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COLLAPSE LOADS FOR CRACKED PIPING ELBOWS UNDER INTERNAL PRESSURE AND IN-PLANE MOMENT

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ABSTRACT

Elbow is a type of components widely used in a piping system, and is particularly important from the point of view of structural behaviour. In practice, the existence of cracks in excess of the defect limits can not be excluded. So it is very important to know the effect of cracks on the collapse loads of elbows for integrity assessment of the piping system. The existing closed-form limit loads for elbows with meridional defects are either too conservative or inadequate, therefore, the present studies focus on plastic limit loads of elbows with both part-through and fully through axial cracks with the crack configurations are assumed to be constant-depth rectangle, subjected to internal pressure and in-plane opening/closing bending moments respectively. The crack-like defects are postulated to be in extrados, crown or intrados of elbows with different crack sizes. Based on the von-Mises yield crierion, the simplified estimation formulas for plastic limit loads of cracked elbows without effects of tangent straight pipes, are derived by using of theoretical analysis. The limit collapse loads and weakening influences of defects on the plastic load carrying capacity of cracked elbows with the connecting tangent pipes are investigated in detail, by use of three-dimensional (3D) non-linear finite element (FE) analyses, assuming elastic-perfectly-plastic material behavior and taking geometric nonlinearity into account. The results from comprehensive parametric studies indicate that the limit load capacity of cracked elbows may reduce with the increasing of crack length and crack depth, these trends are more serious for long radius elbows. Generally speaking, the thickness-radius ratio t/r_m of elbow have insignificant effects on crack weakening factor (P_L/P_{L0} or M_L/M_{L0}).Base on extensive collapse load numeric data and comprehensive parametric studies, the revised closed form equations for cracked elbows involving effects of tangent pipes, under pressure or in-plane close/open moments, are established respectively. The formula's suitability and feasibility applied in engineering practice are also validated with experimental results available in the open literature in the paper.

Keywords: cracked elbows, limit loads, FE analyses, internal pressure, in-plane moments

1. INTRODUCTION

Elbows are one type of components widely used in various technological applications in conventional and nuclear power generating plants, chemical processing plants and petroleum refineries. They are often the most flexible members in a piping system and hence are forced to accommodate disproportionate displacements arising from differential movements. The resistance to excessive deformation will decrease rapidly the load-carrying capacity and may lead to the malfunction or failure of the system, and hence the piping elbows are particularly important from the point of view of structural behavior. In the piping system, elbows endure complex loads, internal pressure and bending moments are the main types of load. Due to the complicacies of loading and geometric structure, elbows are one of the piping components that are prone to failure in the piping system. Not only at the manufacturing/installation stage, the crack-like defects could be introduced, but also they could develop as a result of continued operation, with load cycling and material deterioration being the major causes for defect initiation and propagation, and can significantly reduce the failure load of piping elbows. So their integrity assessment is a key point in safeguard task of the pressurized piping system. As the materials used in piping system generally have high ductility and are often used in elevated temperature pressure circuits, the failure of piping components is frequently dominated by plastic collapse (ASME,1989). Hence, it is essential to predict how the plastic collapse load of an elbow is affected by cracks. At the same time, in the fracture mechanics analysis, the reference stress approach requires the limit load to define the reference stress and thus to approximately estimate non-linear fracture mechanics parameters, such as the J-integral. Thus the plastic limit load calculation of cracked elbow holds an important position in the integrity assessment technology.

However, the simplified closed form formulas for plastic limit loads of elbow, which can fulfill with the engineering application, have not yet been well established due to its structural and loadings complexity. Up to now, besides the experiments on cracked piping elbows were carried out by Griffiths(1979), Yahiaoui et al(2000) and J.chattopadhyay et al(2001,2003), there is very little experimental database and range of limit load solutions for cracked elbows available in the open published literatures. It is safe to say that closed form equations of plastic limit loads for cracked elbows under internal pressure or in-plane bending moment, are contained in the publications up to the late 1980s by Miller(1988) and Zahoor(1991). Both of these draw heavily on the limited experimental study performed by Griffiths(1979) in the late 1970s. But these limit load equations proposed by Miller(1988) and Zahoor(1991) have limitations: the present studies and Yahiaoui et al's(2002) experimental and analytical research have indicated that these formulas are excessively conservative when cracked elbows subjected to internal pressure or in-plane bending moment, and on occasions, nonapplicable to the cases for which they intended; and they do not differentiate between in-plane opening and closing mode of bending, which the responses of elbows are markedly different under these two different modes of bending moment. So it is very imperative to established closed form plastic collapse load equations for cracked piping elbow based on "fitness-for-use" principia. In order to fully exert the material potential of plastic load-carrying capacity, and obtain the simplified estimation formulas of elbow containing axial crack-like defects, which could be met with the need in engineering practice and be precisely evaluate the limit load of cracked elbow, the present studies focus on the research for plastic limit loads of cracked elbows with part-through surface and through-wall axial cracks, subjected to internal pressure or in-plane bending moments, by use of theoretical analysis based on equilibrium stress fields and the von-Mises yield crierion, and three-dimensional (3D) non-linear finite element (FE) analyses using elastic-perfectly-plastic material behavior. The crack-like defects are postulated to be located respectively in extrados, crown and intrados of elbows with different crack sizes. In order to convenient for the studies, the crack configurations are assumed to be constant-depth rectangle section for part-through and fully through axial cracks, as shown in Fig.1 and 2.

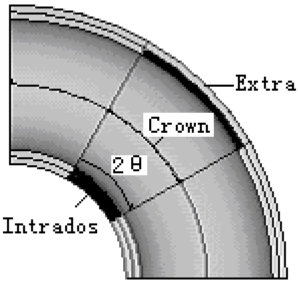


Fig.1 the sketch of cracked elbow (used in theoretical analysis)

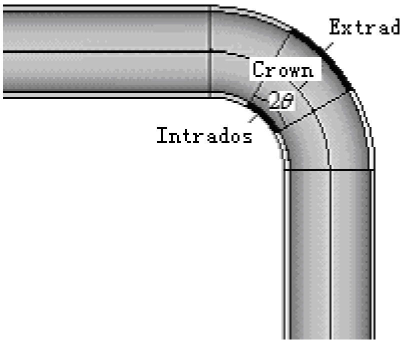


Fig.2 the sketch of cracked elbow (used in FE analysis)

2. LIMIT LOAD OF CRACKED ELBOW WITHOUT CONNECTING STRAIGHT PIPES

To emphasize the main factors and facilitate the limit load analysis, the structure of piping elbow and its materials are simplified as following: (1) neglecting the ellipticity of thin-wall elbow and machining error and residual stresses. (2) An elastic-perfectly-plastic material is specified. In order to close to the work-hardening capacity of real materials, adjustments are made by replacing the yield strengthby the flow strength σ_f of the material. The flow strength is generally accepted to be the mean value of the yield strength σ_y and ultimate strength σ_u, obtained in a conventional tensile test. (3) It is assumed that no elliptic deformation arise before the collapse failure of elbows under pressure or in-plane moment, i.e. the small deformation assumption. (4) ignoring the constraining effects on limit load of tangent straight pipes attached to elbow, namely, taking 90^0 elbow as a quarter of thin-wall torus shell with uniform wall-thickness.

To evaluate the weakening influences of crack sizes on limit load, the crack weakening factor P_L / P_{L0} or M_L / M_{L0} is defined as the normalized limit loads of cracked elbow by the collapse load of the equivalent defect-free elbow respectively. It should be pointed out that the following formulas developed by theoretical analysis are not strictly theoretical expressions for plastic limit loads because some assumptions and approximations are introduced.

2.1 LIMIT PRESSURE OF CRACKED ELBOW UNDER INTERNAL PRESSURE

As internal pressure is the durative action load on piping elbow, and also is the principal load considered in the elbow strength design stage. Hence, the plastic limit pressure of elbow with axial crack located in intrados, crown or extrados is researched first.

For straight pipe with axial crack under internal pressure, the most commonly used collapse formulation is an empirical expression due to Kiefner et al(1973). This is based on an adaptation of the bulking deformation factor M of pipe deduced by Folias(1965) to account for the effect of hoop stress. The experiments carried out by Guo C.X. (1999) and non-linear FE analyses indicate that the bulking deformations of axial cracked elbow with various degree at the crack site are found at the plastic limit pressure, Hence, by using of the Folias' bulking deformation factor, the limit pressure and crack weakening factor empirical equations are presented, based on the von-Mises yield crierion.

The limit pressure expression of defect-free elbow without the tangent straight pipes established by Guo C.X.(1999) is adopted here. This expression is deduces based on von-Mises yield and extended net section plastic collapse (NSC) criterion, and can be written as bellow.

$$P_{L0} = 2\sigma_f t \Big/ \left(r_m \sqrt{(R/(R-r_m))^2 + R/(R-r_m) + 1}\right) \tag{1}$$

Where, R, r_m and t are the mean bend radius, mean cross-sectional radius and wall thickness, respectively.P_{L0} is the limit load of defec-free elbow without the effect of tangent straight pipes.

The Folias' bulking deformation factor M is calculated by the following equation.

$$M = \sqrt{1 + 1.61 c^2 / r_m t} \tag{2}$$

Where, c is half crack length.

2.1.1 LIMIT PRESSURE OF ELBOW WITH CRACK LOACTED IN INTRADOS

axial crack reduce the intensity of elbow and the stiffness of the crack net section, and thus the section containing crack become the weakest part of the cracked elbow. In fact, most elbows failure in practice are due to the plastic hinge formed along the feeblest site at which displacements become unbounded. Accordingly, based on the von-Mises yield and extended net section plastic collapse (NSC) criterions, the plastic failure of cracked elbow can be specified as the Mises stress intensity in the axial section containing lognitudinal cracks at intrados of elbow equivalent to the flow stress and a plastic hinge has been formed along the crack net section. Simultaneity, the Mises stress intensity at the crack net section needs to satisfy the yield condition and force equilibrium, and thus the Mises limit load solution can be easily developed based on equilibrium stress fields.

The final result for elbow with through-wall axial crack at intrados is approximated by Eq. (3).

$$P_L / P_{L0} = 1/M \tag{3}$$

Where, M is the bulking deformation factor of pipe, calculated from the Eq. (2), P_L is the limit pressure of elbow with axial through-wall crack at intrados.

For the surface axial crack located at intrados, the limit pressure approximate solution can be written:

$$P_L / P_{L0} = 1 - a/t + a \Big/ \left(t\sqrt{1 + 1.61 c^2 / (r_m a)}\right) \tag{4}$$

Where, a is the crack depth.

2.1.2 LIMIT PRESSURE OF ELBOW WITH CRACK LOACTED IN CROWN

When the elbow without defect is in elastic stress state, the elastic hoop normal stress on the axial section at intrados is highest, whenas the stress state at crown of elbow is independent of the bend radius R, and is equal to the equivalent straight pipe. With the increasing pressure, the axial section at intrados is the earliest starting yield area and occurring plastic collapse failure. However, the case is diverse when the axial crack is at the crown of elbow. Due to the presence of crack, the stress field in crown of elbow without crack is destroyed and the stress concentration region at the crack tip is formed. With the crack length increasing, the degree of stress concentration becomes more and more great. As the crack length come to a critical value, the stress intensity of crack net-section just equal to the von-Mises stress in the intrados, and when the crack length exceeds the crack threshold value, with the internal pressure increasing, the Mises stress in crack net-section at crown reaches the yield strength first, and then come to the plastic collapse failure. Thus it can be seen that only the length of crack at crown surpass the crack length critical value, the plastic load-carrying capacity of elbow with axial crack at crown is weaken by axial crack.

Based on the von-Mises yield and extended net section plastic collapse (NSC) criterions, by using stress equilibrium condition, the approximate limit pressure of elbow with axial surface and through wall crack at crown can be established.

For through wall axial crack:

$$\frac{P_L}{P_{L0}} = \begin{cases} \sqrt{[(R/(R-r_m))^2 + R/(R-r_m)+1]/3} \Big/ M & P_L < P_{L0} \\ 1 & P_L \geq P_{L0} \end{cases} \qquad (5)$$

For part-through wall axial crack:

$$\frac{P_L}{P_{L0}} = \begin{cases} (1 - a/t + a\Big/(t\sqrt{1+1.61c^2/(r_m a)}))\sqrt{[(R/(R-r_m))^2 + R/(R-r_m)+1]/3} & P_L < P_{L0} \\ 1 & P_L \geq P_{L0} \end{cases} \qquad (6)$$

2.1.3 LIMIT PRESSURE OF ELBOW WITH CRACK LOACTED IN EXTRADOS

Similar to the axial crack at crown, when the length of crack at extrados exceeds the crack threshold value, the plastic limit pressure of cracked elbow is affected by crack. Otherwise, the crack has no influence on plastic limit load of cracked elbow. Similarly, Based on the von-Mises yield and extended net section plastic collapse (NSC) criterions, the approximate plastic limit pressure of elbow with axial surface and through wall crack at extrados of elbow can be proposed.

For through wall axial:

$$\frac{P_L}{P_{L0}} = \begin{cases} \sqrt{[(R/(R-r_m))^2 + R/(R-r_m)+1]/[(R/(R+r_m))^2 + R/(R+r_m)+1]} \Big/ M & P_L < P_{L0} \\ 1 & P_L \geq P_{L0} \end{cases} \qquad (7)$$

For part-through wall axial crack:

$$\frac{P_L}{P_{L0}} = \begin{cases} (1 - \frac{a}{t} + \frac{a/t}{\sqrt{1+1.61c^2/r_m a}})\sqrt{[(R/(R-r_m))^2 + R/(R-r_m)+1]/[(R/(R+r_m))^2 + R/(R+r_m)+1]} & P_L < P_{L0} \\ 1 & P_L \geq P_{L0} \end{cases} \qquad (8)$$

2.2 LIMIT BENDING MOMENT OF CRACKED ELBOW UNDER IN-PLANE MOMENT

For the thin-wall elbow without crack under in-plane moment, the von-Mises bending stress is the highest in the region of crown compared with the other two locations, Guo C.X.(1999), Xu Z.X.(2003) and Yahiaoui(1996) have confirmed this, by theoretical analysis, non-linear FE analysis and experiment research, respectively. Thus the emphasis of present analysis is centre on the plastic limit in-plane moment of elbow with axial surface and through-wall crack at crown.

Based on the von-Mises yield criterion, the extend NSC criterion is used to analyze the failure of cracked elbow under in-plane moment by assuming a uniform bending-stress distribution with the opposite character at the both sides of neutral axis in the axial net-section containing crack. When the elbow reaches collapse failure under in-plane moment, the net-section stress of elbow equals the flow stress σ_f. From the viewpoint of the effective loaded area, the limit moments of elbow with axial crack are determined.

For through wall axial crack in crown section:

$$M_L / M_{L0} = 1 - \theta / 45 \qquad (9)$$

Where, θ is the half crack angle subtended by the axial crack at the centre of elbow bending radius (deg.), M_{Lo} is the limit in-plane moment of uncracked elbow, which can be evaluated by the closed form equation (10) established by Calladine(1974)

$$M_{Lo} = 0.935 D_m^2 t \sigma_f h^{2/3} \qquad (10)$$

For part-through wall crack:

$$M_L / M_{Lo} = 1 - a\theta/(45t) \qquad (11)$$

Where, a is the crack depth.

The applicability range of the above two equations is : $h \le 0.5$, and $h\ (= Rt/r_m^2)$ is the elbow factor.

From the Eq. (9) and (11), it is shown that with the length and depth of the axial crack increasing, the plastic load-carrying capacity and crack weaken factor of elbow are both decreasing, and the more crack length and depth of crack are, the more depressed the plastic load-carrying capacity and crack weaken factor are.

3 FINITE ELEMENT (FE) ANALYSIS FOR CRACKED ELBOWS WITH TANGENT PIPES

In the pressurized piping system, It is inevitably that the plastic load-carrying capacity of elbows is effected by the constraint of connecting tangent straight pipes, flange and other equipments. The researches of Yahiaoui(2000) and chain(1984) have revealed that the tangent pipes connected with piping elbow strengthen the stiffness and intensity of elbow and thus increase the plastic limit load of elbow. Due to the complexity of the structure, the geometrical non－linear effects of elbow which cause ovalisation of elbow cross-section, should be regard and induce the plastic collapse failure together with the mutual influence of non－linear characteristic of material. It is very hard to consider these complicated factors in theoretical analysis, but using non-linear FE numerical analysis technology can solve these problems easily. Hence, in the present studies, the elastic-plastic finite-element analysis method has been used to evaluate the collapse loads of cracked elbows with connecting tangent straight pipes under internal pressure or in-plane bending moment. There are two modes of in-plane moment: opening and closing. As the responses of elbows are markedly different under these two difference modes of bending moment, consequently, in the present study, both opening and closing bending moments are considered separately. Both geometric and material non-linearity is considered in the analysis, and it has been seen in the course of the non-linear FE analysis that consideration of geometric non-linearity is very important for precise determination of elbow collapse loads. The collapse load obtained from the load vs. displacement plots using the ASME(1986) twice-elastic-slope (TES)criterion. The extensive non-linear FE modals with various sizes of axial cracks (axial crack angle: $2\theta = 0-90$ deg, depth-thickness ratios: $a/t = 0.5-1.0$),different wall thickness ($t/r_m = 0.05-0.2$), and various elbow bend radii($R/r_m = 2{,}3$) have been considered in the analysis. The locations of cracks are postulated at intrados, crown and extrados respectively. Regarding crack shape, constant-depth rectangle section configurations are considered, as shown in Fig.2.At both ends of the elbows, there are connecting straight pipes of the length equal to three times the mean cross-section diameter, thus the influences of tangent pipes on collapse loads can be steady and the effects of boundary conditions applied to the end plane of straight pipes can be eliminated.

3.1 FE MODELS AND ANALYSES TECHNIQUES

The non-linear finite element analysis technique is used to conduct the investigation of collapse loads of elbows with various geometric dimensions of crack and elbow, under internal pressure or in-plane opening/closing mode of bending moment. Both geometric and material nonlinearity are considered in the analysis. The ANSYS (1999, version 5.6) FE analysis package is employed for calculating the limit load of elbow from load-displacement curves by TES method. Since the aim of the present work is to assess limit loads, elastic-perfectly plastic material modal is used in this study, with the other material properties chosen: the Young's modulus $E = 2.1\times10^5 MPa$, Yield stress $\sigma_f = 320MPa$ and Poisson's ratio ν=0.3.Due to symmetry in geometry and loading, symmetry conditions are fully utilized in the FE models to reduce the number of elements and computing time. Half of the modal with crack at intrados or extrados is modeled. While when crack is located at crown, the full modal should be used. The meshes are prepared using pre-processing program of ANSYS: 20 nodes three-dimensional isoparametric solid elements (solid 95) are employed for elbow and 8-nodes isoparametric solid elements (solid 45) for the attached pipes. It is proved that crack tip elements are not deemed necessary for limit load analysis. The meshes are refined in the defect tip section for simulating the local geometry more accurately and obtaining reliable and precise FE results. The meshes are selected as a result of a series of convergence studies, the details of which are omitted for brevity. Fig. 3 shows typical FE meshes for cracked elbow.

Internal pressure is applied as a distributed load to the inner surface of the FE modal, together with an axial tension equivalent to the internal pressure applied at the end plane of the pipe to simulate the closed end and for the crack face, no pressure is applied on it. For bending moment, the load is applied via rotation of a master node attached to the nodes at the end of the tangent pipes. Boundary conditions of the FE modal are designed closing to the working condition of elbow. When elbow subject to internal pressure, the axial displacement at the end plane of the tother tangent pipe are constrained, and the symmetric boundary conditions are applied on the symmetric plane. For bending moment, the end plane of tangent pipe with no bending moment applied is fixed.

Using twice-elastic-slope (TES) method, the collapse loads have been evaluated from the load vs. end displacement curves generated through nonlinear finite element analysis, as shown in Fig.4.

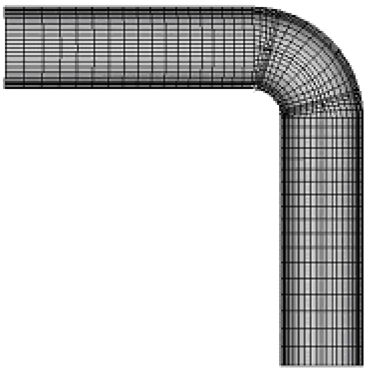


Fig.3 Typical FE mesh of an elbow with crack

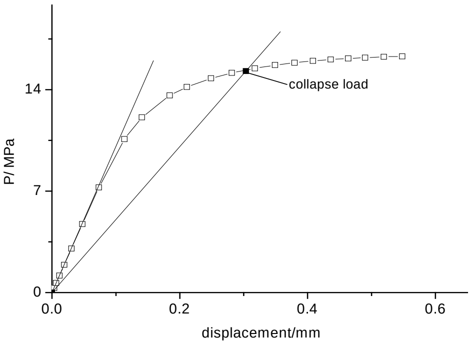


Fig.4 load vs. displacement curve and collapse load

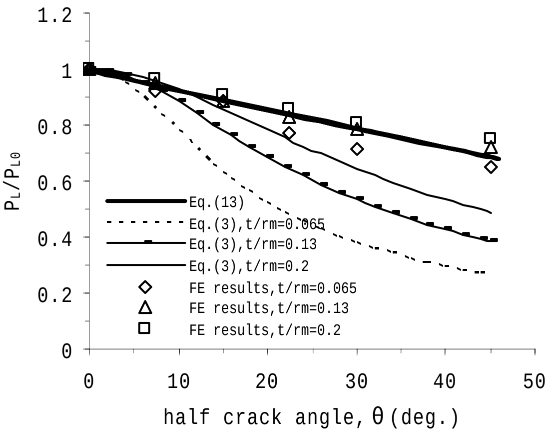
3.2 FE RESULTS AND DISCUSSION

The extensive cracked elbows connecting pipes are studied through elastic plastic finite-element analyses, and by TES method, the collapse limit loads are evaluated from the load vs. displacement plots obtained by elastic-plastic FE analysis. The deformation parameter is employed based on the load type: for internal pressure, the radial displacement of the node at the end plane of pipe is chosen, and for bending moment, the end rotation angle is employed as deformation parameter.

3.2.1 FE RESULTS OF ELBOW UNDER PRESSURE AND DISCUSSION

Figure5-16 show all the normalized collapse pressure data obtained from non-linear FE analysis for cracked and defect free elbows. The weakening factors P_L / P_{L0} due to the presence of axial cracks are quantified respectively by evaluating the ratio of collapse limit pressure of cracked and defect free elbows with or without tangent pipes. The main conclusions from the normalized collapse pressure data are as follows:

1. No matter what sizes of elbows and cracks, and the position of cracks are, the weakening factors are decreasing with the increasing of the crack length and depth; correspondingly, the plastic load-carrying capacities of cracked elbows are also decreasing. The influences of short or shallow cracks ($a/t \le 0.5$) at intrados, crown, and extrados respectively, on collapse limit pressure and weakening factor are very limited, and for shallow surface cracks, no matter how long they are, the reduction of limit pressure is so minor that it could be ignored in engineering practice. Even for the short and deep cracks (e.g., $a/t = 0.8$) or for short through wall cracks, the decrease in loading capacities and weakening factors of the cracked elbows are not yet apparent. But the large and deep axial cracks or large through wall axial cracks can significantly reduce the collapse load of elbows. When the surface and through wall cracks are located at crown or extrados, there are threshold crack angles below which there are no weakening effects on collapse pressure of cracked elbows. The load-carrying capacity of a cracked elbow is decreased on the condition that the crack angle exceed threshold crack angle. For through wall crack, the value for threshold crack angle of long radius elbow is less than that of short radius elbow. 2. For the given size and location of crack, the normalized collapse pressure data demonstrate that the



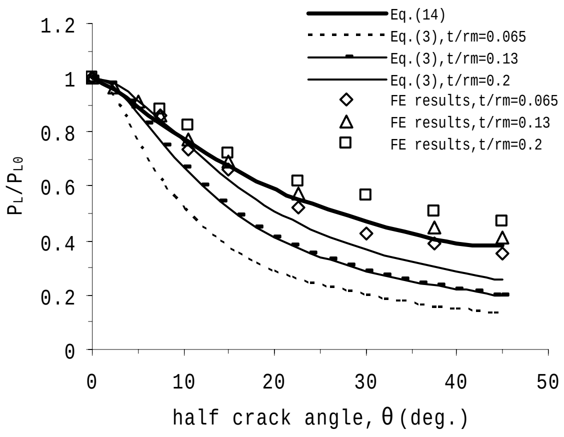


Fig.5 weakening factors (intrados, a/t=1.0,R= Dm) *Fig.6 weakening factors (intrados, a/t=1.0,R=1.5 D_m)*

degree of crack weakening on collapse load is more remarkable for long radius elbow than it is for short radius elbow. 3. The weakening factors are increasing with the higher values of thickness-radius ratio t/r_m of cracked elbows. That is, for the thinner wall elbows, the weakening effects are more obvious than that for thick wall elbows. However, Except for cracks at crown of elbows, the influences of thickness-radius ratio on crack weakening factor are unconspicuous within the range studied.

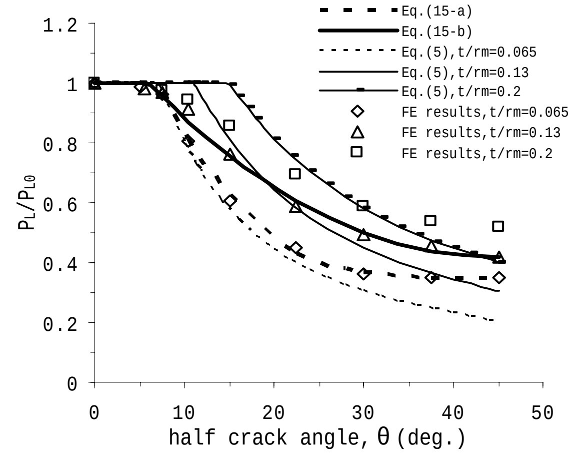


Fig.7 weakening factors (crown, a/t=1.0,R= D_m)

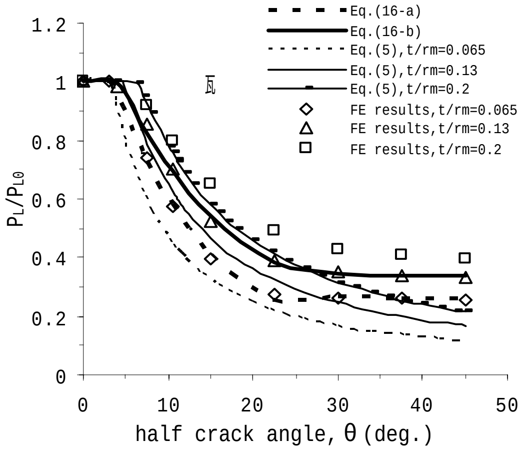


Fig.8 weakening factors (crown, a/t=1.0,R=1.5 D_m)

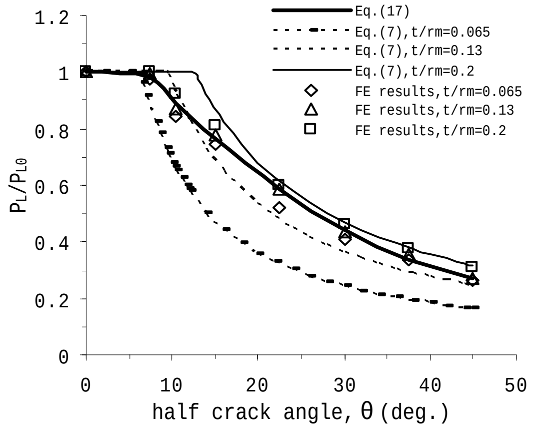


Fig.9 weakening factors (extrados, a/t=1.0,R= D_m)

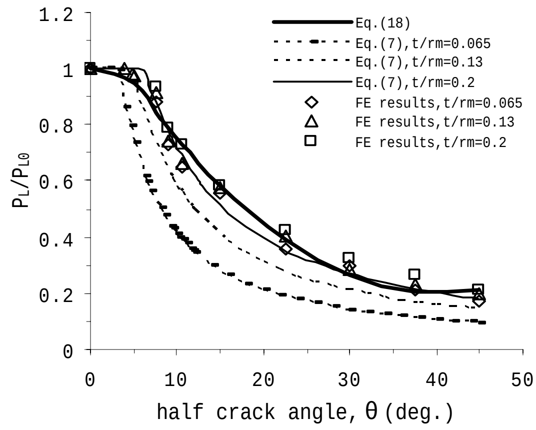


Fig.10 weakening factors (extrados, a/t=1.0,R=1.5 D_m)

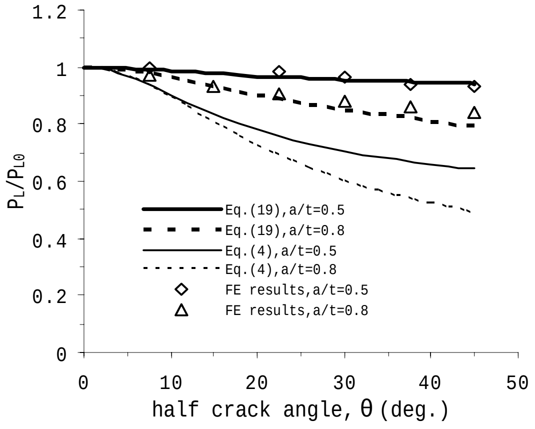


Fig.11 weakening factors (intrados, R= D_m)

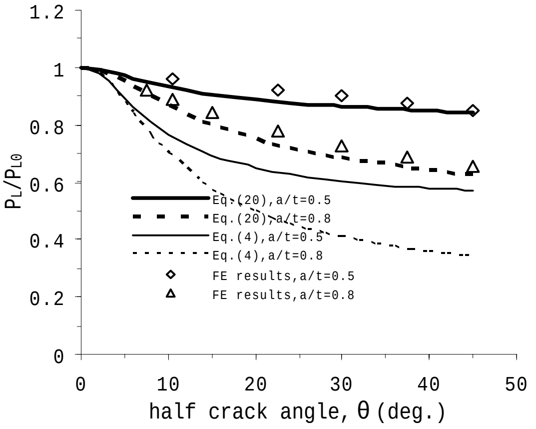


Fig.12 weakening factors (intrados, R=1.5 D_m)

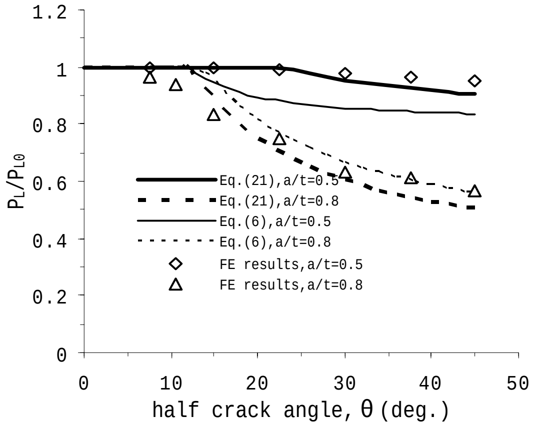


Fig.13 weakening factors (crown, R= D_m)

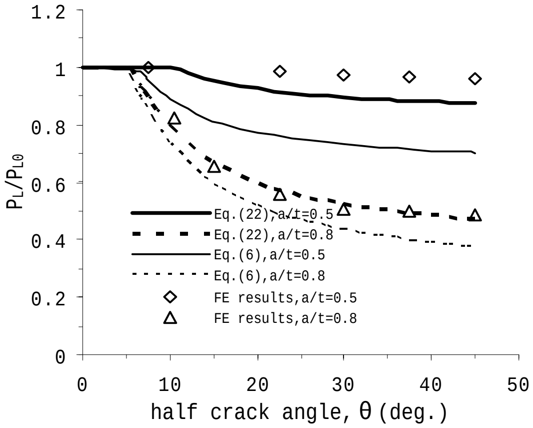


Fig.14 weakening factors (crown, R=1.5 D_m)

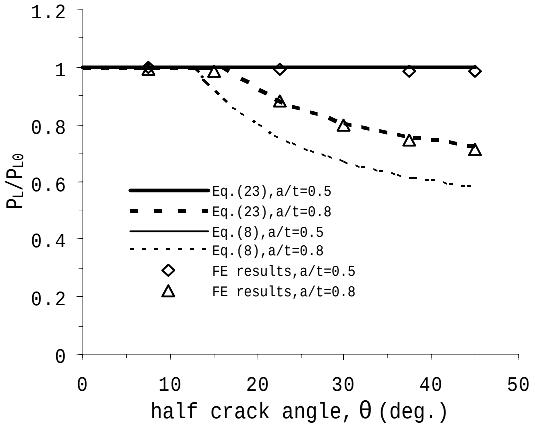


Fig.15 weakening factors (extrados, R=D_m)

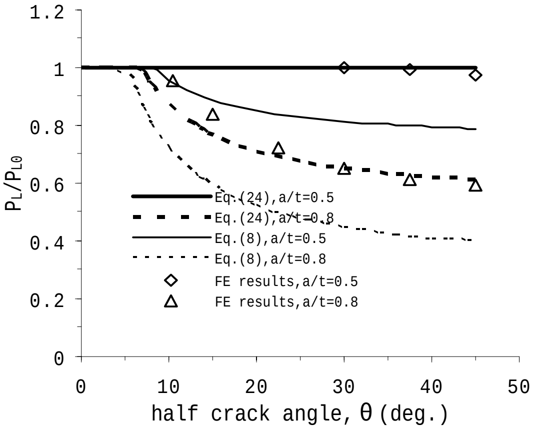


Fig.16 weakening factors (extrados, R=1.5D_m)

The predictions for crack weakening factor of elbows without tangent pipes obtained by theoretical analysis are also shown in Fig.5-16 and compared with the numeric results for crack weakening factor. As a whole, the theoretical predictions are conservative in contrast with FE results and the value of thickness-radius ratio is lower, the degree of conservativeness is greater. The main cause is that the predictions from Eq.3-8 do not include the strengthening effects of straight pipes attached to elbow.

3.2.2 FE RESULTS OF ELBOW UNDER IN-PLANE MOMENT AND DISCUSSION

Using non-linear FE analysis method, the results for collapse limit in-plane opening/closing mode of bending moments of elbows with axial through wall cracks at crown, are obtained and as shown in Fig. 17 and 18. The weakening factors M_L/M_{L0} of axially cracked elbow are defined as normalizing with respect to that of the defect free elbow with and without tangent pipes respectively. The main conclusions of the numerical results can be gotten. An elbow with a certain size of throughwall axial crack subjected to opening mode bend moment has higher collapse load than an elbow with the same size crack subjected to closing moment. This phenomenon is well known associated with the ovalisation of the cross section of the elbow. The ovalisation of the elbow cross section plays an important role in its collapse behaviour, and is more prominent in case of thinner wall elbow. Due to the curvature of the elbow, the axial stress in any fibre will have a component stress deviated from the neutral axis for the case of an opening moment, but directed towards the neutral axis for the case of a closing moment. This, in turn, generates a component of force that tries to ovalize the circular cross-section of the elbow in such a way that increases the diameter across the intrados-extrados of elbow subjected to in-plane opening mode of bending moment, and reduces the diameter across the intrados-extrados of elbow subjected to closing mode bend moment. Thus an elbow subjected to opening mode of bending moment increases the cross-section moment of inertia due to ovalisation and therefore stiffens the elbow, and the case for elbow subjected to closing mode of bending moment is reverse, the cross-section ovalisation weaken the stiffness and load-carrying capacity of elbow. The FE numerical analyses also indicate that for the given crack size, the influences of crack on weakening factor are more serious for long radius than they are for short radius elbow under the same bending mode. However, the influences of the bend radius, thickness radius ratio and the bending mode on crack weakening factor are all not obvious and can be ignored in the engineering practice applications. The solutions obtained from Eq.9 are overly conservative compared with the FE results, and thus indicate the strengthening effects of pipes. The strengthening effects of pipes on collapse moments are higher than the strengthening effects of pipes on collapse pressures.

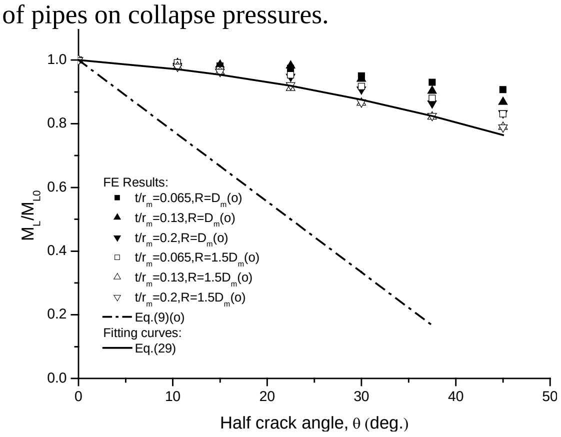


Fig.17 weakening factors (crown, open mode)

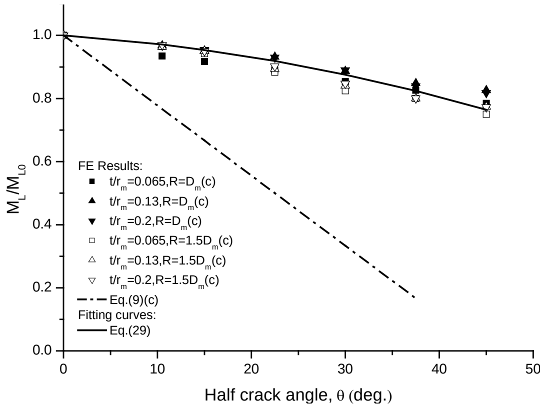


Fig.18 weakening factors (crown, closing mode)

4. DEVEPMOMENT OF CLOSED FORM COLLAPSE LOAD EQUATIONS FOR CRACKED ELBOW WITH TANGENT PIPES

From the above discusses, it is found that the predictions for collapse limit load and crack weakening factor of cracked elbow ignoring the influences of tangent straight pipes, are conservative, and can be used to evaluate the collapse limit loads of cracked elbows conservatively when elbows service in the critical conditions. In order to precise assess the plastic load-carrying capacity of cracked elbow for exerting the load-carrying potential fully, it is necessary to establish the closed form formulas for collapse limit load of elbow with tangent pipes.

4.1 DEVEPMOMENT OF CLOSED FORM COLLAPSE PRESSURE EQUATIONS

For cracked elbow under internal pressure, the influences of tangent pipes attached to elbow are obvious. Hence, based on the non-linear FE analyses results of elbows with axial cracks at intrados/crown/extrados under pressure, the closed form collapse pressure equations are proposed. The solutions from Eq. (13-24) are agreement with results from FE analysis quite well, as shown in Fig.(5-16). The collapse limit pressures of defect free elbows with tangent pipes can be evaluated from Eq.(12), proposed in the present research.

4.1.1 CLOSED FORM COLLAPSE PRESSURE EQUATIONS OF DEFECT FREE ELBOW

Based on the non-linear FE analysis results, the closed form collapse limit pressure of defect free elbow, with considering the strengthening effect of tangent pipes is obtained as bellow:

$$P_{L0} = 0.924\sigma_f t/r_m \qquad (12)$$

The applicability range is: $0.05 \le t/r_m \le 0.20$, $R \le 1.5D_m$ .The comparisons solutions evaluate from Eq.12 with the non-linear FE results and experiment data are made and shown in Fig.19. The solutions from the newly proposed equation 12 are in quite reasonable agreement with results from non-linear FE analysis and experiment data accordingly. The form of Eq.12 is simple and more suitable for engineering using.

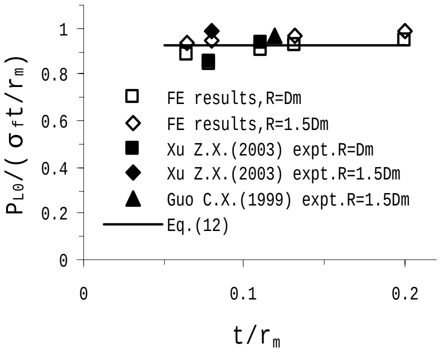


Fig.19 normalized limit pressure without crack

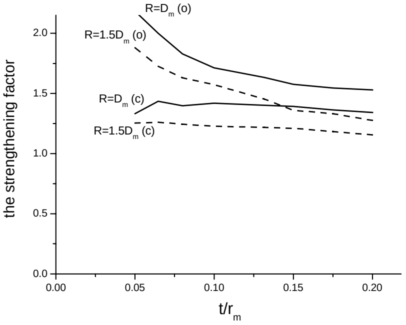


Fig.20 normalized limit moment without crack

4.1.2 CLOSED FORM COLLAPSE LIMIT PRESSURE EQUATIONS WITH THROUGH WALL CRACK

Based on the numerical solutions, the closed form collapse pressure of cracked elbow can be proposed, and in all of the following equations, the limit pressure of defect free elbow can be evaluated form Eq.12.

When the axial through wall crack is at intrados of elbow, the effect of geometric parameter t/r_m is slight on the crack weakening factor and can be ignored. Thus, based on the results of elastic-plastic finite-element analyses, the closed form expressions of cracked elbow connected tangent pipes proposed are as follows.

For short radius elbow (R/ D_m =1):

$$P_L/P_{L0} = -0.0211\theta/\pi + 0.99 \qquad 0^\circ \le 2\theta \le 90^\circ,\ 0.05 \le t/r_m \le 0.2 \tag{13}$$

For long radius elbow (R/ D_m =1.5):

$$P_L/P_{L0} = 0.0029(\theta/\pi)^2 - 0.0862\theta/\pi + 1.023 \qquad 0^\circ \le 2\theta \le 90^\circ, 0.05 \le t/r_m \le 0.2 \tag{14}$$

When the axial through wall crack is at crown of elbow, the geometric parameter t/r_m has certain effect on the crack weakening factor and the weakening factor is the function of the parameter t/r_m and R/D_m . Thus, the closed form expressions of cracked elbow connected tangent pipes can be obtained.

For short radius elbow (R/ D_m =1):

$$\begin{aligned} P_L/P_{L0} &= 1 && 0^\circ \le 2\theta < 10^\circ \\ &= 0.0123(\theta/\pi)^2 - 0.23\theta/\pi + 1.4452 && 10^\circ \le 2\theta \le 60^\circ, \quad 0.05 \le t/r_m \le 0.1 \\ &= 0.35 && 60^\circ < 2\theta \le 90^\circ \end{aligned} \tag{15-a}$$

$$\begin{aligned} P_L/P_{L0} &= 1 && 0^\circ \le 2\theta < 11^\circ \\ &= 0.0039(\theta/\pi)^2 - 0.11\theta/\pi + 1.1956 && 11^\circ \le 2\theta \le 90^\circ \end{aligned}, \quad 0.1 < t/r_m \le 0.2 \tag{15-b}$$

For long radius elbow (R/ D_m =1.5):

$$\begin{aligned} P_L/P_{L0} &= 1 && 0^\circ \le 2\theta < 6^\circ \\ &= 0.0145(\theta/\pi)^2 - 0.2375\theta/\pi + 1.2129 && 6^\circ \le 2\theta \le 60^\circ, \quad 0.05 \le t/r_m \le 0.1 \\ &= 0.26 && 60^\circ < 2\theta \le 90^\circ \end{aligned} \tag{16-a}$$

$$\begin{aligned} P_L/P_{L0} &= 1 && 0^\circ \le 2\theta < 8^\circ \\ &= 0.0107(\theta/\pi)^2 - 0.1945\theta/\pi + 1.2292 && 8^\circ \le 2\theta \le 60^\circ, \quad 0.1 < t/r_m \le 0.2 \\ &= 0.33 && 60^\circ < 2\theta \le 90^\circ \end{aligned} \tag{16-b}$$

When the axial through wall crack is at extrados of elbow, the geometric parameter t/r_m has little effect on the crack weakening factor. Thus, the closed form expressions of cracked elbow connected tangent pipes can be obtained based on non-linear FE results.

For short radius elbow (R/ D_m =1):

$$\begin{aligned} P_L/P_{L0} &= 1 && 0^\circ \le 2\theta < 15^\circ \\ &= 0.0033(\theta/\pi)^2 - 0.1151\theta/\pi + 1.2389 && 15^\circ \le 2\theta \le 90^\circ \end{aligned} \tag{17}$$

For long radius elbow (R/ D_m =1.5):

$$\begin{aligned} P_L/P_{L0} &= 1 && 0^\circ \le 2\theta < 10^\circ \\ &= 0.0059(\theta/\pi)^2 - 0.1514\theta/\pi + 1.1723 && 10^\circ \le 2\theta \le 90^\circ \end{aligned} \tag{18}$$

The applicability range of Eq. (17) and Eq.(18) is: $0.05 \le t/r_m \le 0.20$ .The predictions of all above closed form equations for elbows with intrados/crown/extrados throughwall axial cracks are shown in Fig5-10.

4.1.3 CLOSED FORM COLLAPSE LIMIT PRESSURE EQUATIONS WITH SURFACE CRACK

Based on the non-linear FE analysis, by revising the equations for limit pressure of surface crack elbow without tangent straight pipes, the closed form formulas adaptive for engineering applications are proposed account of strengthening influence of tangent straight pipes.

When the surface cracks locate at intrados of elbow, the approximate expressions for collapse limit pressure of cracked elbow are given.

For short radius elbow (R/ D_m =1):

$$\frac{P_L}{P_{L0}} = 1-(\frac{a}{t})^3 + \frac{(a/t)^3}{\sqrt[4]{1+1.61c^2/r_m a}}, \qquad 0.05 \le t/r_m \le 0.2 \,, \tag{19}$$

For long radius elbow (R/ D_m =1.5):

$$\frac{P_L}{P_{L0}} = 1-(\frac{a}{t})^2 + \frac{(a/t)^2}{\sqrt[4]{1+1.61c^2/r_m a}}, \qquad 0.05 \le t/r_m \le 0.2 \,, \tag{20}$$

When the surface cracks locate at crown of elbow, the approximate expressions for collapse limit pressure of cracked elbow are obtained.

For short radius elbow (R/ D_m =1):

$$\begin{aligned} P_L/P_{L0} &= 1 & P_L \ge P_{L0} \\ &= [1-\frac{a}{t} + \frac{a/t}{[1+1.61c^2/(r_m a)]^{0.575}}]\sqrt{\frac{1}{3}[(R/(R-r_m))^2 + R/(R-r_m)+1]} & P_L < P_{L0} \end{aligned} \tag{21}$$

For long radius elbow (R/ D_m =1.5):

$$\begin{aligned} P_L/P_{L0} &= 1 & P_L \ge P_{L0} \\ &= [1-(\frac{a}{t})^{3/2} + \frac{(a/t)^{3/2}}{\sqrt{1+1.61c^2/(r_m a)}}]\sqrt{\frac{1}{3}[(R/(R-r_m))^2 + R/(R-r_m)+1]} & P_L < P_{L0} \end{aligned} \tag{22}$$

When the surface cracks locate at extrados of elbow, the approximate expressions for collapse limit pressure of cracked elbow are established as bellow.

For short radius elbow (R/ D_m =1):

$$\begin{aligned} P_L/P_{L0} &= 1 & P_L \ge P_{L0} \\ &= [1-(\frac{a}{t})^{3/2} + \frac{(a/t)^{3/2}}{\sqrt{1+1.61c^2/(r_m a)}}]\sqrt{\frac{[(R/(R-r_m))^2 + R/(R-r_m)+1]}{[(R/(R+r_m))^2 + R/(R+r_m)+1]}} & P_L < P_{L0} \end{aligned} \tag{23}$$

For long radius elbow (R/ D_m =1.5):

$$\begin{aligned} P_L/P_{L0} &= 1 & P_L \ge P_{L0} \\ &= [1-(\frac{a}{t})^{2} + \frac{(a/t)^{2}}{\sqrt{1+1.61c^2/(r_m a)}}]\sqrt{\frac{[(R/(R-r_m))^2 + R/(R-r_m)+1]}{[(R/(R+r_m))^2 + R/(R+r_m)+1]}} & P_L < P_{L0} \end{aligned} \tag{24}$$

The applicability range for these equations is: $0.05 \le t/r_m \le 0.20$ .The predictions of all above closed form equations for elbows with intrados/crown/extrados surface axial cracks are shown in Fig11-16.

4.2 DEVEPMOMENT OF CLOSED FORM COLLAPSE MOMENT EQUATIONS

4.2.1 CLOSED FORM COLLAPSE MOMENT EQUATIONS OF DEFECT FREE ELBOW

The plastic limit in-plane opening/closing bending moments of defect free elbow with tangent pipes are normalized with the existing prediction from Eq. (10). As the ratios mainly reflect the influences of tangent straight pipes on collapse limit moments of elbow, and hence are defined as the strengthening factor of tangent pipe, and shown in Fig. 20. The strengthening influences of straight pipes on plastic load-carrying capacity of elbow are obvious under in-plane opening/closing bending moments. In order to satisfy the needs of engineering applications, the closed form collapse limit loads of defect free elbow with tangent pipes are established respectively with the opening and closing bend mode moment.

Opening mode:

For short radius elbow (R/ D_m =1):

$$M_{L0}/4r_m^2 t\sigma_f = 1.9434t/r_m + 0.3352 \tag{25}$$

For long radius elbow (R/ D_m =1.5):

$$M_{L0}/4r_m^2 t\sigma_f = 2.0191t/r_m + 0.4086 \tag{26}$$

Closing mode:

For short radius elbow (R/ D_m =1):

$$M_{L0}/4r_m^2 t\sigma_f = 2.4263t/r_m + 0.1739 \tag{27}$$

For long radius elbow (R/ D_m =1.5):

$$M_{L0}/4r_m^2 t\sigma_f = 2.6135 t/r_m + 0.2213 \quad (28)$$

The applicability range is: $0.05 \le t/r_m \le 0.20$ .Figure 21and 22 show the predictions of these equations, and it may be seen that these equations fit almost exactly with the finite element results.

In order to validate the applicability in the engineering practice of above equations obtained, the comparisons are made with experimental data for defect free elbow obtained by Yahiaoui(2000), J. E. Griffiths(1979) and Xu Z.X(2003) respectively, as shown in Fig.21and 22. The solutions from these newly proposed collapse limit moment equations agree well with the non-linear FE results, and are very close to the experimental data with the relative errors within the range of $\pm 10\%$. Hence these equations can be used to evaluate the collapse limit moment of defect free elbow in engineering practice.

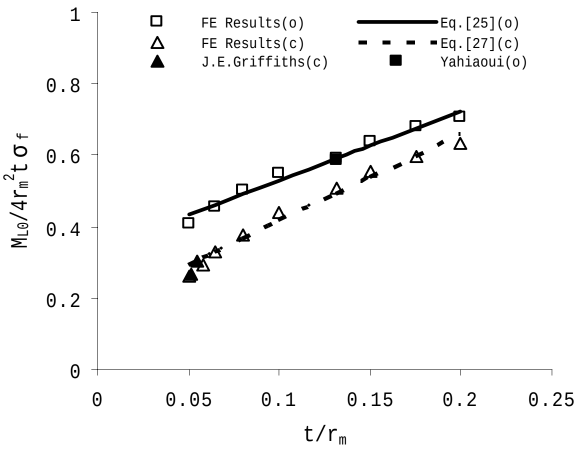


Fig.21 normalized limit moment without crack(R= D_m)

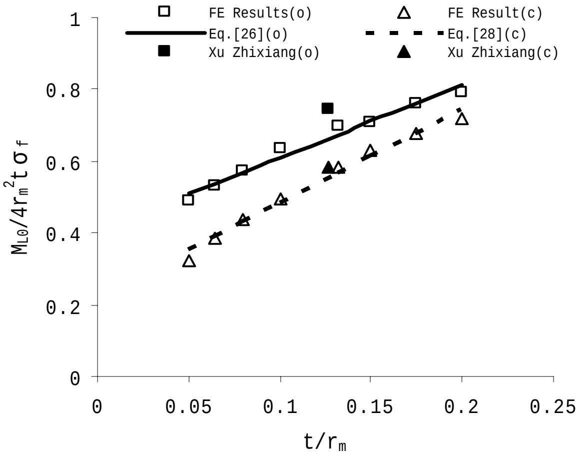


Fig.22 normalized limit moment without crack(R= 1.5D_m)

4.2.2 CLOSED FORM COLLAPSE MOMENT EQUATIONS WITH AXIAL THROUTH WALL CRACK

Based on the collapse limit moments obtained by non-linear FE analysis, the crack weakening factors M_L/M_{L0} without the influences of pipes, are revised by introducing the crack modified factor $\xi(\theta/\pi)$, which is the function of half crack angle, and obtained the collapse limit moment equations with the form $M_L/M_{L0} = 1 - \xi\theta/(45\pi)$, to be easily used in practical situations.

Due to the geometric parameter of elbow t/r_m and R/D_m, the bending modes of in-plane moment have the slight influences on the crack weakening factor and thus can ignore the effects of these parameters to simplify the equation for evaluating the collapse limit moment. Hence, these data from non-linear FE analyses have been reasonably fitted to obtain closed form crack weakening factor M_L/M_{L0} equation conservatively.

$$M_L/M_{L0} = 1 - (0.0322\,\theta/\pi + 0.2784)\theta/(45\pi) \quad (29)$$

The Eq.(25)-(28) can be chosen to evaluate the collapse limit moments of defect free elbows with tangent pipes by the values of t/r_m and R/D_m, and also the bending modes of in-plane moment. Fig. 17 and 18 indicate that the solutions from Eq. 29 are consistent with the FE results.

5. COMPARISON OF SOLUTIOONS FROM EQUATIONS WITH TEST RESULTS

5.1 COMPARISON OF SOLUTIOONS FROM EQUATIONS UNDER PRESSURE

For the short radius elbow (t/r_m) with surface crack, the predictions from the Eq.19, 21 and 23 are compared with the corresponding FE results and solutions based on the equations established by Miller(1988) and Zahoor(1991) respectively, as shown in Fig. 23,24 and 25. Both these solutions based on the equations by Miller(1988) and Zahoor(1991) are overly conservative with respect to the present non-linear FE results and predictions, and for the thinner wall elbows, this trend is more obvious. With the cracks located at crown and extrados of elbow, the suggestions of Miller that ignoring the effects of bend radius on collapse limit pressure are rather conservative, and underestimate heavily the collapse load of cracked elbow under internal pressure.

5.2 COMPARISON OF SOLUTIOON FROM EQUATIONS WITH TEST RESULTS UNDER MOMENT

In order to validate the applicability in the engineering practice of the equation 29, the comparisons are made with experimental data for elbow with axial through wall crack, obtained by Yahiaoui(2001) and J. E.

Griffiths(1979) respectively, as shown in Fig.26. Besides the experiments carried out by Begley (1978), the predictions from Eq. 29 close to the experiment data and are conservative slightly. However, it should be noted that the Begley's(1978) experiment data for long radius elbow differ from the predictions and non-linear FE results newly obtained. It is worth noting that the experiments carried out by Begley (1978) indicate that axial penetrating cracks can substantially reduce the limit moment of long radius elbows, particularly cracks greater than one pipe diameter in length. This is in contrast to the present study which shows that for cracked elbow, the maximum reduction is within 15% in the closing mode of bending, and axial defects do not substantially reduce the plastic moment for both short radius and long radius elbows. As the details about Begley's experiments are not available, the reason of this conflict case is not clear, and the farther study should be done.

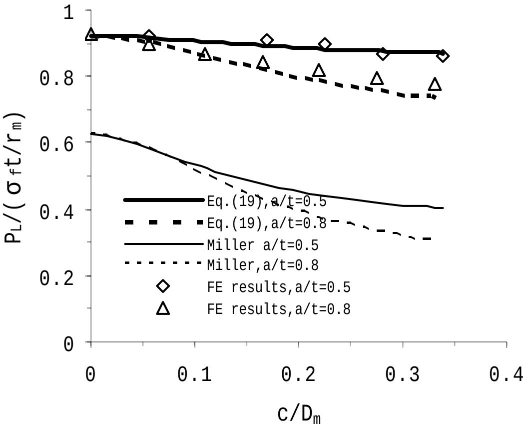


Fig.23 normalized limit pressure with crack(intrados)

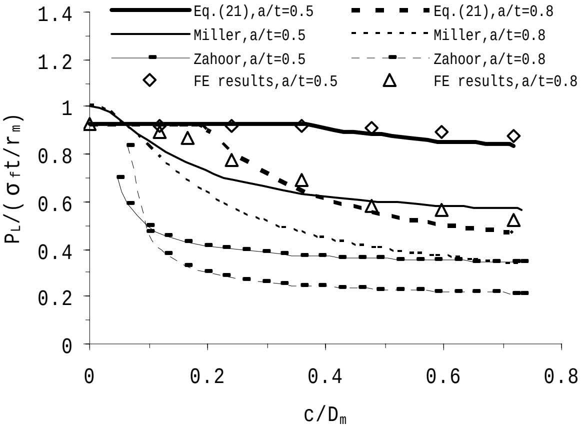


Fig.24 normalized limit pressure with crack(crown)

For the axial throughwall crack located at the crown of elbow, the Miller's(1988) predictions are grossly conservative compared with the experiment data and the solutions of present studies. Because the predictions for long radius elbow proposed by Zahoor(1991) are based on the Begley's(1978) experiment data, inevitably, the predictions of Zahoor(1991) are close to the experiment data of Begley quite well, hence, deviate to the results obtained in this paper.

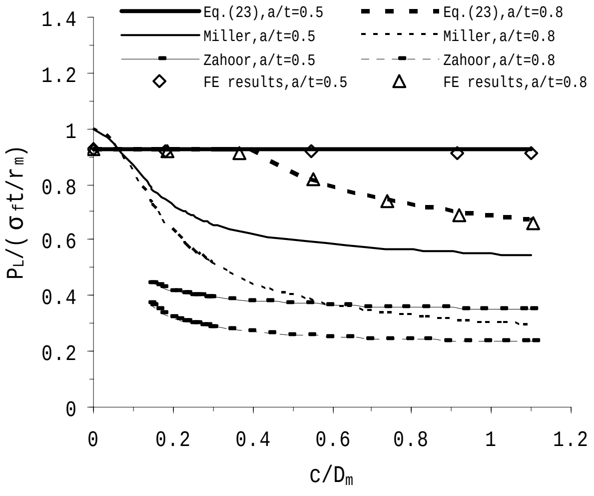


Fig.25 normalized limit pressure with crack(extrados)

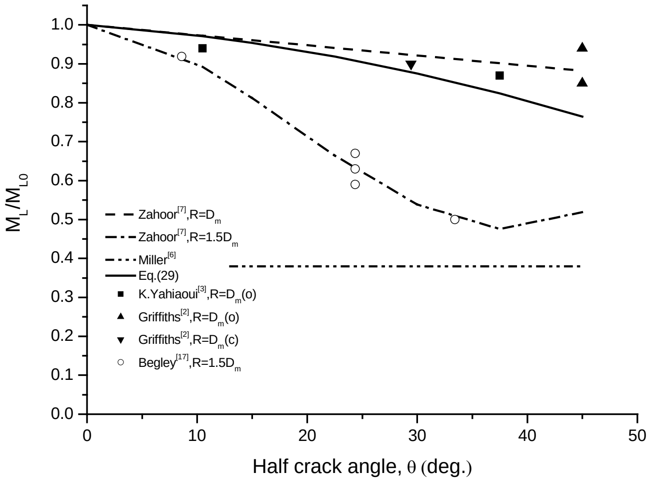


Fig.26 normalized limit moment with crack(crown)

6. CONCLUSION

The plastic limit loads of cracked elbow are studied systemically by using of theoretical analysis and non-linear FE analysis technology. The main conclusions are as bellow:

(1) Based on the von-Mises yield and extended net section plastic collapse (NSC) criterion, the simplified estimation formulas for plastic limit pressure or in-plane moment of cracked elbows without effects of tangent

straight pipes, are derived. These formulas can be used to evaluate the limit load of cracked elbow conservatively.

(2) The strengthening effects of tangent pipes on plastic limit loads of elbows with and without cracks are both significant, and the level of tangent constraint is more pronounced for short radius than it is for long radius elbows.

(3) The weakening effect of cracks on collapse pressure is not obviously dependent on elbow thickness-radius ratio, and can be ignored with crack at intrados or extrados. Under in-plane moment, there are insignificant differences for weakening effects of axial crack at crown on collapse moment with different elbow geometric dimensions and bending modes of moment. But for long radius elbow, the weakening factors obtained in present research are distinctly different from the experiment data of Begley, thus the collapse limit moment of long radius elbow with axial crack at crown should be emphasis in after studies. For the same size of crack, the crack influence on collapse load of long radius elbow is more critical than of short radius elbow under pressure or in-plane opening/closing moment.

(4) There is a critical crack angle below which there is no weakening effect on collapse pressure with the throughwall/part-throuth crack at crown or extrados. The value of critical crack angle of short radius elbow is higher than that of long radius elbow, and is irrespective of the changing of thickness-radius ratio.

(5) The equations of Miller(1988) and Zahoor(1991)are too conservative, and on occasions, nonapplicable to the engineering applications.

(6) Base on the extensive numerical results obtained from detailed non-linear FE analyses and comprehensive parameters research, the equations of cracked elbow with connecting tangent, subjected to internal pressure or in-plane opening/closing mode of bending moment are proposed in closed-form, and thus could be easily used in practical situation. As there is very little information available for the experiment data on collapse limit pressure of cracked elbows, the applicability of the newly proposed equations for collapse limit pressure of cracked elbow are failed to be validate. So the correctness and reasonableness of these closed-form equations are further validated by more experimental work, they could be utilized in the integrity assessment of pressured pipeline.

REFERENCES

ASME XI IWB-3650 and Appendix H,(1989) Flaw evaluation procedures and acceptance criteria for ferritic piping. New York : ASME.

J.E.Griffiths，(1979), The Effect of Cracks on the Limit Load of Pipe Bends Under In-Plane Bending：Experimental Study，Int. J. Mech. Sci. 21 pp.119-130.

Yahiaoui K, Moffat D G, Moreton D N, (2000), Piping elbows with cracks part 2: global finite element and experimental plastic loads under opening bending. J. Strain Analysis, Vol.35, No.1 pp.47-57.

Chattopadhyay, J., Dutta, B.K., and Kushwaha, H.S., (2001), Derivation of γ Parameter From Limit Load Expression of cracked Components to Evaluate J-R Curve, Int. J. Pressure Vessels Piping, 78 pp. 401-427.

Chattopadhyay, J., Pavankumar, T. V., Dutta, B.K., and Kushwaha, H.S., (2003), J-R Curves From Throughwall cracked Elbows Subjected to In-Plane Bending Moment, ASME J. Pressure Vessel Technol., 125, pp. 36-45.

Miller.A.G.，(1988)，Review of Limit Loads of Structure Containing Defects ，International Journal of Pressure Vessels and Piping, 32 pp.197-327.

Zahoor, A, (1991), Ductile Fracture Handbook, Vol. 3, EPRI-NP-6301-D, N14-1, Research Project 1757-69, Electric Power Research Institute.

Yahiaoui K, Moreton D N., and Moffat D G., (2002), Evaluation of Limit Load Data for Cracked Pipe Bends Under Opening Bending and Comparisons With Exiting Solutions, Int. J. Pressure Vessels Piping, 79, pp. 27-36.

Kiefner J F, Maxey W A, Eiber R J et al, (1973), Failure stress levels of flaws in pressurized cylinders: in progress in flaw growth and fracture toughness testing. ASTM STP536, pp. 461-481.

Folias E S, (1965),An axial crack in a pressurized cylindrical shell. Int. J. Frac. Mech., 7, pp. 104-113.

Guo C.X., (1999), Plastic limit loads for surface defect pipes and bends under combined loads of tension, bending moment, torsion and internal pressure [Ph D Thesis]. Shanghai: East China University of Science and

Technology, (In Chinese).
Xu Z.X., (2003), Plastic limit loads for surface defect pipes and bends under combined loads of tension, bending moment, torsion and internal pressure [M Thesis]. Shanghai: East China University of Science and Technology, (In Chinese).
Yahiaoui.K.，Moffat D.G and Moreton.D.N，(1996)，Damage Assessment of Piping ElbowsLoaded by Steady Internal Pressure and Dynamic In-Plane or Out-plane Bending，Proc 8th Int. Conf on Pressure Vessel Technology，Montreal，ASME, 1, pp.361-375.
C.R.Calladine，(1974)，Limit Analysis of Curved Tubes，J. Mech. Eng. Sci., Vol.16, No.2, pp.85-87.
Chain K L C, (1984), Approximate limit loads for pipe bends with end constraints. Applied Solid Mechanics，pp.275-285.
ASME, Boiler and Pressure Vessel Code, Section XI, (1986), Rules for Inservice Inspection of Nuclear Power Plant Components, Division 2.
J.A. Begley, W.K. Wilson and A.A. Sheinker, (1978), I. Mech. E. Conf. on Tolerance of Flaws in Pressurized Components, London