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An Analytical Thermal Fatigue crack growth approach

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1 ABSTRACT

This paper presents recent works on the thermal fatigue crack growth approach in structure integrity analysis proposed by nuclear standard codes such as A16 Appendix of RCC-MR. The proposed approach for crack growth is used to study the mechanisms leading to cracking of piping as a result of thermal loading in mixed flow zones. To accurately determine the mechanical response of a complex component such as a pipe or a mixing zone due to thermal loading, it is necessary to be able to take into account in modelling the effects due to the structure, i.e. the effects due to the boundary conditions, the geometry and the additional loadings (pressure). The first part of our proposed approach consists of the description of our analytical model of the thermo-mechanical problem. By using this analytical model, a study to determine the critical range of frequency of the thermal loadings for a particular thermal fatigue problem is done. And the second part treated the used of analytical model to estimate the crack growth under a spectrum temperature signal.

2 INTRODUCTION

The operating conditions in industrial facilities subject structural materials to a wide range of challenges of thermal and mechanical origins, exposing them to the phenomenon of thermal fatigue. Thermal fatigue first became an issue with high-temperature fast-breeder reactors (FBR), then with certain parts of pressurized water reactors (PWR), such the decay heat removal system loop mixing zones.

The design of current power plants is based on construction codes that use simplified rules and uniaxial test fatigue criteria. Many of the complex cases involved are dealt with using simplified methods. For thermal fatigue, use is made of analytical methods and 1D models for evaluating stress variation through wall thickness as a function of surface temperature variation. The incident at Civaux Power Plant showed that such rules could prove inadequate in the presence of complex geometry. But it was clearly demonstrated that thermal fatigue is not properly understood.

In this field, primary role of CEA Laboratory for Structural Integrity and Standards (LISN) is to analyse the initiation and propagation of cracks under thermal loading. This article describes an analytical thermal fatigue crack growth approach. The finality of this approach will be to introducing in the design code like the RCC-MR.

3 BASIS OF THE APPROACH PROPOSED

The approach is based on fracture mechanics and uses the methodology described in RCC-MR, Appendix A16 [RCC-MR]. This appendix is a guide for leak-before-break analysis and defect assessment. A real or postulated crack is studied to determine the corresponding crack growth due to thermal fatigue loads. Certain hypotheses are adopted; the minimum crack length is 10% (with a minimum of 1 mm) of the thickness until 80% of the thickness. The analytical model involves exact solution of a problem corresponding to the conduction of an infinite plate subjected to sinusoidal thermal loading.

3.1 Standard model

Consideration is given to the problem of a part subjected on the inside surface to continuous thermal loading of sinusoidal form with a mean value. Two cases are possible for thermal loading. First, the temperature of the internal wall is known and the analytical model starts at equation (4). Second, the temperature of the fluid is known and it is needed to determine the temperature of the inside wall.

The temperature of the fluid is not transferred to the inside surface in totality. The transfer of energy between fluid and wall is attenuated and characterized by coefficient of exchange h. If there is assumed to be a coefficient of exchange that is constant in time, the transfer function transfer characterizing the energy exchange of the fluid towards the wall can be expressed as follows:

for the attenuation:

(1)

for dephasing:

(2)

with a characteristic thermal diffusivity and results such that: a and λ the thermal conductivity.

In a simplistic way, one can thus determine the temperature transferred to the structure via the energy exchange to the wall by the following relationship:

(3)

Solving the unidimensional conduction problem for the plate makes it possible to obtain the response in temperature over time throughout the plate, such that:

$$T(x,t) = \Delta T_{solid}.e^{i\omega t}A(\omega,x) \qquad (4)$$

Where ω represents the pulsation of the loading and ΔT the temperature amplitude.

Using elastic plate theory, one can determine the expression of the stress in the thickness according to the temperature and by assuming that all of the thermal strain is transformed into mechanical strain (the total deflection is null). In a direct way, one has then the following expression, in the case of a planar strain problem:

$$\sigma(x,t) = -\frac{\alpha E}{1-\nu}\Delta T.e^{i\omega t}A(\omega,x) \qquad (5)$$

Parameters α, E et ν are, respectively, thermal expansion factor, modulus of elasticity and Poisson's ratio. In the case of a planar stress problem, the term $(1-\nu)$ becomes 1.

This model is established by considering that the structure or the plate considered cannot deform freely in the perpendicular direction. It may be necessary to extend this model to two- or three-dimensional structures in order to obtain satisfactory estimates of stress.

3.2 The “improved” analytical model

To accurately determine the mechanical response of a complex component such as a pipe or a mixing zone due to thermal loading, it is necessary to be able to take into account in modelling the effects due to the structure, i.e. the effects due to the boundary conditions, the geometry and the additional loadings (pressure). Figure 1 shows that the mechanical response according to the thickness depends on the geometry. Whereas at the inside wall, the mechanical response is similar in the two cases, the stress profile along the thickness is different and has a substantial impact on the propagation of a crack, for instance. In one case, the crack will be arrested (infinite plate) and the stress will become null. The depth of the crack depends on the frequency of the thermal loading. In the other case, the crack will be continue to grow until it extends all the way through the pipe wall, with considerable membrane stress remaining.

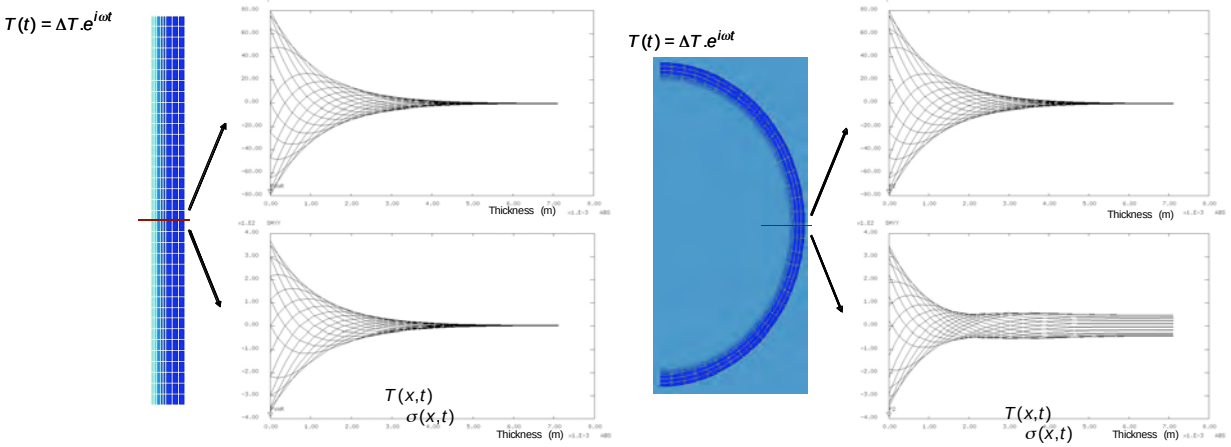

Figure 1. The effect of geometry on mechanical response under the same thermal loading

For infinite plates, many thermomechanical analytical models exist in the literature and have been used with varying degrees of success. However, the industrial problems encountered in practice do not correspond to cases involving infinite plates subjected to fluctuations in temperature.

This being the case, coefficients depending on the geometry and the boundary conditions are being introduced into “improved” analytical modelling. Various authors already used this improvement, notably N. Kasahara [KAS-02].

This model is established by considering that the structure or the plate in question cannot deform freely in the perpendicular direction. If it is desired to be able to extend this model to two- or three-dimensional structures in order to obtain a satisfactory estimate of the stresses, it is necessary to introduce terms of correction. These terms of correction must make it possible to take into account the influence of the boundary conditions on the distribution of the stress through the thickness. One of the ideas selected consists in breaking up the temperature into different terms, such as “membrane temperature” and “bending”. This division is similar to that employed to treat stresses in the methodologies described in Appendix A16 of RCC-MR [RCC-MR].

[redacted] *(6)*

The definitions of the terms “membrane temperature” and "bending" are:

[redacted] *(7)*

A complete expression of the stress can then be written as follows:

$$\sigma_{11} = \sigma_{22} = \frac{\alpha E}{1-\nu}\left[-T(x) + (1-Am)\frac{1}{h}\int_0^h T(x)dx + (1-Ab)\frac{12}{h^3}\left(x-\frac{h}{2}\right)\int_0^h\left(x-\frac{h}{2}\right)dx\right] \quad (8)$$

Four types of mechanical boundary conditions are treated in order to highlight the various correction terms introduced in the stress expression. The plate is assumed to be rigidly held at the bottom. The boundary conditions relate to the higher face (cf figure 2), such as:

• Type a: Rigidly held at the top

• Type b: Blocking of rotation, only perpendicular displacement being allowed,

• Type c: The top is perfectly rigid and the medium point is constrained along the perpendicular direction,

• Type d: The top is perfectly rigid.

Figure 1. Mechanical boundary conditions of the plate

A mechanical interpretation of coefficients Am and Ab can enable us to understand their effect on mechanical response, as:

• Am is directly related to the presence of membrane stress in the structure, when Am = 0, the membrane stress is null, true when $0 < Am \leq 1$.

• Ab is directly related to the presence of bending stress in the structure, when Ab = 0, the bending stress is null, true when $0 < Ab \leq 1$.

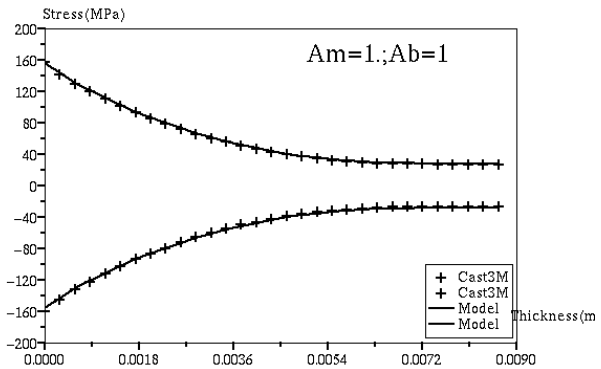

Figure 2. Stress profile envelop for Type a boundary conditions

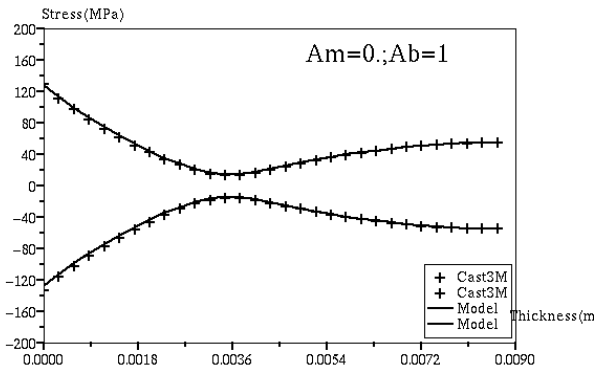

Figure 3. Stress profile envelop for Type b boundary conditions

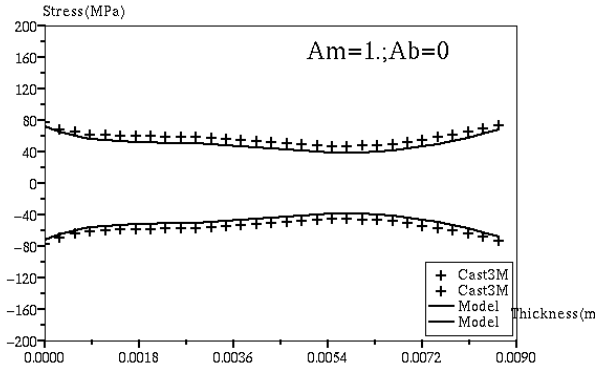

Figure 4. Stress profile envelop for Type c boundary conditions

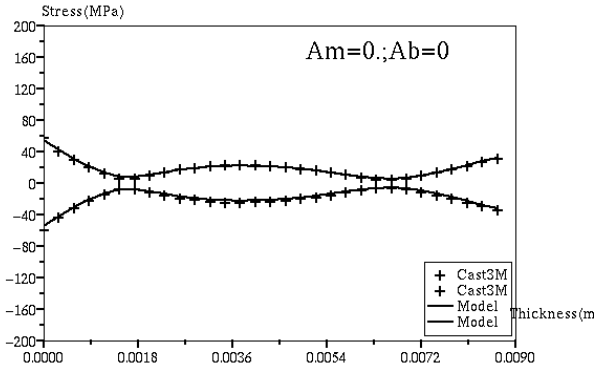

Figure 5. Stress profile envelop for Type d boundary conditions

Parametric studies relating to generic geometries (straight sections of pipes) confirm the mechanical interpretation of coefficients Am and Ab. These studies highlight the independence of these coefficients compared to loading fluctuation, making it possible to determine a range of values corresponding to the average loading (depending on the boundary conditions) and the geometry of the structure.

3.3 Crack Growth calculation

The purpose of this step is to estimate the geometry of the defect at instant tf given its geometry at instant to and the number of occurrences of each cycle. The contribution of any phenomena other than fatigue is disregarded. Determination of the crack propagation rate or da/dN requires determination of the stress intensity factor for the geometry of the defect in question.

3.3.1 Stress intensity factor K calculation

The stress intensity factor (K_I) is determined via the influencing functions on the basis of the size of the defect and the stresses associated with the situation under analysis. The stress intensity factor formulas (K_I) use coefficients of influence i_0, i_1, i_2, i_3 and i_4, which are applied the normal nominal stress in the plane of the postulated defect, along the segment of support describing the section analyzed .

The nominal stress normal to the plane of the postulated defect is expressed in polynomial form according to the position along the segment of support. The polynomial is expressed in terms of the reduced variable (u/L), where L represents the length of field of smoothing which can be used to restrict the size of the defect to achieve better precision ($a \leq L \leq h$) with variable u in this field being $0 \leq u \leq L$.

$$\sigma\left(\frac{u}{L}\right) = \sigma_o + \sigma_1 \cdot \left(\frac{u}{L}\right) + \sigma_2 \cdot \left(\frac{u}{L}\right)^2 + \sigma_3 \cdot \left(\frac{u}{L}\right)^3 + \sigma_4 \cdot \left(\frac{u}{L}\right)^4 \qquad (9)$$

$$K_I = \left[\sigma_0.i_0 + \sigma_1.i_1.\left(\frac{a}{L}\right) + \sigma_2.i_2.\left(\frac{a}{L}\right)^2 + \sigma_3.i_3.\left(\frac{a}{L}\right)^3 + \sigma_4.i_4.\left(\frac{a}{L}\right)^4\right].\sqrt{\pi.a} \qquad (10)$$

"The values of the coefficients of influence are calculated for the specimen geometry and defect geometry indicated in [RCC-MR].

The values , σ_0, ..., σ_4 are obtained by a smoothing polynomial of the Lagrange type at every stress solution instant through the thickness.

3.3.2 Crack propagation rate calculation

The procedure consists in determining crack propagation rate or da/dN . This requires determination of the variation of the effective value of the stress intensity factor from the stress intensity factor perpendicular to

the thickness (K_I) and which takes account either the effect of closing (R<0) or of the effect of mean stress (R>0), the stress intensity factor for the geometry of the treated defect, such that:

The procedure is the following:

• for each cycle of Type i:

o determine the range ΔK_I during the cycle

o calculate the effective stress intensity factor range ΔK_{eff} accounting for plasticity and crack closure:

$$\Delta K_{eff} = q \cdot \sqrt{E^* \cdot \Delta J_s} \qquad (11)$$

where

R = load ratio

q = closure (R<0) and mean stress (R>0) coefficient, the value of q depending on parameter $R = \frac{K_{I\min}}{K_{I\max}}$ such that

$$\text{if } R<0, \text{ then } q = \left(\frac{1-0{,}5R}{1-R}\right) \quad \text{and if } R>0, \text{ then } q = \left(\frac{1}{1-0{,}5R}\right) \qquad (12)$$

• depending upon the number of occurrences n_{ir}, the propagation of the defect $(da)_i$ is determined using the propagation law for the material specified:

$$\frac{da}{dN} = C \cdot [\Delta K_{eff}]^n \quad \text{then} \quad \delta a_i = \sum_{N=0}^{n_{ir}} C \cdot [\Delta K_{eff}]^n \qquad (13)$$

• the size of the final defect at instant t_f is obtained by adding the propagation corresponding to the different types of cycles specified.

At the end of this procedure, the final length and the final height of the defect at instant tf are thus determined and define the size of defect on which the analysis of fast rupture is to be performed.

3.4 Case of spectrum of random fluctuations

Development of the thermomechanical analytical model from sinusoidal cyclic thermal loading makes it possible to envisage treating random temperature signals by Fourrier series breakdown. From a random signal x(n) discretized by N points on the inside wall of the structure, k different harmonics can be determined such that:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk} \qquad (14)$$

Then, temperature Ts(x,t) and stress σ(x,t) in the thickness are determined by the Fourrier recombination, such that:

$$Ts(x,t) = \sum_{K=0}^{\tilde{N}-1} Xn(k) e^{\left(j\frac{2\pi}{N}nk+\Theta n\right)} A(\omega_k, X)$$

$$\sigma(x,t) = \sum_{K=0}^{\tilde{N}-1} Xn(k) e^{\left(j\frac{2\pi}{N}nk+\Theta n\right)} G(\omega_k, X, A_m, A_b) \qquad (15)$$

The terms Θn, Xn and ωk are specified, according to the order of the harmonics:

$$\Theta n = \arg(Xn)$$
$$Xn = |Xn|$$
$$\omega_k = \frac{2\pi k}{N} \qquad (16)$$

The Fourrier breakdown, by a procedure involving the use of calculation FFT, gives N harmonics in a fast and conventional way. For the determination of the thermomechanical answer, it is possible to limit the number of harmonics to be treated according to the nature of the signal, thus one a : $\tilde{N} \leq N$.

To determine the crack propagation rate, the standard RAINFLOW procedure for cycle counting is used [A03-406]. This counting procedure gives the different ΔKeff values greater than ΔKlimit. Each ΔKeff value considered gives a corresponding crack propagation rate, and the final crack propagation rate for the sequence is obtained by summing all of them.

4 APPLICATION TO THE CIVAUX CASE

4.1 Specify the critical range of frequency of the thermal loading

The goal of the frequential study is to determine what frequency range should be taken into account in studying the major mechanisms involved in the rapid propagation of cracks in the mixing zone. At high frequencies, temperature penetration is low as the conduction phenomenon filters out the temperature variations and they remain localized at the surface of the part. The region affected is therefore smaller, and mechanical loading outside the area is too small to cause significant crack propagation. However, at very low frequencies, the penetration of a thermal shock is greater, which gives rise to greater stress variations at a depth in the component. This type of low frequency loading can cause rapid propagation (in terms of number of cycles) of a macroscopic crack.

To determine the critical frequency range, the model was identified and applied it to the case encountered at Civaux case to enable study of propagation of a crack subjected to sinusoidal loading. By investigating a wide range of frequencies extending from quasi static (0.0001 Hz) to high (100 Hz), it is possible to narrow the range to be studied.

For different locations in the mixing tee and for each type of stress, the analytical model could determine how stress varied as a function of the frequency of the load. The results are shown in Figures 7 and 8 for the case of the outside of the bend and for the thick part of the mixing tee.

For the thickness corresponding to the outlet bend (9.3 mm), the peak and bending stress variations were at maxima at 0.04 Hz and 0.4 Hz respectively. And from 10 Hz, the total stress variation thereafter represented only 22% of the maximum value obtained at 0.001 Hz. In addition, the bending and membrane stresses cancelled each other out (see figure 7).

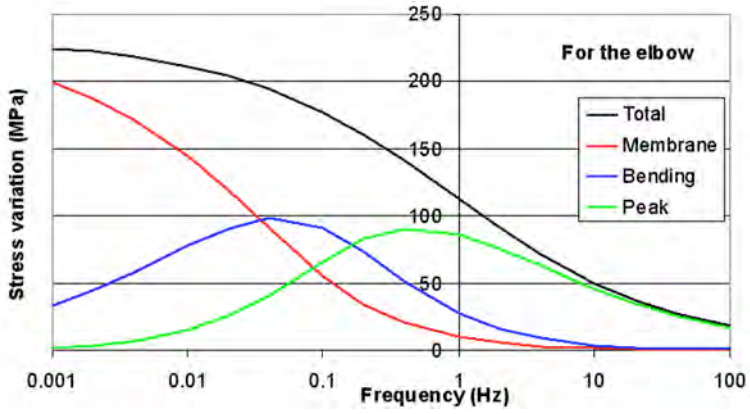

Figure 7. Change in stress variation as a function of frequency at the outside surface of the bend

For the thickness corresponding to the tee, the stress variations or frequencies were at a maxima at 0.004 Hz for bending and at 0.1 Hz for peak stress. Beyond 10 Hz, the total stress variation no longer represented more than 20% of its maximum value (see figure 8).

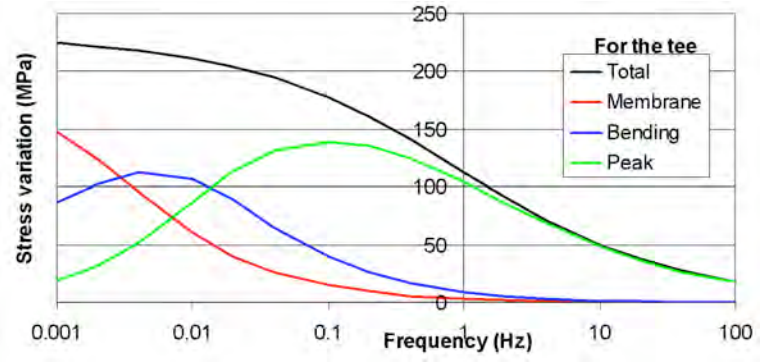

Figure 8. Change in stress variation as a function of frequency in the case of the tee

4.2 Determination of the crack propagation rate for the Civaux 1 case.

The work on this part stage is based on experimental temperature observation with the most realistic smoothing possible between the findings. This thermal loading obviously has little physical reality as it is a mathematical construct, but it offers the advantage of being rapidly set up. We will use the same tools and procedure described on the description of the analytical model. The particularity is the use of spectral temperature fluctuation. Crack propagation rate is determined with the time sequence used, making it possible to estimate the lifespan of the component.

The temperature variation at the inside surface of the Civaux elbow was determined by the thermohydraulic computations described in [CHAP-05]. This is shown in figure 9..

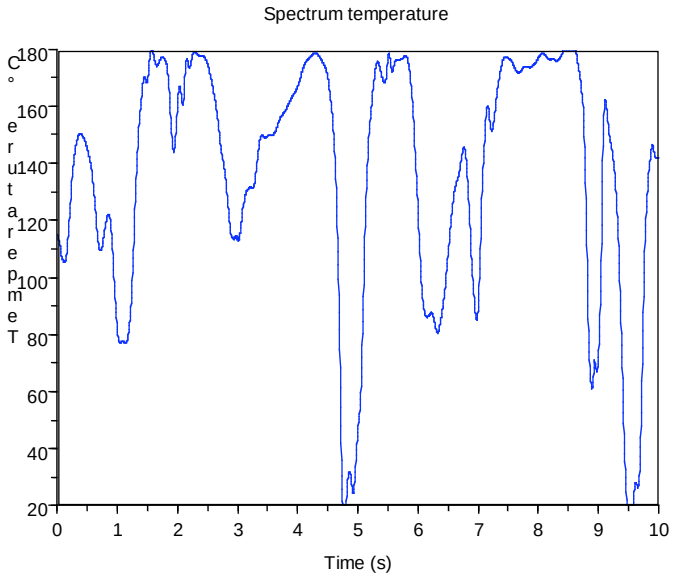

Figure 9. Temperature variation in the Civaux elbow

Crack propagation rate was determined for two kinds of cracks.

• Tube containing a longitudinal part-throughwall infinite internal defect (LDII) (see [RCC-MR])

• Tube containing a circumferential part-throughwall axisymmetrical internal defect (CDAI) (see [RCC-MR])

Figures 10 and 11 show determination of the stress intensity factor. This was calculated with using the analytical model.

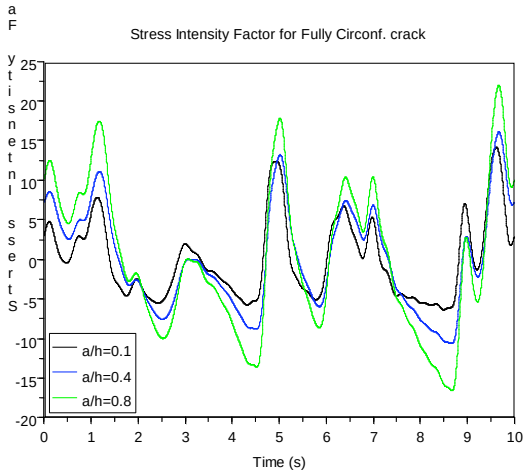

Figure 10. Stress intensity factor for fully circumferential defect at different depths

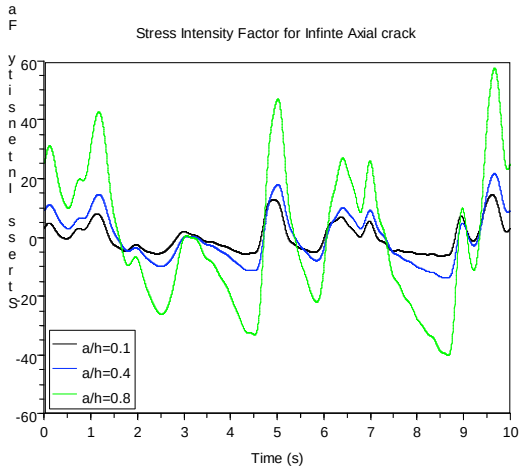

Figure 11. Stress intensity factor for infinite longitudinal defect at different depths

The table summarizes the results obtained with using the analytical model. The number of cycles corresponds to the crack propagation between 10% (0.1) and 80% (0.8) of the thickness. Outside these bounds, the methodology is not accurate enough.

Table 1. Number of cycles to the crack propagation

Type pf defects	LDII	CDAI
Number of total cycles (from 0.1 to 0.8)	24567 Cycles	57 798 Cycles
Time (f=0.1Hz)	68 Hours	160 Hours

5 CONCLUSIONS OF THE APPLICATION WITH THE ANALYTICAL THERMAL FATIGUE CRACK GROWTH APPROACH

Rapid crack propagation necessitates substantial variation of membrane bending and peak stresses. However, it was observed that there was a substantial reduction in the peak stress and null variations of the bending and membrane stresses at frequencies above 10 Hz.

In the light of these results, the frequency of 10 Hz represented an upper bound for variations of the thermal stresses to be analysed to investigate the propagation issue. It was considered that phenomena occurring at high frequencies could not explain the rapid propagation of the cracks observed in the former configuration of the mixing tee.

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