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Measurement of residual stress in plates by tongue-cutting strain-gage method

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ABSTRACT

A new technique has been developed to measure residual stress which is uniformly distributed over the thickness of the plate. The computational model for the tongue-cutting method is based on theoretical solution of elasticity for deformation of a rectangular plate which is subjected to an arbitrary surface traction at the ends of the plate. An experiment is carried out with butt-welded steel plates. The result obtained by the new method agrees well with the one obtained by the X-ray methods.

1. INTRODUCTION

The presence of residual stresses can influence the reaction of a material to externally applied stresses. Not all residual stresses are considered to be damaging. In general, the stability and resistance to deformation of materials and fatigue life are increased by the presence of compressive residual stress and are decreased by the presence of tensile residual stresses. The effect of residual stresses on fatigue or stress corrosion cracking performance is a well-recognized phenomenon. Reactor internal attachment welds affected by stress-corrosion cracking have relatively little applied load. Rather, the mechanical driving force for the crack propagation is primarily welding residual stress. Some materials used in reactor components are susceptible to stress-corrosion cracking. Therefore measurement of residual stresses is of great importance in many engineering application. The estimation of residual stress fields due to welding or other sources has received a great deal of attention in both experimental and computational studies.

The X-ray diffraction techniques can be used for determination of residual stresses. Recently, Bourke et al.[1] reported that the non-destructive measurement of residual stress has been accomplished by using the neutron diffraction technique. But, the measurements of the stresses by these diffraction techniques require usually very expensive equipment. The mechanical techniques for measuring stress variations of a pipe or plate require relatively inexpensive equipment. They usually involve release of residual stresses in the region of interest. After removal of material or separation of parts of the specimen the relationship between released stresses and measured strains is used to compute residual stresses. The repetitive process of removing or slicing material generally increases the chance of introducing new residual stresses and makes residual stress measurement expensive. Wolf and Bohm[2] presented a ring-core method of measuring residual stresses near the core. Schajer[3] has developed an exact hole-drilling method for determining non-uniform residual stress fields. Cheng and Finnie[4] have developed a crack compliance method for estimation of residual stresses near the surface of the plate. This method can be applied only to shallow cut with a depth of $0.05t$; t is thickness of the body measured. The crack compliance method only estimates the normal stress and shear stresses along the plane where the cut is made while hole drilling allows the estimate of complete biaxial surface stress state at the location of the hole. These techniques provide information about surface residual stresses. Its main limitations are that its accuracy tends to diminish with the depth of cut. With this motivation a new technique has been developed to measure residual stress which is uniformly distributed over the cross section of the plate. The method makes use of parallel thin cuts of progressively increasing depth to release normal stress along the plane of the cuts. The computational model for tongue-cutting method is based on theoretical solution of elasticity for deformation of a rectangular plate which is subjected to an arbitrary traction at the ends of the plate. The great advantage of determining the residual stresses by the tongue-cutting and strain measurements is that this can be done relative easily and with inexpensive equipment. A residual stress distribution due to welding is generally localized. From St. Venant's principle the effect of releasing the stress is expected to decay at a rapid rate. Thus, the gradient of the measured strains will tend to decrease as the width of the tongue increases. The limitations are that flat plate-type specimens are required and the residual stresses have to be uniformly distributed over the thickness of the plate.

2. ANALYSIS

We suppose that a butt-welded plate is wide enough and weld is made along the x-axis. We also suppose that the residual normal stresses σ_x , σ_y and shearing residual stress τ_{xy} are functions of x and y, and are uniform in the z-direction. Figure 1 shows a plate with a uniform thickness and a tongue of depth $2b$ and width $2a$ which can be provided in experiments by thin slits made by a saw. Let the center of the tongue be the origin O of a

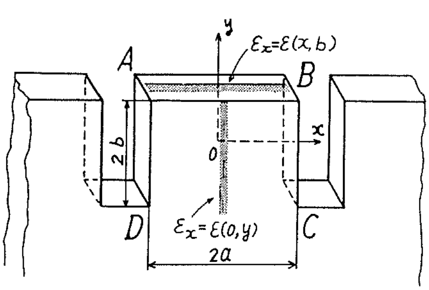

Fig.1 Coordinate system for measuring stress in a plate and shape of a tongue.

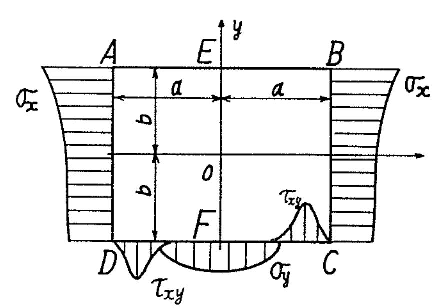

Fig.2. Rectangular plate loaded with distributed forces.

xy-coordinate. The significant stresses are those parallel to the weld direction, designated σ_x, and those transverse to it, designated σ_y. Stress distributions across two adjoining section, such as sections A-D and B-C, are essentially the same[5]. In other words, changes of stresses in the x-direction, except in areas near both ends of the rectangular plate, are far less than those in the y-direction. The stress distribution shown above is an ideal for one-dimensional stress analysis. When no external forces act on a plate, the residual stresses balance across a cross section. The balance is disturbed when a part is removed. The relation of the stress change measured at the upper side and the y-axis may be determined by the force existing in the parts before slitting to the vertical sides of the plate. The strains measured by the strain gages shown in Fig.1 can be used to estimate the residual stress σ_x in the x direction. On this basis an analysis is developed to obtain the deformation due to releasing residual stresses by introducing slits of progressively increasing depth with a final value of 2b. On removing the parts, the forces which have been removed from each square unit of the cross section are the original residual stress σ_x. Removing the force σ_x, which is by convention considered a tensile force, is equivalent to applying an equal compressive force $-\sigma_x$. Rectangular plate A, B, C and D loaded with arbitrary distributed forces, can be approximated in experiments, Fig.2. Suitable boundary conditions in this residual stress analysis are expressed as follows:

$$\left.\begin{array}{ll} -\sigma_x = a_0 + \sum_{n=1}^{\infty} (a_n \cos \alpha_n y + b_n \sin \alpha_n y) & \text{for } x = \pm a \\ \tau_{xy} = 0 & \text{for } x = \pm a \\ \sigma_y = \tau_{xy} = 0 & \text{for } y = b \\ \sigma_y \fallingdotseq 0 \text{ or } \tau_{xy} \fallingdotseq 0 & \text{for } y = -b \end{array}\right\} \quad (1)$$

where, $\alpha_n = n\pi/b$ (n=1,2,3,...) and a_0, a_n and b_n are coefficients of Fourier series. Exact solutions for stresses in rectangular plates under any boundary conditions expressible by Fourier series can be obtained by the Fourier method are solved by Picket[6]. The stress function for this two-dimensional problem may be taken as

$$\begin{aligned} \Phi = \frac{A_0}{2} y^2 &- \sum_{n=1}^{\infty} \frac{A_n}{\alpha_n^2} [\alpha_n x \sinh \alpha_n x - (1+\alpha_n a \coth \alpha_n a) \cosh \alpha_n x] \cos \alpha_n y \\ &- \sum_{n=1}^{\infty} \frac{B_n}{\alpha_n^2} [\alpha_n x \sinh \alpha_n x - (1+\alpha_n a \coth \alpha_n a) \cosh \alpha_n x] \sin \alpha_n y \\ - \sum_{m=1}^{\infty} \frac{1}{\beta_m^2} &[E_m \sinh \beta_m y + F_m \beta_m y \sinh \beta_m y \\ &+ G_m \sinh \beta_m y + H_m \beta_m y \cosh \beta_m y] \cos \beta_m x \quad (2) \end{aligned}$$

where $\alpha_n = n\pi/b$ and $\beta_m = m\pi/a$ and A_n, B_n, E_m, F_m, G_m and H_m are arbitrary constants. The constants of Eq. (2) are determined from the boundary conditions (1) as follows:

$$a_0 = A_0 - \sum_{m=1}^{\infty} (-1)^m \frac{\sinh \beta_m b}{\beta_m b} [E_m + F_m (1 + \beta_m b \coth \beta_m b)] \quad (3)$$

$$\begin{aligned} a_n = -A_n \left(\frac{\alpha_n a}{\sinh \alpha_n a} - \cosh \alpha_n a \right) - \sum_{m=1}^{\infty} \frac{2(-1)^{m+n} \beta_m}{(\alpha_n + \beta_m)^2 b} \{ E_m \sinh \beta_m b \\ + F_m [(2 + \frac{\alpha_n^2 - \beta_m^2}{\alpha_n^2 + \beta_m^2}) \sinh \beta_m b + \beta_m b \cosh \beta_m b] \} \quad (4) \end{aligned}$$

$$b_n = -B_n\left(\frac{\alpha_n a}{\sinh \alpha_n a} - \cosh \alpha_n a\right) - \sum_{m=1}^{\infty} \frac{2(-1)^{m+n} \alpha_n}{(\alpha_n + \beta_m)^2 b}\{G_m \sinh \beta_m b$$

$$+H_m\left(\frac{2\alpha_n^2}{\alpha_n^2+\beta_m^2}\sinh \beta_m b + \beta_m b \cosh \beta_m b)\right] \qquad (5)$$

$$\sum_{m=1}^{\infty} \frac{4A_n(-1)^{m+n}\alpha_n \beta_m^2}{(\alpha_n^2+\beta_m^2)^2 a}\sinh \alpha_n a = E_m \cosh \beta_m b + F_m \beta_m b \sinh \beta_m b$$

$$+G_m \sinh \beta_m b + H_m \beta_m b \cosh \beta_m b \qquad (6)$$

$$\sum_{m=1}^{\infty} \frac{4B_n(-1)^{m+n}\alpha_n^2 \beta_m}{(\alpha_n^2+\beta_m^2)^2 a}\sinh \alpha_n a = E_m \sinh \beta_m b + G_m \cosh \beta_m b$$

$$+F_m(\sinh \beta_m b + \beta_m b \cosh \beta_m b) + H_m(\cosh \beta_m b + \beta_m b \sinh \beta_m b) \qquad (7)$$

By the assumption that the lower edge($y=-b$) of the plate(Fig.2) is free from external forces, the following relation will hold for the stress at the edge ;

$$\sigma_y = 0 \text{ and } \tau_{xy} = 0 \qquad \text{for } y=-b$$

The following equations are obtained by the above conditions.

$$G_m \sinh \beta_m b + H_m \beta_m b \cosh \beta_m b = 0 \qquad (8)$$

$$E_m \sinh \beta_m b + F_m(2\sinh \beta_m b + \beta_m b \cosh \beta_m b) = 0 \qquad (9)$$

Assuming that strains are being measured in the x direction at the upper side and the y-axis and that these readings change by $\varepsilon_x(x, b)$ and $\varepsilon_x(0, y)$ respectively. From the boundary condition that the upper edge($y=b$) of the plate is free from external forces, the following relation hold for the stress and the strain at the edge ;

$$\sigma_x = E\varepsilon_x(x, b) \qquad \text{for } y=b$$

where, E is Young's modulus.

Strain $\varepsilon_x(x, b)$ and $\varepsilon_x(0, y)$ may be expressed by Fourier series and may be measured by strain gages. At the edge($y=b$) of the plate, the stress component in terms of the stress function(2) are

$$[\sigma_x]_{y=b} = A_0 + \sum_{n=1}^{\infty} A_n(-1)^n[\alpha_n x \sinh \alpha_n x - (1+\alpha_n a \coth \alpha_n a)\cosh \alpha_n x]$$

$$-\sum_{m=1}^{\infty}[E_m \sinh \beta_m y + F_m \beta_m y \sinh \beta_m y + G_m \sinh \beta_m y + H_m \beta_m y \cosh \beta_m y]\cos \beta_m x$$

$$= E\varepsilon_x(x, b) = s_0 - \sum_{m=1}^{\infty} s_m \cos \beta_m x \qquad (10)$$

The coefficients of the above Fourier series can be obtained in the usual manner.

$$A_0 - \sum_{n=1}^{\infty} 2A_n \frac{(-1)^n}{\alpha_n a}\sinh \alpha_n a = s_0 \qquad (11)$$

$$\sum_{m=1}^{\infty} \frac{4A_n(-1)^{m+n}\alpha_n^3}{(\alpha_n^2+\beta_m^2)^2 a}\sinh \alpha_n a + [E_m \cosh \beta_m b + F_m(2\cosh \beta_m b + \beta_m b \sinh \beta_m b)$$

$$+G_m \sinh \beta_m b + H_m(2\sinh \beta_m b + \beta_m b \cosh \beta_m b)] = -s_m \qquad (12)$$

Under condition of plane stress and at the center line EOF ($x=0$) of the plate (Fig.2), the following relation hold for the stress and the strain at the line ;

$$\sigma_x + \nu \sigma_y = E \varepsilon_x (0, y) \qquad \text{for } x=0$$

where, ν is Poisson's ratio.

At the center line($x=0$), the stress component in terms of the stress function(2) are

$$[\sigma_x + \nu \sigma_y]_{x=0} = A_0 - \sum_{n=1}^{\infty} A_n [(1-\nu)+(1+\nu)\alpha_n a \coth \alpha_n a] \cos \alpha_n y$$

$$- \sum_{n=1}^{\infty} B_n [(1-\nu)+(1+\nu)\alpha_n a \coth \alpha_n a] \sin \alpha_n y$$

$$- \sum_{m=1}^{\infty} (-1)^m \{E_m (1+\nu) \cosh \beta_m y + F_m [2\cosh \beta_m y + (1-\nu)\beta_m y \sinh \beta_m y]$$

$$+ G_m (1+\nu) \sinh \beta_m y + H_m [2\sinh \beta_m y + (1+\nu)\beta_m y \cosh \beta_m y]\}$$

$$= E \varepsilon_x (0, y) = p_0 - \sum_{m=1}^{\infty} [p_n \cos \alpha_n y + q_n \sin \alpha_n y] \tag{13}$$

The coefficients of the above Fourier series can be obtained in the usual manner.

$$A_0 - (1+\nu) \sum_{m=1}^{\infty} (-1)^m \frac{\sinh \beta_m b}{\beta_m b} [E_m + F_m (\frac{1-\nu}{1+\nu} + \beta_m b \coth \beta_m b)] = p_0 \tag{14}$$

$$A_n [(1-\nu)+(1+\nu)\alpha_n a \coth \alpha_n a]$$

$$+2(1+\nu) \sum_{m=1}^{\infty} \frac{(-1)^{m+n} \beta_m}{(\alpha_n^2 + \beta_m^2) b} \sinh \beta_m b \{E_m + F_m [\beta_m b \coth \beta_m b$$

$$+ (\frac{2}{1+\nu} + \frac{\alpha_n^2 - \beta_m^2}{\alpha_n^2 + \beta_m^2})]\} = -p_n \tag{15}$$

$$B_n [\frac{1-\nu}{1+\nu} + \alpha_n a \coth \alpha_n a] - \sum_{m=1}^{\infty} \frac{2(-1)^{m+n} \alpha_n}{(\alpha_n^2 + \beta_m^2) b} \sinh \beta_m b \{ G_m$$

$$+ H_m [\frac{\alpha_n^2 - \nu \beta_m^2}{\alpha_n^2 + \beta_m^2} \cdot \frac{2}{1+\nu} + \beta_m b \coth \beta_m b]\} = -q_n \tag{16}$$

Substituting the coefficients of the Fourier series expressed by Eqs. (10) and (13) into the simultaneous Eqs. (6), (7), (8), (9) and (15) or (16), arbitrary constants A_0, A_n, B_n, E_m, F_m, G_m and H_m are determined by measured strain. The distribution of applied forces is then expressed by Eqs.(3), (4) and (5). The residual stress distribution σ_x is obtained as follows:

$$\sigma_x = -[a_0 + \sum_{n=1}^{\infty} (a_n \cos \alpha_n y + b_n \sin \alpha_n y)]$$

Experiment and Results

An experiment was carried out with butt-welded plates for determining residual stresses distributed in the direction of parallel to the weld joint. The material used was a low carbon steel JIS SS400 supplied as a 6mm thick. The mechanical properties are given in Table 1. Hot-rolled plates with a width 65mm or 100mm were machined to a thickness 5.5mm and a length 600mm. After annealing at 973K, they were then butt-welded to two types specimens with width 130mm and 200mm under the welding condition of table 2.Two specimens (designated A and B), were used. The shapes of the specimens are shown in Fig.3. Residual stress was measured by a parallel beam X-ray diffractometer with a scintillation counter.

Table 1. Material properties of SS400 steel at room temperature

Thickness X Width(mm)	6 X 65	6 X 100
Young's modulus, E(GPa)	215	210
Poisson's ratio, ν	0.3	0.28
Yield point(MPa)	333	294
Tensile strength(MPa)	441	402
Elongation at failure(%)	39	45

Table 2. Welding conditions

Welding current	135A
Welding voltage	20V
Welding speed	8mm/sec
Sealing gas	CO_2
Electrode wire	1.2mm

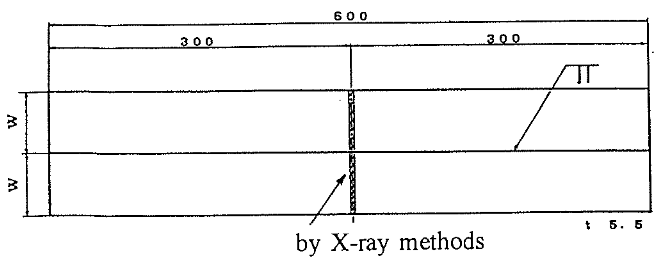

Fig.3. Shape of Specimen and Locations measured by the X-rays. w=65 (specimen A). w=100(specimen B).

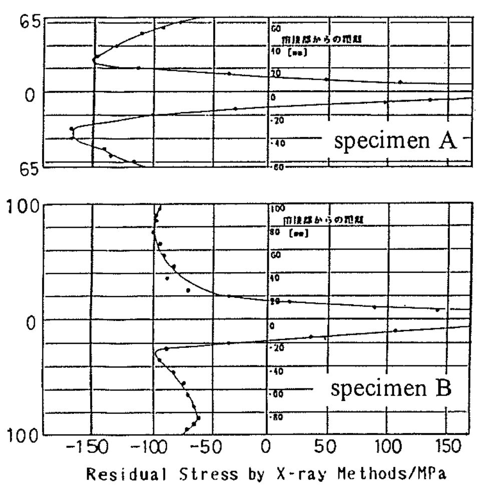

Fig.4. Residual stress distributions obtained by the X-ray methods.

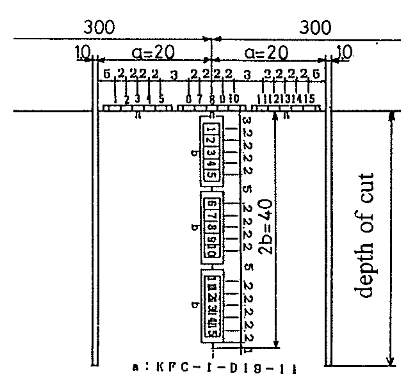

Fig.5. Locations of multiple strain gages.

The X-ray techniques were used to verify the calculated results for the same test pieces. The residual stress distributions by the X-ray methods, at the center of the specimens, are shown in Fig.4. Multiple strain-gages are placed on the upper edge and the center of the plate, as shown in Fig.5. Each specimen was notched perpendicular to the plate surface to a depth of 60mm. Figure 6 shows distributions of measured strains $\varepsilon_x(x, b)$ for eight values of depths of cut. Distributions of measured strains $\varepsilon_x(0, y)$ for the same eight depths are

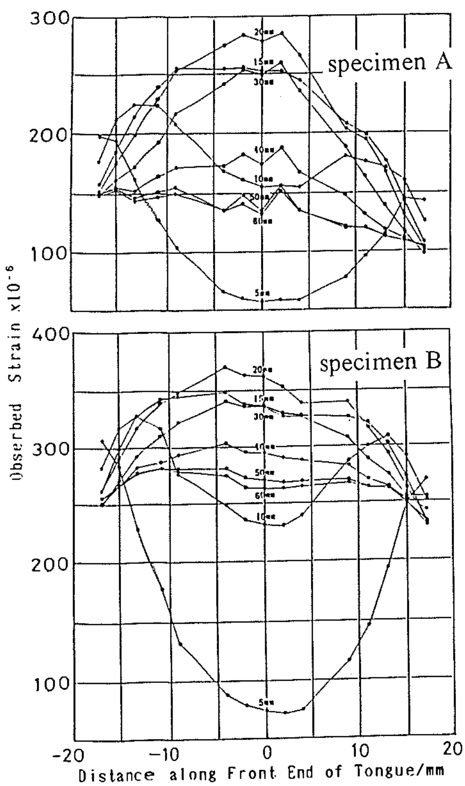

Fig.6. Distributions of strain $\varepsilon_x(x, b)$ measured at the upper end of the plate.

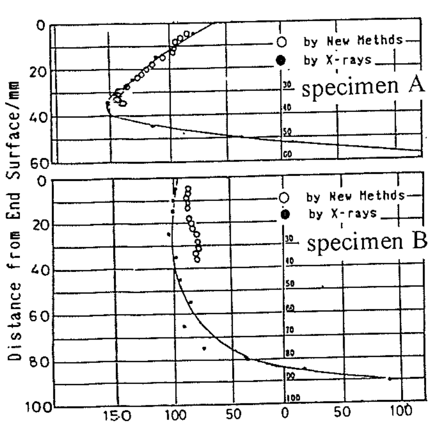

Fig.8. Comparison of residual stresses obtained by the new method and X-ray methods.

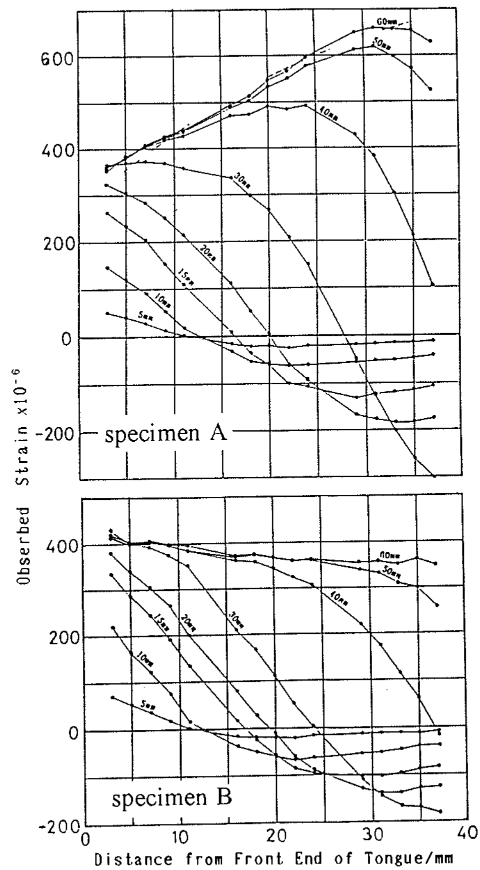

Fig.7. Distributions of strain $\varepsilon_x(0, y)$ measured at the center of the plate with various depth of cut.

shown in Fig.7. Figure 8 shows a comparison in residual stresses between the new methodproposed in this paper and the X-ray methods. The results obtained by the new methods agrees well with the one obtained by the X-ray methods.

DISCUSSION

The X-ray diffraction techniques can be used to measure the residual stresses. But there are certain well-known limitations, such as excessively large grain size and effects of sever plastic deformation, detract from universal acceptance of the technique. The method proposed in this paper, has no metallurgical limitations such as X-ray method. The method makes use of parallel thin cuts of progressively increasing depth to release normal stress along the plane of the cuts. The computational model for tongue-cutting method is based on theoretical solution of elasticity for deformation of a rectangular plate which is subjected to an arbitrary traction at the ends of the plate. Saint-Venant's principle might prevent measured strains from perfectly representing details of removed stresses. From his principle the effect of releasing the stress is expected to decay at a rapid rate. Thus, the gradient of the strains $\varepsilon_x(x, b)$ measured at upper end will tend to decrease as the depth $2a$ of the tongue increases. The distributions of the strains $\varepsilon_x(0, y)$ are not exactly representing the removed stresses. This fact is shown Figs. 6 and 7. The boundary conditions that $\sigma_y = 0$ and $\tau_{xy} = 0$ for the lower edge($y= -b$) of the plate(Fig.2) are somewhat controversial, it may approximately hold when a plate has not steep gradients in the stress distribution.

CONCLUSIONS

The new method was applied to a butt-welded plates. X-ray techniques were used to verify the calculated results for the same test pieces. The results obtained by the new methods agrees well with the one obtained by X-ray methods. This indicated that good results can be expected from the new metnod. The great advantage of determining the residual stresses by the tongue-cutting and strain measurements is that this can be done relative easily and with inexpensive equipment. The limitations are that flat plate-type specimens are required and the residual stresses have to be uniformly distributed over the thickness of the plate.

References

1. Bourke, M. A., Smith, D. J., and Webster, G. A.,1990, "Neutron Diffraction Measurements of Residual Stresses in Plastically Deformed Cracked Beams," *Proceedings of the 9th International Conference on Experimental Mechanics*, pp.1196-1206.

2. Wolf, V.,H. & Bohm, W., 1971, "Das Ring-Kern Verfarren Zur Messung von Eigenspannungen und seine Anwendung bei Turbinen und Generattorwellen", *Archiv. fur Eisenhuttenwesen,* Vol.42, pp.195-200.

3. Schajer,G. S.,1998,"Measurement of Non-uniform Residual Stresses Using the Hole-Drilling Method. Part 1-Stress Calculation Procedures", *ASME Journal of Engineering Materials and Technology*, Vol.110, pp338-343.

4. Cheng,W., Finnie,I. and Vardar,O., 1991,"Measurement of residual Stresses Near the Surface Using the Crack Compliance Method," *ASME Journal of Engineering Materials and Technology*, Vol.113, pp199-204.

5. Masubuchi,K., 1983, "Residual Stresses and Distortion", *Metals Handbook*, 9th ed., Vol6, Welding, Brazing, and Soldering, ASM, pp.865-895.

6. Pickett, G., 1944, "Application of the Fourier Method to the Solution of Certain Boundary Problems in the Theory of Elasticity," *ASME Journal of Applied Mechanics*, pp.A176-A182