus the author (Smith, 1987) has considered the stability of a system in which a circumferential cross-section contains two symmetrically situated through-wall cracks, and shown that the tendency towards instability can be appreciably greater than if the section contains a single symmetrically situated through-wall crack. This result suggests that a single crack, asymmetrically oriented with respect to the bending axis, might be more unstable than a single crack that is symmetrically oriented with respect to the bending axis. The objective of this paper's analysis is to confirm this viewpoint.

THEORETICAL ANALYSIS

The geometry of the cracked circumferential section of the pipe is shown in Figure 1. The section contains a through-wall crack whose size and position are as indicated, the pipe radius as measured to the middle of the wall being R, and the pipe thickness being t. The pipe is constrained to bend about a horizontal axis and consequently the crack is asymmetrically situated with respect to this axis. It is assumed that plastic deformation is confined to the cracked cross-section which is fully yielded, with the yielded material being perfectly plastic while the remainder of the pipe behaves elastically. The cracked cross-section is therefore subject to the plastic limit moment M_p, and the stress distribution across this section can be described by a tensile stress σ_o acting above the neutral axis, while a compressive stress of similar magnitude acts in the region below the neutral axis (σ_o is often referred to as the flow stress). With $t/R<<1$, the location (defined by the angle α) of this neutral axis about which plastic rotation occurs, is readily shown by balancing forces to be given by the expression

$$4\alpha = \beta - \theta \qquad (1)$$

provided there are no tensile forces acting perpendicular to the cracked cross-section. This expression is valid provided the neutral axis is below the crack tip, i.e. for values of β from θ to $(2\pi-\theta)/3$. Since this covers a wide range of cracking geometries, this condition is not unduly restrictive. Moment equilibrium provides the relation:

$$\frac{M_p}{4\sigma_o R^2 t} = \frac{1}{4} \sin\theta - \frac{1}{4}\sin\beta + \cos\alpha \qquad (2)$$

$$= \frac{1}{4}\sin\theta - \frac{1}{4}\sin\beta + \cos\left[\frac{\beta-\theta}{4}\right] \qquad (3)$$

using relation (1). Following similar arguments to those used by Tada, Paris and Gamble (1980), the magnitude of the J integral for the upper crack tip for a given plastic rotation Φ_{PL} about the neutral axis and a given crack area

$$A = Rt(\beta-\theta) \qquad (4)$$

is $$J = -\frac{\partial M_p}{\partial A}\Phi_{PL} = \sigma_o R\Phi_{PL}\left[\cos\theta + \sin\left(\frac{\beta-\theta}{4}\right)\right] \qquad (5)$$

using relation (3)

In examining the stability of the system as regards circumferential crack growth at the upper crack tip (growth will occur at this tip first since the crack tip displacement is greatest), consider like Tada, Paris and Gamble (1980) the case where a rotation Φ (about a horizontal axis) is imposed at the fixed ends of a pipe of length L, now regarded as a beam. The rotation Φ is separable into a plastic contribution Φ_{PL}, given by relation (5), and an elastic contribution Φ_{EL}, given by the relation

$$\Phi_{EL} = \frac{M_p L}{EI} \qquad (6)$$

where E is Young's modulus and $I = \pi R^3 t$ is the second moment of area of the pipe. Circumferential crack growth is assumed to be unstable if, with the rotation Φ fixed, the applied tearing modulus T_{APP}, defined by Paris and co-workers (1979) via the relation

$$T = \frac{E}{\sigma_o^2}\frac{dJ}{ds} \qquad (7)$$

where δs is an increment of crack length, exceeds some critical value T_{MAT}, the initial slope of the material's J_R curve, which is assumed to be a material property. T_{APP} is given by relations (3), (5), (6) and (7), and it therefore follows that the resulting condition for instability at the onset of crack extension ($J = J_{IC}$) is

$$T_{APP} = \frac{L}{R} F_1(\theta) + \frac{EJ_{IC}}{\sigma_o^2 R} F_2(\theta) > T_{MAT} \tag{8}$$

with $F_1(\theta)$ and $F_2(\theta)$ being given respectively by the expressions

$$F_1(\theta) = \frac{1}{\pi} \left[\cos\theta + \sin\left(\frac{\beta-\theta}{4}\right) \right]^2 \tag{9}$$

$$F_2(\theta) = \frac{\left[\sin\theta + \frac{1}{4}\cos\left(\frac{\beta-\theta}{4}\right) \right]}{\left[\cos\theta + \sin\left(\frac{\beta-\theta}{4}\right) \right]} \tag{10}$$

As with the Tada, Paris and Gamble (1980) analysis for a single through-wall crack symmetrically situated with respect to the bending axis, the second term in the expression for T_{APP} may be ignored when $T_{MAT} \gg EJ_{IC}/\sigma_o^2 R$, as is the case with a material, like 304 stainless steel, that has a high crack growth resistance. In this case the instability condition simplifies to

$$T_{APP} \doteq \frac{L}{\pi R} \left[\cos\theta + \sin\left(\frac{\beta-\theta}{4}\right) \right]^2 > T_{MAT} \tag{11}$$

The corresponding result for the extension (at the outer tips) of a single through-wall crack of included angle 2Ψ symmetrically situated with respect to the bending axis is (Paris et al, 1980)

$$T_{APP} = \frac{L}{R} \cdot \frac{2}{\pi} \left[\sin\frac{\Psi}{2} + \cos\Psi \right]^2 \tag{12}$$

This latter expression is a maximum when $\sin(\Psi/2) = (1/4)$, the maximum value being

$$T_{APP} = \frac{L}{R} \cdot \frac{81}{32\pi} \tag{13}$$

Accordingly the ratio of the applied tearing modulus for the asymmetric situation to the maximum applied tearing modulus for the symmetric situation is given by relations (11) and (13) as

$$\frac{T_{APP}\ (\mathrm{ASYMM})}{T_{APP}\ (\mathrm{SYMM,\ MAX})} = \frac{64}{81} \left[\cos\theta + \sin\left(\frac{\beta-\theta}{4}\right) \right]^2 \tag{14}$$

The value of this ratio, which is a measure of the effect of asymmetry on the instability criterion, is shown in Table I for different values of β and θ. The greater the value of this ratio, provided it exceeds unity, the greater is the tendency for instability of the asymmetrically cracked system in comparison with the symmetrically cracked system.

CONCLUSIONS

The preceding section's analysis (see results in Table I) has shown that if a circumferential cross-section of a pipe contains a crack that is asymmetrically situated with respect to the bending axis, the tendency for unstable fracture can be significantly greater than if a single through- wall crack is symmetrically situated with respect to the bending axis. Crack asymmetry is therefore a factor that should be taken into account in piping system integrity evaluations since, at least on the basis of the present paper's calculations, the pipe-run lengths required for unstable failure in the asymmetric case are lower than that for the symmetric case.

REFERENCES

H. Tada, P.C. Paris and R.M. Gamble, Proceedings of the Twelfth National Symposium on Fracture Mechanics, ASTM STP 700, July 1980, p. 295.

P.C. Paris, H. Tada, A. Zahoor and H. Ernst, Proceedings of Symposium on Elastic-Plastic Fracture, ASTM STP 688, March (1979), 5.

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TABLE I

The ratio of the two applied tearing moduli: $T_{APP}(ASYMM)/T_{APP}(SYMM,MAX)$

	$T_{APP}(ASYMM)/T_{APP}(SYMM,MAX)$				
β	θ				
	-60°	-30°	0°	30°	60°
60	-	1.24	1.26	0.78	-
90	0.98	1.48	1.52	1.00	0.32
120	1.16	1.72	1.78	-	-
150	1.32	-	-	-	-

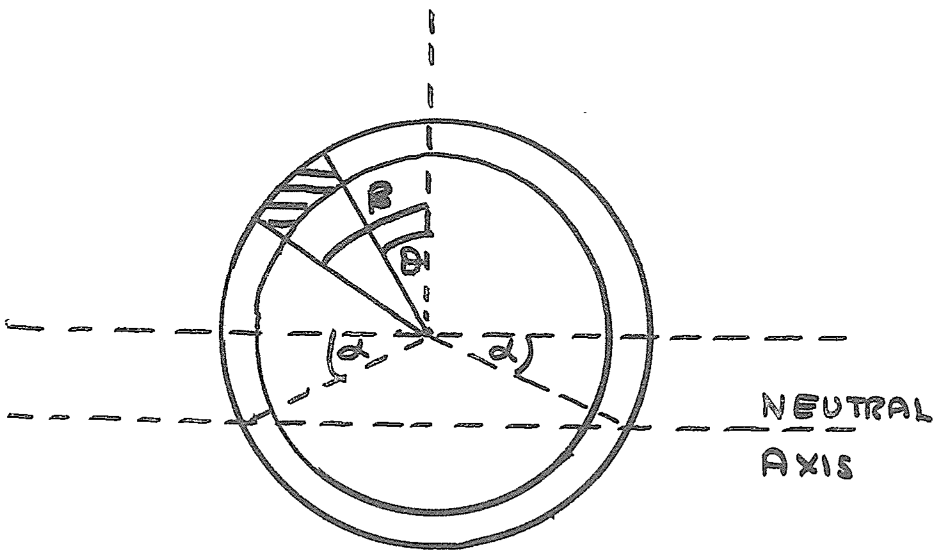

FIGURE 1 The geometry of the cracked section of the pipe. This section contains a through-wall crack, the pipe radius as measured to the middle of the wall is R, and the pipe thickness is t.