



SEMIANALYTICAL SOLUTION METHOD OF DIFFERENTIAL- INTEGRAL EQUATION OF FLAT CRACK PROBLEM

Anatolii Batura¹, Igor Orynyak², and Andrii Oryniak³

¹PhD, Senior Scientist, G.S.Pisarenko Institute for Problems of Strength of National Academy of Science of Ukraine, Kiev, Ukraine (batura@ipp.kiev.ua)

²Professor, Head of Department, G.S.Pisarenko Institute for Problems of Strength of National Academy of Science of Ukraine, Kiev, Ukraine (or@ipp.kiev.ua)

³PhD student, National Technical University of Ukraine “Kyiv Polytechnic Institute“, Kiev, Ukraine

ABSTRACT

The exact analytical approach for stress intensity factor calculation for an arbitrary shape mode I crack loaded by the polynomial stresses is proposed. The approach is based on the calculation of the crack faces displacement at given loading. At that the key problem is the solution of well-known inverse task: obtaining the stresses field at the crack faces on the base of a given displacements field. Multiply solution of such task for a whole set of certain displacements base functions (e.g., for the polynomial terms) allows to get analytical expression which connects stresses and displacements fields.

The original semi-analytical technique for integration with subsequent differentiation of well-known singular integral equation of the flat crack problem is developed. The excellent accuracy of the method is confirmed for an elliptic crack as well as for a rectangular one in the infinite 3D body. New results are given for an inner semi-elliptic crack in the infinite body which surfaces are loaded by polynomial stresses up to the 6th order.

INTRODUCTION

The key problem of the assessment of Nuclear Power Plant safety and residual lifetime is the estimation of allowable shift of the critical brittle temperature for the Reactor Pressure Vessel (RPV), in other words – the assessment of the brittle strength of reactor (thick-walled cylinder vessel) at different operation conditions, especially – at accidents. Obviously, that such calculations are impossible without Stress Intensity Factor calculation (SIF) for the postulated crack in the wall of RPV.

There is a lot of SIF calculation methods at present for the cracks of “canonical” shapes, e.g. semielliptical surface cracks, internal elliptical cracks. A large amount of interpolational expression of high accuracy was got for those cracks and such expression are widely used in the strength-assessment standards for nuclear, petrochemical, transport industry, etc. Also, such methods as Weight Function Method (e.g., see Orynyak et al. (1994)) allows to extend existing results for “canonical” cracks for the case of arbitrary loading.

However, according to the last approaches in the nuclear industry it is necessary to calculate SIF for the cracks with lack of theoretical base solutions, such as “eye-shaped” cracks in the rounding of the nozzle, internal (under-cladding) semielliptical cracks, etc (e.g., see VERLIFE (2008)). The most usual solution, included in the normative documents, is the interpolative tables, obtained by the Finite Elements Method (FEM). But question of their verification is still open, because there are no enough test data for the comparison.

So, our goal is the development of “universal” SIF calculation method, which should have strong theoretical basis, high accuracy and should be applicable to the cracks of any shape and loaded by the arbitrary stresses fields.

The relation of SIF and crack faces displacements field at crack contour zone is the well-known result of the theory of elasticity, e.g. Panasyuk (1988), eqn. 1.24. This relationship for the close proximity of crack contour can be written in the following form:

$$U = K_I \sqrt{\frac{2\Delta}{\pi}} \frac{(1-\mu)}{G}, \quad (1)$$

where U - is the crack face opening in certain point (x, y) , Δ - the length of the normal from (x, y) to the point of crack contour, μ - Poisson's ratio, G - shear modulus, K_I - SIF in the correspondent point of crack contour.

From the form of (1) the quite simple conclusion can be made: an expression for the K_I is the limit transition of expression like $U/\Delta \cdot const$ at $\Delta \rightarrow 0$. So, the main goal of the presented work is to find the expression for crack faces displacements field $U(x, y)$ on the base of arbitrary stress field in the form applicable to such transition.

THE IDEA OF THE METHOD

The method is based on the well-known theoretical solution of the task, which is an inversion of current: finding of the stresses field for the crack faces on the base of certain displacements (crack face opening) field. The singular differential-integral equation for this task is known (see Panasyuk (1968)):

$$p(x, y) = -\frac{E}{4\pi(1-\mu^2)} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \iint_{(S)} \frac{U(\xi, \eta) d\xi d\eta}{\sqrt{(x-\xi)^2 - (y-\eta)^2}}, \quad (2)$$

where (x, y) and (ξ, η) are Cartesian coordinates in the crack plane related to the some point of the coordinates origin, say point O (the example, elliptical crack, is shown on Fig. 1); $p(x, y)$ is the load acting on the crack faces; S is the crack area; E is the module of elasticity.

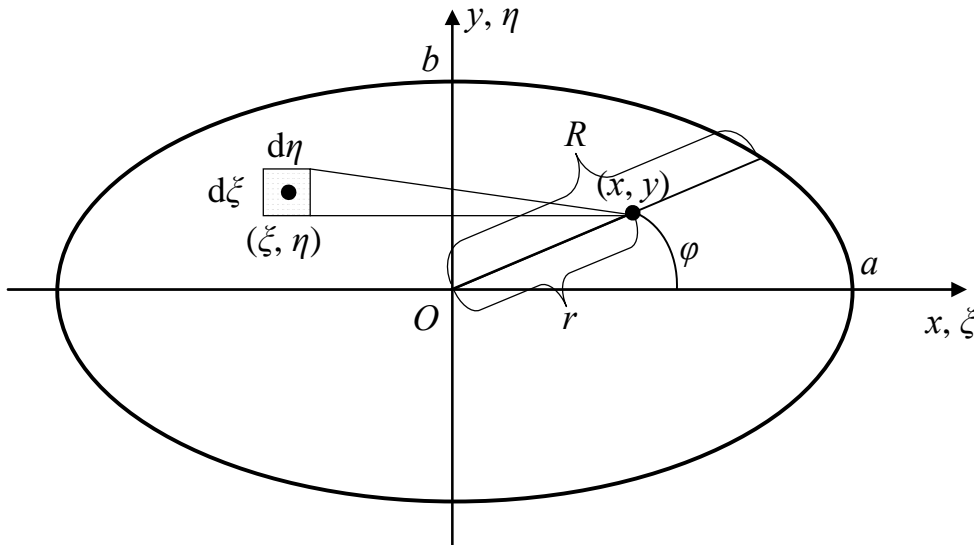


Figure 1. Coordinate system and nomenclature for the elliptical crack.

In spite of this relatively simple formulation of the flat crack problem (2) there are no universal approaches to its *direct* solution for *any crack form*. Main drawbacks of existing methods consist in that

they either employ a very complicated techniques, for example, Fourier transform (Stadnik and Gorbachevskiy (1981)) or the main (singular) part is extracted from the above hypersingular integral equation which is solved analytically while the remained part of it is solved numerically (Qin and Noda (2003), Tang and Qin (1993)). Furthermore, obtaining of the general *inverse* (which is actual for our goal) solution of (2) in the close form seems to be impossible.

Thus, the idea of the solution method is that: expression (2) is replaced by its analytical interpolative function in the form that allows *easy obtaining* of the *inversion*. Then obvious step should be made. This method requires determination of (2) values through the *S* to find the interpolation.

A polynomial interpolation of (2) is proposed to use. It can be relatively easy obtained and transformed into the inversion. Also the accuracy of calculation can be increased by obtaining the interpolation for the integral part of (2) and further analytical application of Laplacian (double differentiation in (2)).

The dimensionless shape function Ω is commonly used at crack problem calculations for taking into account asymptotic character of crack face opening at contour proximity, which is proportional to $\sqrt{\Delta}$ according to the common solution of fracture mechanics (see expression (1)). Note, that such character can be hardly replicated by the polynom.

The choice of the Ω form is quite complex and ambiguous task which should be solved individually for different types of the crack shape. The following Ω function was decided to use it this work because of its generality, good correspondence to the real opening shape for the well-known types of crack (e.g. elliptical, rectangular), also limit transition $\lim_{\Delta \rightarrow 0} U/\Delta$ can be trivially obtained for it:

$$\sqrt{1 - \frac{r^2(\varphi)}{R^2(\varphi)}} = \Omega, \quad (3)$$

here φ - is the angular coordinate of a point, $r(\varphi)$ – radius to the point from the crack center (coordinates origin), $R(\varphi)$ – radius function, which describes crack contour (see Fig. 1). Note, that geometrical center of the elliptical and rectangular cracks can be trivially chosen as their coordinates origin, unlike certain other cracks, such as semielliptical, where choosing the geometrical center is ambiguous task.

The expression (3) is not the only possible type of Ω . E.g., in several literature sources another Ω is used for the rectangular crack (see Qin and Noda (2003)):

$$\Omega_1 = \sqrt{1 - \frac{\eta^2}{b^2}} \cdot \sqrt{1 - \frac{\xi^2}{a^2}}. \quad (4)$$

In general, this expression is rather strange, because polynom degree under root sign depends on the proximity to the corner of crack contour and can change from 2 near the center of rectangular side to 4 near the corner or even to a higher value if several corners are close to each other. So, Ω expression in form (3) is more reasonable than (4).

According to the above, the interpolative form of displacements U and stresses p are the following:

$$U(\xi, \eta) = \sum_{m+n=0}^{\infty} U_{m,n}(\xi, \eta) = \frac{\bar{\sigma}_0 \cdot A}{H} \Omega \sum_{m+n=0}^{\infty} Q_{m,n} \left(\frac{\xi}{A} \right)^m \left(\frac{\eta}{A} \right)^n = \frac{\bar{\sigma}_0 \cdot A}{H} \Omega \sum_{t=0}^{\infty} Q_t F_t(\xi, \eta), \quad (5)$$

$$p(x, y) = \sum_{i+j=0}^L p_{ij}(x, y); p_{ij}(x, y) = \bar{\sigma}_0 \cdot p_{ij} \left(\frac{x}{A} \right)^i \left(\frac{y}{A} \right)^j = \bar{\sigma}_0 \cdot p_{ij} F_{i,j}(x, y) = \bar{\sigma}_0 \cdot p_k F_k(x, y), \quad (6)$$

here m, n, i, j – degrees of correspondent polynomial members, H – certain material property (for the solution of (2) H can be written as $\frac{4\pi(1-\mu^2)}{E}$), $Q_{m,n}(Q_t)$ and $p_{ij}(p_k)$ are the correspondent interpolative polynomial coefficients, A – a certain characteristic size of the crack (e.g., maximum of a and b for the elliptical crack), $\bar{\sigma}_0$ – a certain unit loading (e.g., 1 MPa). For the simplification purpose, here we've introduced values t and k instead of pairs, (m, n) and (i, j) correspondently.

It is natural that degree value must be bounded for the practical implementation. If sum $m+n$ ($i+j$) is bounded by L value, the t (k) is bounded by $N = \frac{(L+1)(L+2)}{2} - 1$. At that $t = m + n \cdot L$ ($k = i + j \cdot L$).

Such method of U and p representation allows to find inverse solution of the (2) which is described in the following chapter.

THE FORMING OF THE SOLUTION MATRIX AND GETTING THE INVERSION.

The idea of the inversion method consists of forming of a number of the $p(x, y)$ solutions for N elementary displacement functions $U_t(\xi, \eta) = \frac{A}{H} \Omega \cdot \bar{\sigma}_0 \cdot \bar{F}_t(\xi, \eta)$, further filling of square matrix M_1 by their coefficients and getting the inversion matrix M_2 . Applying M_2 to the coefficients of any stress polynom $p(x, y)$, we actually get the solution – the polynomial part of correspondent $U(\xi, \eta)$ expression.

Let's write this method with the matrix notation. Introduce column-vector \bar{F} , which consists of all monomials, from F_0 to F_{N-1} , with parameters (ξ, η) or (x, y) . Let's call functional from (2) as \mathfrak{R} , its inversion as \mathfrak{R}^{-1} . Let's represent solution of the direct task (result of \mathfrak{R}) for t -th elementary displacement function $U_t(\xi, \eta)$ in the following form:

$$p(x, y) = \sum_{k=0}^{N-1} \bar{\sigma}_0 \cdot p_k^t F_k(x, y) = \bar{\sigma}_0 \cdot \bar{P}^t \bar{F}, \quad (7)$$

here $\bar{P}^t = (p_0^t, \dots, p_{N-1}^t)$ – is the row-vector of stress-polynom coefficients, corresponded to $U_t(\xi, \eta)$. Such row-vectors form matrix M_1 , and inverse matrix $M_2 = M_1^{-1}$ filled with row-vectors of type $\bar{Q}^k = (q_0^k, \dots, q_{N-1}^k)$. So, after obtaining solution for all $U_t(\xi, \eta)$, expression can be written in the form (8). Note, that according to the features of integration and differentiation, \mathfrak{R} (as well, as \mathfrak{R}^{-1}) is linear, so (9) and (10) can be obtained from the (8).

$$\mathfrak{R} \left(\frac{\bar{\sigma}_0 \cdot A}{H} \Omega \bar{F} \right) = \bar{\sigma}_0 \cdot M_1 \bar{F}, \quad (8)$$

$$\mathfrak{R} \left(\frac{\bar{\sigma}_0 \cdot A}{H} \Omega M_2 \bar{F} \right) = \bar{\sigma}_0 \cdot \bar{F}, \quad (9)$$

$$\frac{\bar{\sigma}_0 \cdot A}{H} \Omega M_2 \bar{F} = \bar{\sigma}_0 \cdot \mathfrak{R}^{-1}(\bar{F}). \quad (10)$$

Thus, notation (10) specifies the whole set of solutions for all elementary stress functions, and k -th row of (10) is the displacement field for elementary function $F_k(x, y)$: $U(\xi, \eta) = \frac{\bar{\sigma}_0 \cdot A}{H} \Omega \bar{Q}^k \bar{F}$. The

solution for the arbitrary stress field $p(x, y)$, set by the vector of coefficients $\vec{P} = (p_0, \dots, p_{N-1})$, can be written as linear combination of the solutions for the elementary stress functions:

$$\frac{\bar{\sigma}_0 \cdot A}{H} \Omega \bar{P} M_2 \bar{F} = \bar{\sigma}_0 \cdot \bar{P} \mathfrak{R}^{-1}(\bar{F}). \quad (11)$$

Thus, the main problem is obtaining coefficients \bar{P}^t for each $\mathfrak{R}(U_t(\xi, \eta))$.

If integral part of (2) can be calculated for any point $O^q(x_q, y_q)$ of crack square, the least-squares method can be used to change integral by the interpolation polynom in the following form:

$$p_{\text{int}}(x, y) = \sum_{i+j=0}^{L_{\text{int}}} p_{i,j}^{\text{int}} \left(\frac{x}{A}\right)^i \left(\frac{y}{A}\right)^j = \sum_{k_{\text{int}}=0}^{N_{\text{int}}-1} p_{k_{\text{int}}}^{\text{int}} F_{k_{\text{int}}}(x, y), \quad (12)$$

here $L^{\text{int}} = L + 2$, $N^{\text{int}} = \frac{(L^{\text{int}} + 1)(L^{\text{int}} + 2)}{2}$. Note, that different principles of choosing O^q points for integral calculation can be used, in this work uniform filling of S by O^q is applied. Applying analytical Laplacian procedure to (14) and obtaining result (\bar{P}^t) is very simple. So, the semi-analytical procedure of integration is described in the next chapter. For definiteness purpose integral procedure is applied to the point O^q .

THE INTEGRATION METHOD

The idea of the presented procedure is based on the integration method for equation of contact mechanics which was proposed by Love (1927), (1929). It based on the integration over the area of thin triangle, which uses the local polar system of coordinates connected with the point of interest $O^q(x_q, y_q)$.

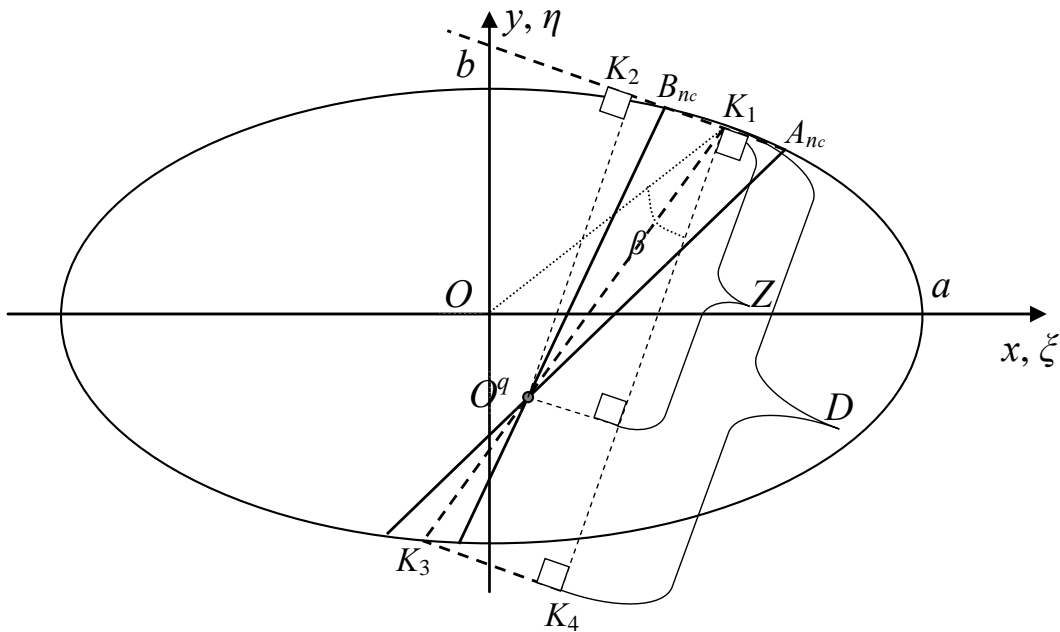


Figure 2. Nomenclature for the triangle segmentation of crack square.

Thus, we need to split crack area into triangles by the rays, outgoing from O^q . Rays crossings with crack contour form third sides of triangles – rectifying segments of crack contour. Angle increment for the segmentation rays should be chosen on the base of necessary accuracy, at that corners of crack contour (if exist) are recommended to take into account. The example of the nomenclature for the segmentation triangle $O^q A_{n_c} B_{n_c}$ is shown on Fig. 2. Note, that integral from (2) is changed to the sum of square integrals for each triangle $O^q A_{n_c} B_{n_c}$, $n_c = 1..N_{cont}$, where N_{cont} - is the number of triangles.

K_1 is the certain point (e.g. the middle) of $A_{n_c} B_{n_c}$ segment, K_2 - point of crossing of $A_{n_c} B_{n_c}$ line and normal from O^q . Note, that K_2 can be situated at both side outside $A_{n_c} B_{n_c}$ segment, as well as inside. For the triangle $O^q A_{n_c} B_{n_c}$ its integral can be changed as algebraic sum of two integrals:

$$\int_{O^q A_{n_c} B_{n_c}} = \text{sign}(A_{n_c}, K_2) \int_{O^q A_{n_c} K_2} + \text{sign}(K_2, B_{n_c}) \int_{O^q K_2 B_{n_c}}, \quad (13)$$

where $\text{sign}(X, Y)$ is the certain rule sets the sing of the integral, “+1” if way from contour point X to Y runs counterclockwise, “-1” – otherwise. For the definiteness purpose integral for $O^q A_{n_c} K_2$ is considered.

Coordinate system in the triangle is transformed from Cartesian with the center in O point into polar one with the center in O^q , ray $O^q K_2$ as polar axis and positive counterclockwise angle α_{O^q} direction. This transformation convert singularity of $1/\sqrt{(x-\xi)^2 - (y-\eta)^2}$ kind into the similar $1/r_{O^q}$ (r_{O^q} - the radial coordinate for $O^q A_{n_c} K_2$), but $d\xi d\eta$ transforms into $r_{O^q} \cdot d\alpha_{O^q} dr_{O^q}$ and Jacobian determinant *reduce the singularity*. So, integral part of (2) can be written in the following “mix-coordinate” form:

$$\int_{O^q A_{n_c} K_2} = \int_0^{\alpha_1} d\alpha_{O^q} \int_0^{\frac{Z}{\cos \alpha_{O^q}}} F(x, y) \sqrt{1 - \frac{r^2(\varphi)}{R^2(\varphi)}} dr_{O^q}, \quad (14)$$

where α_1 is the angle $A_{n_c} O^q K_2$, Z – is the normal length, see Fig. 2. Generally such integral also can't be solved analytically, so integral expression is replaced by interpolative one-dimension function, which can be integrated. Note, that triangles are quite thing, so we *don't need* to use *two-dimension interpolation*. Also specificity of Ω near both sides of contour (at K_1 and K_3 points zone) should be taken into account. The following polynom-based function, which can be integrated by the known reference analytical recurrent expression, is proposed:

$$F_{\text{int}}(z) = \sqrt{\frac{z}{Z}} \sqrt{\frac{D-z}{D}} \cdot \left(C_0 + C_1 \left(\frac{z}{Z} \right)^1 + \dots + C_V \left(\frac{z}{Z} \right)^V \right), \quad (15)$$

where z – is the distance from $A_{n_c} B_{n_c}$. C_v ($v=0..V$) coefficients can be also found by the least-squares method. At that values of expression $F(x, y) \sqrt{1 - \frac{r^2(\varphi)}{R^2(\varphi)}} * \sqrt{\frac{Z}{z}} \sqrt{\frac{D-z}{D}}$ along $O^q K_1$ segment should be used. There is an uncertainty of type $\frac{0}{0}$ at point K_1 . It's easily solved by the limit:

$$\lim_{z \rightarrow 0} \sqrt{1 - \frac{r^2(\varphi)}{R^2(\varphi)}} = \lim_{z \rightarrow 0} \sqrt{2} \sqrt{\frac{z}{R(\varphi) \cos \beta}} = \sqrt{\frac{2}{R(\varphi) \cos \beta}} \cdot \sqrt{z}, \quad (16)$$

where β is an angle between segment OK_1 and normal to $A_{n_c} B_{n_c}$.

So, after transformation the final expression for the $O^q A_{n_c} K_2$ integral is following:

$$\int_{O^q A_{n_c} K_2} = \int_0^{\alpha_1} d\alpha_{O^q} \int_0^{\frac{z}{\cos \alpha_{O^q}}} F_{\text{int}}(Z - r_{O^q} \cos \alpha_{O^q}) dr_{O^q} = \int_0^{\alpha_1} \frac{d\alpha_{O^q}}{\cos \alpha_{O^q}} \int_0^z F_{\text{int}}(u) du = \quad (17)$$

$$\frac{1}{2} \ln \left(\frac{1 + \sin \alpha_1}{1 - \sin \alpha_1} \right) \cdot \sum_{v=0}^V C_v \left(\frac{u}{Z} \right)^v \sqrt{\frac{u}{Z}} \sqrt{\frac{D-u}{D}} du = \frac{C_v}{2} \ln \left(\frac{1 + \sin \alpha_1}{1 - \sin \alpha_1} \right) \cdot \sum_{v=0}^V \int_0^z \left(\frac{u}{Z} \right)^{v+1/2} \sqrt{\frac{D-u}{D}} du.$$

Thus the calculation method for the integration part of (2) was developed.

VERIFICATION OF THE METHOD

Integration And Differentiation For The Ellipse Crack

The first and very important set of tests is a comparison of the integration and differentiation results with the well-known data for the elliptical shape. In this chapter results for the full-numerical integration method are also presented for the comparison. At that additional, finer splitting of crack square near contour was used to take into account features of Ω , because they aren't considered analytically in this case. The modeling parameters are following: $N_r = N_\varphi = 40$ (number of O^q points by radial and angular coordinate), $N_{\text{cont}} = 360$, $V = 10$, $L = 3$.

First test was made for Hertz task, obtaining of the integral from (2) for the loading of the ellipse with the law $p(x, y) = p_0 \Omega$. The theoretical result is well-known:

$$\omega(x, y) = p_0 \pi b \left[K(e) - \frac{x^2}{a^2} \frac{K(e) - E(e)}{e^2} - \frac{y^2}{a^2} \frac{E(e) - (1 - e^2)K(e)}{e^2(1 - e^2)} \right], \quad (18)$$

here $e = \frac{\sqrt{a^2 - b^2}}{a}$ ($a > b$) - is the ellipse eccentricity, $K(e)$ and $E(e)$ - are the complete elliptical integrals.

Both methods demonstrated high accuracy. The maximum errors for $a/b=1$ are 0.1% for numerical approach, 0.0032% - for semianalytical; for $a/b=3$ errors are 1.5% and 0.02% correspondently.

Second test is the stamp task with loading law $p(x, y) = p_0 / \Omega$. The result for the elliptical shape is a constant value. The comparison of theoretical and numerical data is presented in Table 1. Note, that current implementation of presented methods isn't optimized for such singular stress fields, which is the explanation of such error.

Table 1. Calculation results for the stamp task, numerical method

The ellipse geometry	Theoretical result, Lurie (1955)	Result in the center of the ellipse	Maximum error for the whole ellipse square
$a=1, b=1$	$\pi^2 = 9.8696$	9.86	0.5%
$a=3, b=1$	$5.0573\pi = 15.888$	15.836	3%

Next test is the integration and differentiation of (2). Three laws of crack faces opening are analyzed: $U_0(x, y) = 1 \cdot \Omega$, $U_1(x, y) = 1 \cdot \left(\frac{x}{a}\right)\Omega$, $U_2(x, y) = 1 \cdot \left(\frac{y}{b}\right)\Omega$. The theoretical results, presented without material parameters for simplification, are following: $p_0(x, y) = -\frac{2\pi E(e)}{b}$, $p_1(x, y) = 2\pi \left(\frac{x}{a}\right) \frac{(1-e^2)K(e) + (2e^2-1)E(e)}{e^2}$, $p_2(x, y) = 2\pi \left(\frac{y}{b}\right) \frac{(1+e^2)E(e) - (1-e^2)K(e)}{e^2}$. Comparison with our results are shown in Tables 2-3.

Table 2. Calculation results for uniform displacement field U_0

The ellipse geometry	Theoretical result, Panasyuk (1968)	Numerical procedure		Semianalytical procedure	
		Result in the center	Maximum error	Result in the center	Maximum error
$a=1, b=1$	-9.8696	-9.8914	0.3%	-9.8691	0.006%
$a=3, b=1$	-6.9978	-7.0149	1.1%	-6.9974	0.022%

Table 3. Calculation results for linear displacement fields, U_1 and U_2 , $a=3, b=1$, semianalytical procedure

Displacement function	Maximum of absolute value, Orynyak (1998)	Values in the points			Maximum error
		$x=-a$ ($y=-b$)	0	$x=a$ ($y=-b$)	
U_1	8.1091	8.109	$-7 \cdot 10^{-10}$	-8.109	0.026%
U_2	12.8844	12.884	$-9 \cdot 10^{-9}$	-12.884	0.009%

So, integration and differentiation procedures demonstrate high accuracy for elliptical shape.

Examples of SIF calculation

According to the proposed method, SIF for square and semielliptical cracks loaded by uniform stress was obtained. The comparison with Qin and Noda (2003) FEM data for square crack is shown on Fig. 3. The dimensionless SIF, F_1 from Qin and Noda (2003), is presented on the plots. The expression for the crack face opening for the unit stress is following: $\frac{\bar{\sigma}_0 \cdot A}{H} \Omega(0.1123 - 0.0086(x^2 + y^2) - 0.0547(x^2 y^2) + 0.0201(x^4 + y^4))$.

The conclusion can be drawn, that accuracy depends on degree of interpolative polynom and one is quite good for $L=4$. But the question of a result at crack corner zone is still open. The mismatch can be explained by the calculation problem of FEM at such singularity zone. Note, that one of the variants of Qin and Noda (2003) results is closer to our (point "FEM 2" on the plot), but it can be caused by a misprint.

SIF calculation for the semicircular crack of unit radius in the infinite body, having line $y=0$ as straight part of contour was performed. For the semielliptical crack form of Ω is a still open question.

Several Ω functions were used in our calculation: $\Omega_0 = \sqrt{\frac{y}{b}} \sqrt{1 - \frac{r}{R}}$ (point O_0 , the conditional center of

crack, is (0,0) – the circle center), $\Omega_{1,2,3} = \sqrt{1 - \left(\frac{r}{R}\right)^2}$ (centers are the following points: $O_1(0,0.25)$,

$O_2(0,0.5)$, $O_3(0,0.75)$). The obtained SIF results were compared with data of Marie et al. (2005) and our FEM calculations. The best agreement with the FEM data was got for Ω_2 . At that proximity of O to the contour cause certain disagreement of SIF values for nearby points of contour (straight part for O_1 and circular part for O_3). The interpolation degree $L=4$ was used for Ω_0 and $L=6$ was used for $\Omega_{1,2,3}$. Using $L=4$ for $\Omega_{1,2,3}$ causes relatively high disagreements and difference in the character of SIF plots.

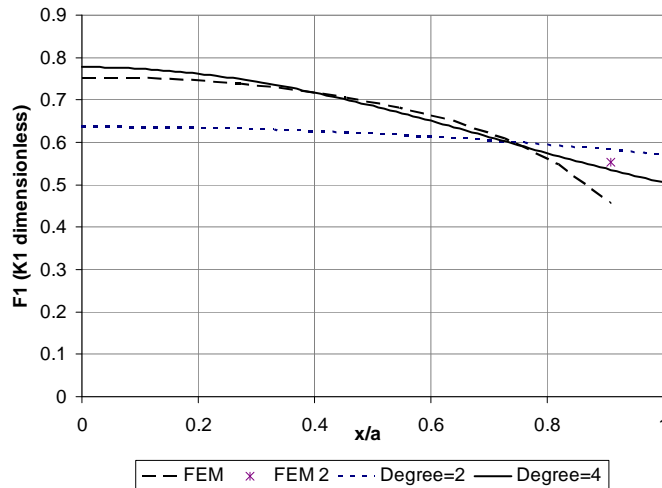


Figure 3. Result for square crack.

The expression for the crack face opening for the stress 1 MPa and case of best results (Ω_2) is following (odd degrees of x are zero, because of crack symmetry):

$$\frac{\bar{\sigma}_0 \cdot A}{H} \Omega \cdot 0.1 \left(0.665 + 0.496y - 0.5x^2 - 4.705y^2 - 0.383x^2y + 16.301y^3 + 6.191x^2y^2 + 0.408x^4 - 27.318y^4 + 1.015x^4y - 8.481x^2y^3 + 22.061y^5 - 2.697x^4y^2 + 3.296x^2y^4 - 0.156x^6 - 6.889y^6 \right), \quad (19)$$

Note, that expression (23) was obtained as first row of M_2 matrix, but all other solutions for the polynomial loading of degrees up to 6th can also be written. The dimensionless SIF parameter $F_I = K_I / \sigma \sqrt{2\pi a}$ obtained for the different Ω is shown on Fig. 4. Results for circular and straight parts of contour are presented.

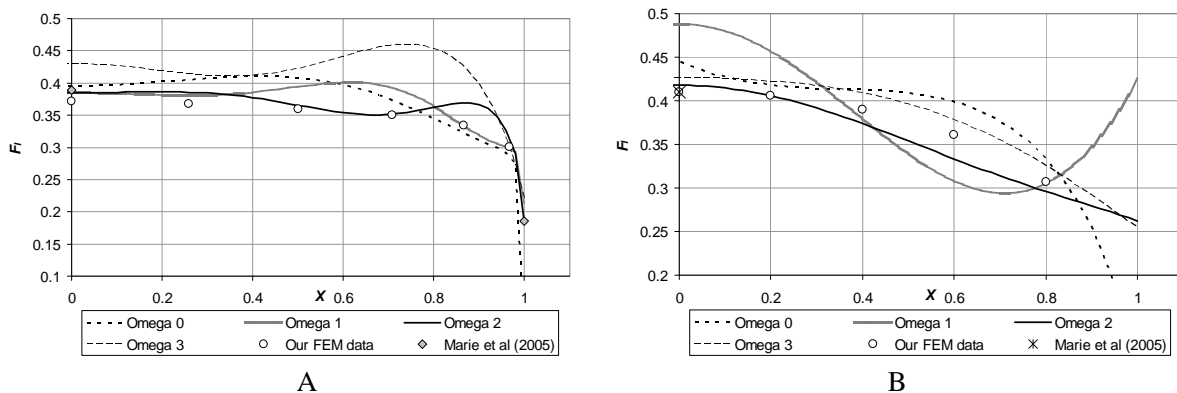


Figure 4. Results for semicircular crack. A – circular part, B – straight part.

CONCLUSION

The original high-effective method of the analytical integration and numerical differentiation of the singular differential-integral equation of flat crack problem was developed. The integration is based on the crack area segmentation with the thin triangles and further analytical integration within triangles square. Then interpolative polynomial is built on the base of the integral values through the crack square and Laplace operator can be easily applied to it, so stresses field can be obtained on the base of given crack faces displacement field.

The polynomial statement allows to solve inverse task and to get fields of the crack face opening for the arbitrary polynomial laws of loading. Having displacements of crack fields in the analytical form the stress intensity factor can be easily calculated at any point of crack contour.

The excellent accuracy was shown for the integration and differentiation procedures. Obtained SIF results for the square and semielliptical cracks show good correspondence with FEM data, at that new SIF data for the polynomial loading laws up to 6th degree were obtained. The importance of the polynomial degree and shape function for the calculation accuracy was demonstrated.

REFERENCES

- Orynyak, I.V., Borodii, M.V. and Torop, V.M. (1994). "Approximate Construction of a Weight Function for Quarter-elliptical, Semi-elliptical and Elliptical Cracks Subjected to Normal stresses," *Engineering Fracture Mechanics*, 49(1), 143-151.
- VERLIFE (2008). *Unified Procedure for Lifetime Assessment of Components and Piping in WWER NPPs*, "VERLIFE" Version 2008, Project co-funded by the European Commission under the Euratom Research and Training Programme on Nuclear Energy within the Sixth Framework Programme (2002-2006).
- Panasyuk, V.V. (1988). *Fracture Mechanics and Strength of Materials. Vol. 2. Stress Intensity Factors for the Bodies With Cracks* (in Russian), Naukova Dumka, Kyiv, Ukraine (former USSR).
- Panasyuk, V.V. (1968). *Ultimate Equilibrium of the Brittle Bodies with cracks* (in Russian language), Naukova Dumka, Kyiv, Ukraine (former USSR).
- Stadnik, M.M. and Gorbachevskiy, I.Ya. (1981). "Limit Equilibrium of Body With a Flat Triangular Crack," *Prickladnaya Mekhanika*, 17(7), 101-105.
- Qin, T.Y. and Noda, N.A. (2003). "Stress Intensity Factors of a Rectangular Crack Meeting a Bimaterial Interface," *International Journal of Solids and Structures*, 40, 2473–2486.
- Tang, R.J. and Qin, T.Y. (1993). "Method of Hypersingular Integral Equations in Three-dimensional Fracture Mechanics," *Acta Mechanica Sinica*, 25, 665–675.
- Love, A.E.H. (1927). *A Treatise on the Mathematical Theory of Elasticity, 4th edition*, Cambridge University Press.
- Love, A.E.H. (1929). "The Stress Produced in a Semi-infinite Solid by Pressure on Part of the Boundary," *Philosophical Transactions of the Royal Society*, London, A228:377.
- Lurie, A.I. (1955). *Spatial Problems of Elasticity Theory* (in Russian), State Publishing House of Scientific-Technical Literature, Moscow, Russia (former USSR).
- Orynyak, I.V. (1998). "Method of Translation a Mode I Elliptic Crack in an Infinite Body. Part I. Polynomial Loading," *International Journal of Solids and Structures*, 35, 3029-3042
- Marie, S., Me'nager, Y. and Chapuliot, S. (2005). "Stress Intensity Factors for Underclad and Through clad Defects in a Reactor Pressure Vessel Submitted to a Pressurised Thermal Shock," *International Journal of Pressure Vessels and Piping*, 82, 746–760.