Transactions, **SMiRT-25**
Charlotte, NC, USA, August 4-9, 2019
Division V

DYNAMIC MODEL OF THE WWER-1000 REACTOR

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ABSTRACT

This paper presents development approaches to dynamic model of WWER-1000 reactor, with emphasis on connected pipes influence and coolant flexibility. Different types of interactions between elements of reactor and attached piping are taken into account. The adequacy of number of nodes and theirs positions is justified. Lagrange equations of the second kind were used to obtain the mathematical model and Duhamel integral for dynamic response. In addition pressure vessel internals FEM model was developed to address the elastic-plastic water hammer calculation during large LOCA accident.

INTRODUCTION

Nuclear Reactor dynamic modeling is a step forward safety analysis of the reactor facility (Zeman and Hlavac, 2008). Of course, the model can be developed using FEA (Palamara et al., 2015), but in this case, nonlinear contacts and FSI will not be addressed, moreover a reliable simple analytical model is highly appreciated in engineering community. The aim of model development consist in obtaining the general system response, i.e. analysis of possible collisions of reactor parts and support elements strength assessment.

The dynamic analysis provides for the use as input data the accelerograms of support construction of the reactor. But, its direct translation to the forces of contact by static method can instead lead to lower loads from seismic effects. In this case, usually, a dynamic model of the reactor type "spring-mass" is needed to build. The masses are the elements of the reactor, and the roles of springs perform real interactions between the elements of the reactor.

On the other side, the reactor pressure vessel is connected with pipes of the main circuit piping and the emergency cooling zone. The effect of those pipes also has to be taken into account in the dynamic analysis of the reactor. But, in this case, direct using connection of "spring-mass" is not applicable because of these pipelines are themselves complex oscillatory systems.

On the basis of the above, this paper aims the following:

- Build a dynamic model of the reactor, which uses as input data accelerograms of support construction of the reactor or spectrum data and allows to take into account the possibility of loss of contact between the elements of the reactor;
- Develop a method of taking into account the influence of associated pipelines and coolant in the dynamic model of the reactor.
- To test the build dynamic model for seismic analysis and large LOCA break
- To give an additional support for the analytical model using a simplified shell-beam FEM model of the pressure vessel internals

The feature of the model is that the main equations describe the static state before dynamic process starts. The peculiarity of the constructed system is taking into account the presence of reactor coolant pump (RCP) by introducing the possible mutual displacements of piping and nozzles of RPV in horizontal and vertical planes at different natural frequencies of RCP. Piping is modelled as a system of concentrated masses with flexible springs acting in different directions. The coolant is treated as an elastic element of RPV.

ANALYTICAL MODEL DESCRIPTION

Mathematical formulation

First of all, natural frequencies of particular reactor elements were found using FEA analysis, it allowed to understand what kind of model should be developed. The model consists of six main elements or subsystems (see Figure 1):

- reactor pressure vessel (RPV), covers and details of the reactor main flange. Rigid shell, FEM analysis of natural frequencies should that the first mode (~62 Hz) – is beam like mode, the next one are shell modes ~ 105 Hz;
- core barrel (CB) together with core baffle and core belt. Core baffle is a very rigid element, and it can be accounted as an added mass. FEM calculations of CB natural frequencies showed that shell frequencies dominates (~41 Hz);
- block of guide tubes (BGT); composed from relative rigid shell and supporting plate. These components are connected by system of 61 protection tubes, 60 protections tubes and perforated shell. Dominant are shell frequencies of perforated shell which is around 61 Hz;
- reactor core (RC), formed by a 163 Fuel Assembly (FA). FA are heavy (around 730 kg of a fresh assembly) and flexible structures, they are prestressed by springs block, at force around 1 ton-force each. They should be modeled carefully because, FA collisions are of high importance for dynamic analysis;
- upper block (UP). Supporting structure of upper block (UB) composed from the three horizontal plates and assembly travers, which are mutually connected by 6 tubes and 6 circular rods placed inside the tubes. UP is a very flexible structure the first frequencies are of bending type (~3 Hz);
- control and protective system formed by a 61 drive assembles with massive electromagnets consisted of pulling, retaining and holding. The drive assembly is composed from a lifting system mechanism which ensures a suspension bar motion with the control element. All components are long cylindrical bodies which can be modelled as beams.

Support elements of RVI, PVI, UP and flexible parts of main components like cylindrical body of the CB, were schematized as springs (see Figure 1). To calculate the support stiffnesses, we used static FEA analysis and analytical methods. For massive parts appropriate masses and inertia characteristics were determined.

The Lagrangian equation of motion of the second king are written in the form

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\mathbf{q}}_i}\right)-\frac{\partial E_k}{\partial \dot{\mathbf{q}}_i}=-\frac{\partial E_p}{\partial \mathbf{q}_i} \tag{1}$$

Here $\mathbf{q}_i -$ generalized coordinates, $\dot{\mathbf{q}}_i -$ generalized velocities, E_k – is the kinetic energy, and E_p – is the potential energy, which can be defined as:

$$E_k\left(\dot{\mathbf{q}}\right)=\frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}}\,, \tag{2}$$

$$E_p\left(\mathbf{q}\right)=\frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q} \tag{3}$$

Here $\mathbf{M}$ – mass matrix, $\mathbf{K}$ – stiffness matrix – both symmetric and positive defined.

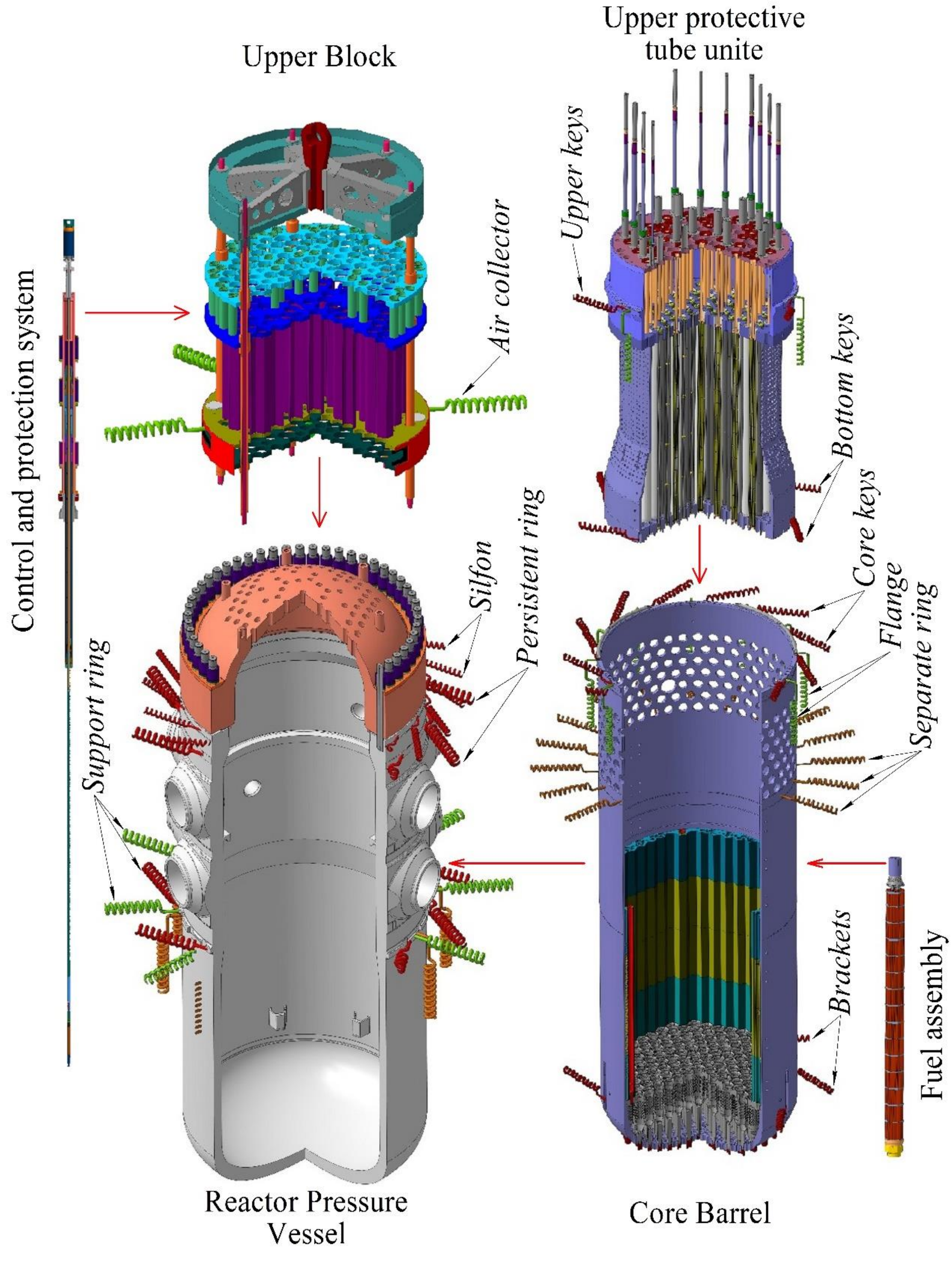

Figure 1. Reactor components

Primary coolant pipelines influence

Let's consider procedure of nozzle influence on the elements of reactor by example of the hot branch vibration of MPC. Suppose, that we have first three natural frequencies $\omega_1, \omega_2, \omega_3$ and corresponding displacement w_1, w_2, w_3 and parameters $\alpha_1, \alpha_2, \alpha_3$:

$$\alpha_i = \frac{M_i}{u_i^2(0)}, \ i = 1...3 \tag{4}$$

Then the equations of vibration process for the hot branch MCP are of the form:

$$\ddot{w}_i + \omega_i^2 w_i = \frac{P_h}{\alpha_i}, \ i = 1...3 \tag{5}$$

Here P_h is the force of interaction of the reactor and the nozzle. For small values of time increment:

$$P_h = A_h + B_h t \tag{6}$$

Here A_h and B_h is the unknown parameters which are found at the end of each time step using conditions of equality of displacement and velocity nozzles and RPV:

$$w_1 + w_2 + w_3 = w_{KR} - w_{KR}^0 \tag{7}$$

$$\dot{w}_1 + \dot{w}_2 + \dot{w}_3 = \dot{w}_{KR} \tag{8}$$

We assume, that the positive direction of the force P_h is responsible to its action on the nozzle down and RPV – up. Similarly to the force P_h determined effort P_c from the cold branch MCP and other nozzles of pipes of ECCS. So, the equation for the RPV equilibrium becomes:

$$-m_{RPV}\ddot{w}_{RPV} - R_{SupportElm\_RPV} + R_{SupportElm\_CB} - F_1 + P_h + P_c = 0 \tag{9}$$

Modified Eq. (9) fully meets the logic of time increment solution algorithm and is used to find the displacements and forces in the elements with the influence of the reactor nozzles.

Influence of the coolant

This section presents a model to analyze the influence of the coolant vibrations under seismic loading. Note, that here we consider the example of seismic vibrations in the horizontal direction. Vibrations in the vertical direction are simpler because, in this case, seismic loading act on one axis. Figure 2 shows a diagram of the model. CB, RPV and coolant between their walls are considered. Seismic loading act along the axis *OX*. Let's construct differential equations to model the behavior of CB, RPV and coolant during seismic loading. The law of volume changing in the elementary section of coolant during seismic effects can be written as:

$$\frac{\partial u_r}{\partial r} + \frac{\partial u_\varphi}{R\partial\varphi} = -\alpha P \tag{10}$$

In Eq. (10) u_r, u_φ – displacements along the normal and tangent to the contour respectively; P – radial force along the normal (positive sign corresponds to the compression of coolant element); α – compressibility of the coolant; $R = \frac{R_1 + R_2}{2}$ – average radius between the outer radius of CB and inner radius of RPV. Eq. (10) is supplemented by the equations of equilibrium of forces:

$$\frac{\partial P}{\partial r} + \rho\frac{\partial^2 u_r}{\partial t^2} + f(r) = 0 \tag{11}$$

$$\frac{\partial P}{R\partial\varphi} + \rho\frac{\partial^2 u_\varphi}{\partial t^2} = 0 \tag{12}$$

Using Eq. (10)-(12), we can find that:

$$\frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{R^2\partial\varphi^2} = \alpha\rho\frac{\partial^2 P}{\partial t^2} \tag{13}$$

Solution of Eq. (13) can be presented in the form:

$$P = P_r(r)\cos\varphi\sin\omega t \tag{14}$$

Here ω – natural frequencies of coolant; $P_r(r)$ – radial component of P; r – coordinate of the coolant element in the radial direction which varies from 0 on the outer surface of CB to h on the inner surface of RPV. Note, that form of P around the circle is chosen as $\cos(\varphi)$ because only in this case the displacement of the whole system (movement of centre) is realized. Substituting Eq. (14) into Eq. (13) we can write that:

$$P_r'' + \left(\alpha\rho\omega^2 - \frac{1}{R^2}\right)P_r = 0 \tag{15}$$

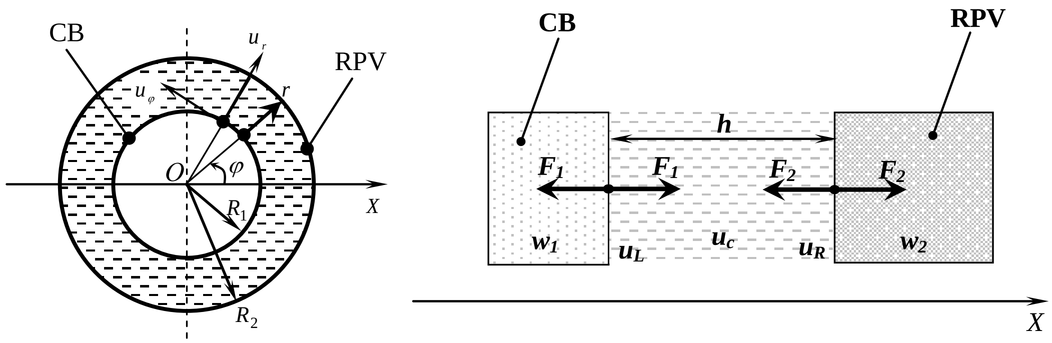

Figure 2. Scheme of interaction between coolant and the walls of CB and RPV

The general solution of Eq. (15) has the form:

$$P_r(r) = P_1\cos\eta r + P_1\sin\eta r \tag{16}$$

Value $\eta^2 = \alpha\rho\omega^2 - \dfrac{1}{R^2}$. Obviously, the free vibrations of coolant provided the following boundary conditions: $P_r(r=0) = P_r(r=h) = 0$. Taking this into account, we obtain that the form of oscillation along the axis *r* is:

$$u_r = \frac{\eta}{\rho\omega^2}P_2\cos\eta r \text{, and } \eta = \frac{\pi}{h}i,\ \ i = 1,2,... \tag{17}$$

On the other hand, the movement of the coolant can be represented as a movement of a whole body, i.e. $u_r(r=0) = u_r(r=h) = 0$ and deformation, which presents by forms of oscillation Eq. (17). Form of oscillation for the movement as a whole body is:

$$u_r = \frac{\eta}{\rho\omega^2}P_1\sin\eta r \tag{18}$$

Let us consider in detail the distribution of forces for the element coolant in radial direction. Positive signs of forces F_1 and F_2 of interactions between coolant and CB and RPV are presented on Figure 2. All positive displacement directions coincide with the positive direction of the axis OX. Then the equilibrium equation for the coolant has the view:

$$-m_C\ddot{u}_C + F_1 - F_2 = 0 \tag{19}$$

On the other hand, we can show that for movement as a whole body, we have at the ends forced $\left(\dfrac{m\cdot\ddot{u}_c}{2}\right)$ and $\left(-\dfrac{m\cdot\ddot{u}_c}{2}\right)$.

From Eq. (11) and Eq. (10) we have that:

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{\partial^2 u_\varphi}{R\partial\varphi\partial r} = \alpha\rho\frac{\partial^2 u_r}{\partial t^2} + \alpha f(r) \tag{20}$$

In view of the above, we can write that:

$$u_r = \left(u_c + \sum_{i=1}^{N} A_i\left(t\right)\cos\frac{\pi}{h}ir \right)\cos\phi \tag{21}$$

Substituting Eq. (21) into Eq. (20) and then multiplying on $\cos\left(\frac{\pi}{h}ir\right)\cos\left(\varphi\right)$ and integration by z (vertical axis, height of interaction is equal to *H*), φ and r we obtain the equation for $A_i\left(t\right)$:

$$A_i''(t) + \frac{1}{\alpha\rho}\frac{\pi}{h}i\left(\frac{\pi}{h}i - \frac{1}{R}\right)A_i\left(t\right) = \frac{2}{\rho hHR}\left(F_1 - F_2\right) \tag{22}$$

Note, that in Eq. (22) $i = 2k+1,\ k = 1,2,...$, because for odd values of i the right side of Eq. (22) with taking into account Eq. (19) is equal to 0. Similarly for dynamic model in vertical direction here we assume, that for small periods of time strength F_1 and F_2 vary linearly. Then Eq. (22) can be rewritten as:

$$A_i''(t) + \lambda_i^2 A(t) = C + Dt \text{ and } \lambda_i^2 = \frac{1}{\alpha\rho}\frac{\pi}{h}i\left(\frac{\pi}{h}i - \frac{1}{R}\right) \tag{23}$$

Solution of Eq. (23) is represented as:

$$A_i\left(t\right) = A_{i1}\cos\lambda_i t + A_{i2}\sin\lambda_i t + \frac{C}{\lambda_i^2} + \frac{D}{\lambda_i^2}t \tag{24}$$

Unknown coefficients are founded from the conditions:

$$A_i\left(t=0\right) = A_i^0;\ A_i'\left(t=0\right) = A_i'^0\ . \tag{25}$$

Finally, we have:

$$A_i\left(t\right) = A_i^0\cos\lambda_i t + A_i'^0\sin\lambda_i t + \frac{C}{\lambda_i^2}\left(1 - \cos\lambda_i t\right) + \frac{D}{\lambda_i^2}\left(t - \frac{\sin\lambda_i t}{\lambda_i}\right). \tag{26}$$

Thus, we can write relations between forces F_1, F_1 and displacements of coolant:

$$w_1 = u_L = u_r\left(r=0\right) = \left(u_c + \sum_{i=1}^{N} A_i\left(t\right)\right)\cos\phi \tag{27}$$

$$w_2 = u_R = u_r\left(r=h\right) = \left(u_c - \sum_{i=1}^{N} A_i\left(t\right)\right)\cos\phi \tag{28}$$

Besides equilibrium equations for CB and RPV are given by:

$$-m_{RPV}\ddot{w}_{RPV} - R_{SupportElm\_RPV} + R_{SupportElm\_CB} - F_1 = 0 \tag{29}$$

$$-m_{CB}\ddot{w}_{CB} - R_{SupportElm\_CB} - R_{RC} + R_{SupportElm\_BGT} + F_2 = 0 \tag{30}$$

Now, we can formulate the basic steps of the algorithm of the dynamic model solution which accounts for the influence of the coolant vibration.

Firstly, time increment Δt from the condition $\Delta t = \min_i\left(\pi/20\lambda_i\right)$ is chosen. Then, the relations between displacements and forces by using equations Eq. (27)-(28) and Eq. (19) are founded. Forces for the next time step are calculated by Eq. (29) and (30).

RESPONCE OF THE MODEL

Natural frequencies

The practical application of the model consist in obtaining seismic response for a reactor as a whole system, and to evaluate stresses in both the reactor supports and internals supports. Moreover, it gives the possibility to analyze the elements collisions.

Using the Lagrange equation of motion (1), we can obtain the conservative model, and thus evaluate natural frequencies and mode shapes:

$$\mathbf{M} \cdot \ddot{\mathbf{q}}(t) + \mathbf{K} \cdot \mathbf{q}(t) = 0 \ . \qquad (31)$$

The summary of this frequencies is given in Table 1. In our analysis we neglected the torsional vibrations, because they are useless for seismic and dynamic analysis. We disregarded the small eccentricities in reactor components, thus there no condition for vibration transition.

Table 1: Natural frequencies of the Reactor

Order of eigen-Frequencies	Value, Hz	Description
1, 2	2.8	Lateral vibrations of the Reactor Core, 1st mode
3, 4	2.98	Lateral of the Control and Protective system drive assembly, 1st mode
5, 6	3.1	Lateral of the Upper Block, 1st mode
7	6.94	Vertical of the Control and Protective system
8, 9	10.8	Lateral vibrations of the Reactor Core, 2nd mode
10, 11	13.0	Lateral of UB rods, 1st mode
12, 13	15.1	Lateral of UB, 2nd mode

Seismic and dynamic response

Firstly, Reactor model was applied to the seismic analysis, which is of general type, and performed due to the lack of accelerogram by linear-spectral method. The pressure distribution generated by the RCP (Orynyak et al., 2015), is used as an external force for the dynamic model in vibration analysis (Figure 3), which is important for high cyclic fatigue. Another model application is for the severe LOCA accident, using pressure distribution obtained in Dubyk et al. (2018), were the supports reliability is of a great importance, and elements collision (see Figure 54). Only minimum results are presented in Figure 3 and Figure 4, generally our dynamic model allowed us to find forces and moments in supports elements, and what is more important to analyze the possible contact or elements collision.

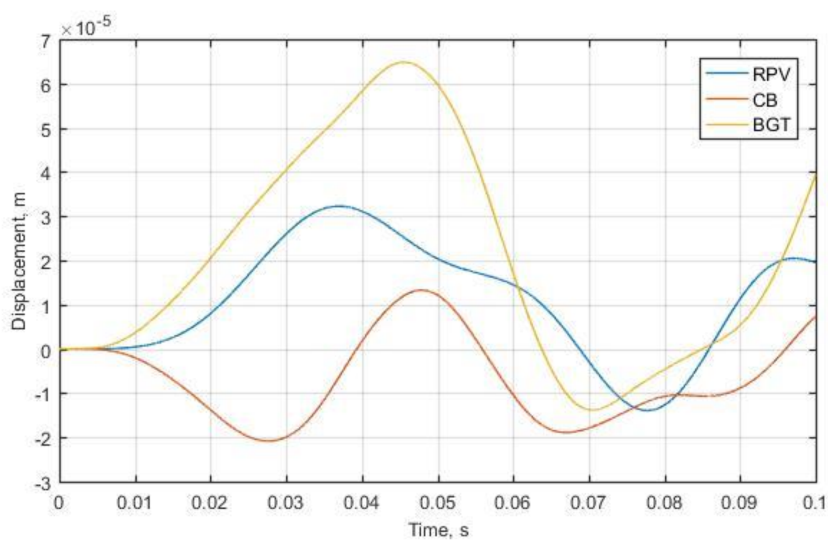

Figure 3. Displacement of model components due to the RCP pulsations

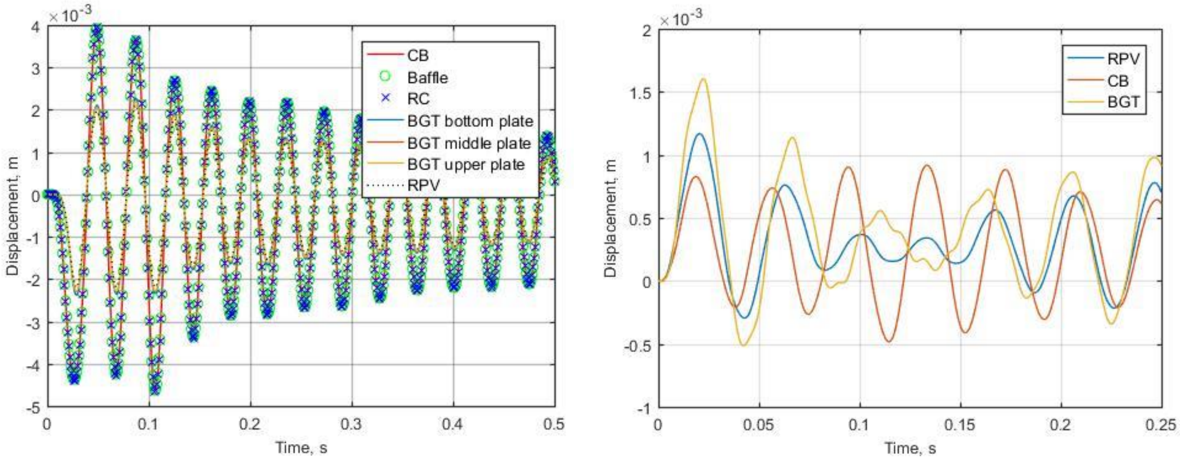

Figure 4. Displacement due to large LOCA accident: right – horizontal displacements, left vertical

FEM MODEL OF THE REACTOR PRESSURE INTERNALS

General description of the model

For further assessment Pressure Vessel Internal (PVI) at the accident and to mitigate computational time for elastic-plastic calculations, we created a beam-shell Finite Element Model. The model includes: Core Barrel (CB) (including a flange, support pipes and a core basket belt), Upper protective tube unit (including three plates, which connected by shells, protective tubes and tubes of system in-Core control). The model also includes elements of the CB for a fixation upper protective unit, reactor elements for a fixed CB, that is elastic tubes CB, separating ring, brackets and keys.

We modelled exactly the perforation in the upper part of CB, the perforation of the bottom of the CB was simplified: the Core Bottom without holes was modelled with a lower Young`s modulus. The same approach we created for a simplify shell and three plates of Upper protective tube unit (Figure 5).

The fuel assemblies and bottom fuel assemblies nozzles are modelled by beam elements and 163 spring elements, which simulating the top fuel assemblies. Upper protective unit tubes are modelled using beam elements too.

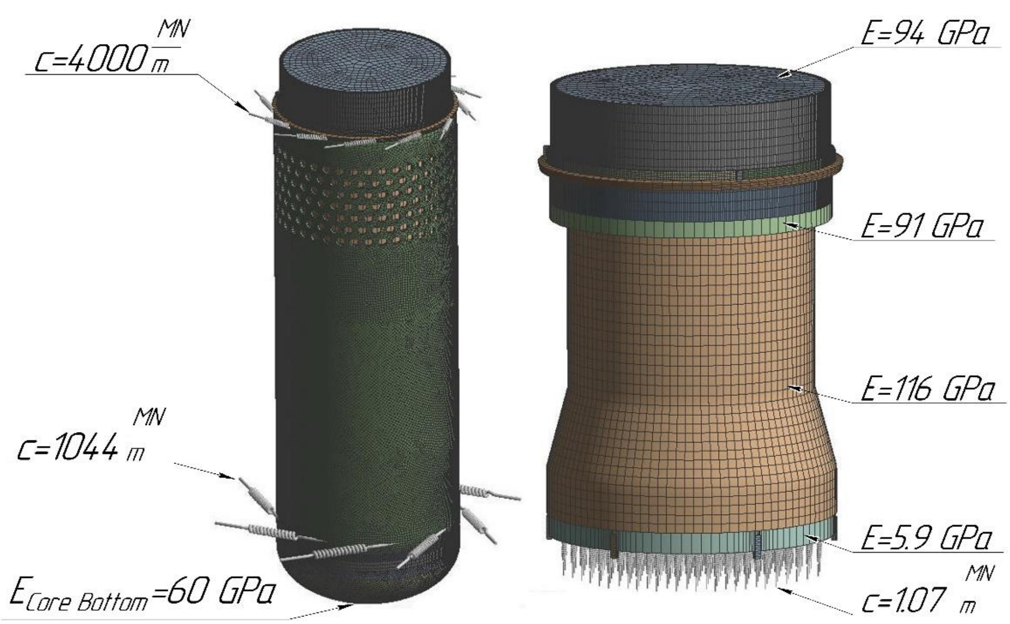

Figure 5. CB and Upper protective unit models; show Young`s modulus and spring stiffness

For fuel assemblies and bottom fuel assemblies nozzles, which we modeled as tubes, the equivalent densities were calculated. Top fuel assemblies modeled as springs, and were preloaded by a vertical force of magnitude 10111 N. The reactor vessel consider as a rigid body. The Core Baffle was replaced by an equivalent mass applied on the core basket belt. The flange of the CB and the top keys of the CB are fixed in vertical direction. The keys and brackets modeled as springs are fixed in the tangential direction.

All elements of the PVI are made of Stainless Steel Type 321 (X6CrNiTi18-10S). For the elastic-plastic calculation CB material had a power hardening law. The full FEA-model consist of 57116 shell elements and 5417 beam elements.

Elastic-plastic analysis of PVI for large LOCA

Load in the model are static and dynamic. Static loadings are vertical preload of fuel assemblies and own weight of the PVI.

To assess the impact of dynamic loads at break of the cold loop of the MCP with the formation a water hammer, a thermal hydraulic calculation was performed using a CFD (Dubyk et al., 2018). The resulting pressure drop over time was applied on the surfaces of the CB, plates and shells upper protective tube unit.

We solved 0.1s of large LOCA with very detailed time step. The resulting displacements and stresses in front of the inlet nozzle, and maximum deformation in the CB, which occurred after the event are shown in Figure 6.

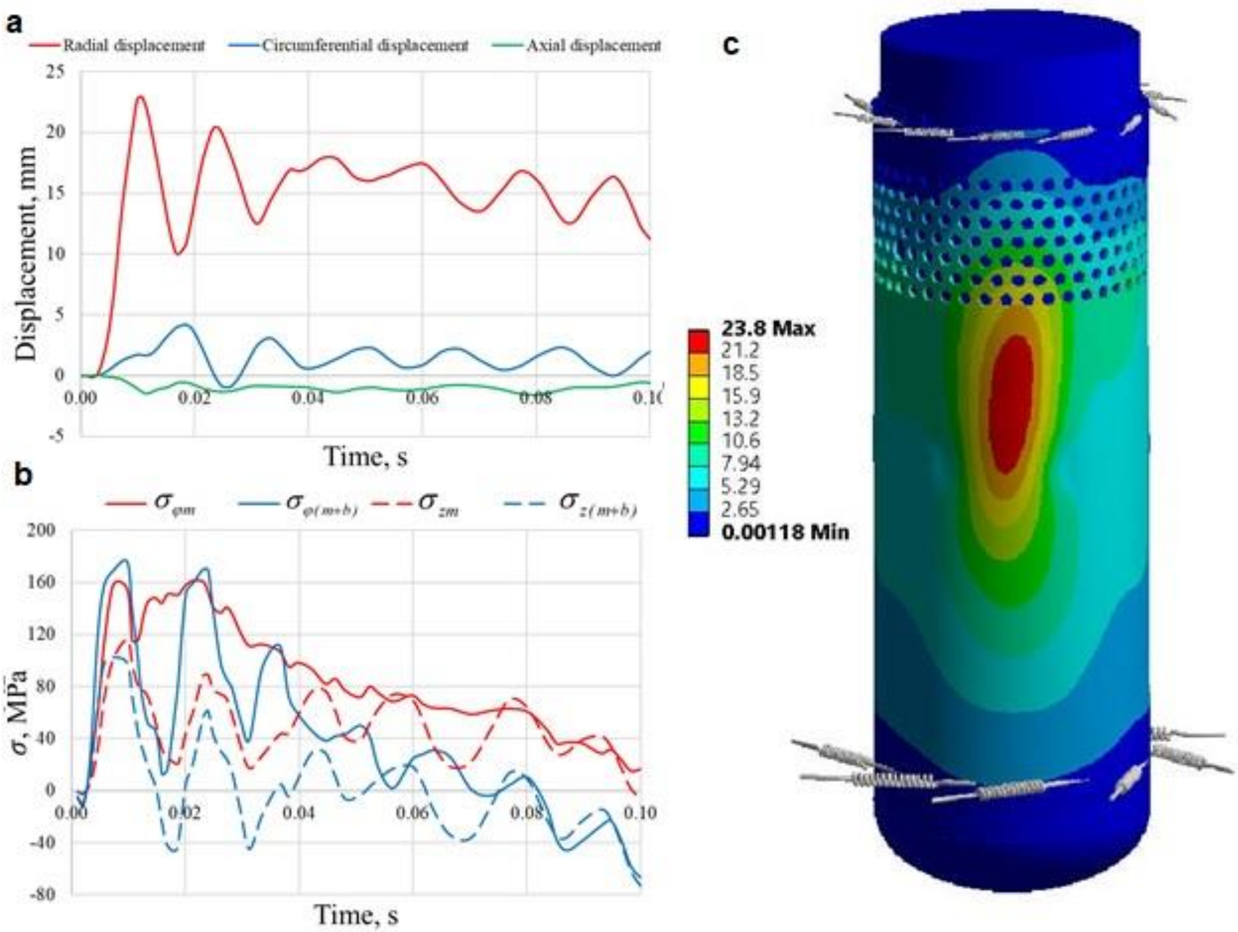

Figure 6. Displacement (a) and stresses (b) in front of the inlet nozzle in time; maximum deformation (mm) on the external surface at 9.786 ms (moment of maximum value in the area of the inlet nozzle) (c)

Stresses and displacements in elastic-plastic are lower compared (Dubyk et al., 2018), that`s why we used conservatively elastic result in fracture calculations. Equivalent plastic strain are not large (3.7%), maximum values are in the area at the down keys of the upper protective tube unit, so the CB and upper protective tube unit can be pulled out after the accident.

CONCLUSION

The dynamic model of RPV of WWER-1000 for seismic, vibration and impact analysis was developed. The peculiarity of the constructed system consists in taking into account the presence of MCP and ECCS by introducing the possible mutual displacements of pipelines and nozzles of reactor in horizontal and vertical planes at different natural frequencies of MCP and ECCS.

The main scientific result is that piping (system of distributed masses) was presented as a system of concentrated masses with flexible springs in different directions. The same approach was used to taking into account the influence of coolant vibrations. The advantages of the models of RPV, piping and coolant are their unified approach for dynamic analysis. So, the developed model can be easily extended (or reduced) by other elements without any additional restrictions.

The advantages of the developed RPV dynamic model is accounting for nonlinear contacts and collision analysis of components. Model is used to assess the water hammer event during LOCA accident, stresses for vibration calculations and seismic impact.

REFERENCES

Dubyk Y., Filonov V., Ishchenko O., Orynyak I. and Filonova Y.(2018) “Dynamic assessment of the Core Barrel during Loss of Coolant Accident,” *Proc., of the ASME 2018 Pressure Vessels and Piping Conference PVP2018-84762*, Prague, Czech Republic, 10p.

Orynyak I., Dubyk I. and Batura A.(2015) “Analysis of dynamic stress in the reactor core barrel due to the pressure pulsations,” *Proc. of the ASME 2015 Pressure Vessels and Piping Conference PVP2015-45291*, Boston, USA, 7 p.

Palamara M., Smith S, Walker A., Basel R. and Meyer G. “Development of an Advanced PWR Reactor Internals System Finite Element Model for Flow-Induced Vibration Analyses,” *Proc. of the ASME 2015 Pressure Vessels and Piping Conference PVP2015-45278*, Boston, USA, 10 p.

Zeman V. and Hlavac Z. (2008) “Dynamic Response of VVER 1000 Type Reactor Excited by Pressure Pulsations,” *J. Engineering Mechanics*, 15(6), 435–446.