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INCORPORATION OF ALTERNATING PLASTICITY IN THE SHAKEDOWN METHOD

D.E. Buckthorpe[1] and P.S. White[2]

[1]NNC Ltd., Risley, UK
[2]European Gas Turbines, Engineering Research Centre, Whetstone, UK

1 INTRODUCTION.

Shakedown is the process whereby behaviour becomes wholly or predominantly elastic and in particular no further change in the dimensions of a structure occurs after a few cycles of loading. If the loading is increased progressively, a level is reached where shakedown does not occur and the dimensions of the structure continue to change. This is the condition known as ratcheting.

Structural materials and weldments are of limited ductility and therefore ratcheting is not acceptable in plant where large numbers of load applications occur. All successful plant must therefore operate within the shakedown limit.

The principles of shakedown have been accepted for many years but their use in design has been hindered by difficulty of performing analysis. The shakedown design method developed in the UK overcomes this difficulty and is used for guarding against progressive deformation or ratcheting and for assessment and fatigue and creep/fatigue damage.

A paper [1] outlines the method and test cases [2] were presented at a previous SMIRT. These are extended to LMFBR problems [3] covering comparisons with elastic code methods such as RCC-MR and cyclic inelastic analysis.

The method uses the elastic analysis of a load cycle and requires the estimation of a constant (in time) residual stress field which is used to obtain a reference stress for creep damage estimates and local estimates of fatigue damage. The post-processor ADAPT is the basic special computational tool within the shakedown method for performing this estimation.

The emphasis of this paper concerns an improvement to the ADAPT algorithms to give better estimates in cases of overall shakedown where a substantial elastic core is maintained but small regions upto 20% of a section may be beyond yield.

The present methodology is based on lower bound shakedown theory and in the case of overall shakedown evaluates an approximate self equilibrating residual stress field which minimises the region of plasticity. The new treatment estimates a representative time independent field through a criterion based on approximate symmetrisation of the local stress cycle.

The basis behind the improvements is outlined and some results of initial performance trials are described. Comparisons are made with the original version and with results of cycle elastic plastic analysis.

2 ALTERNATING PLASTICITY VERSION OF ADAPT.

Within the shakedown design method the material is treated as elastic perfectly plastic. An appropriately chosen yield stress is used to allow a suitable degree of cyclic hardening and to ensure that finding shakedown also corresponds to a stability of strain accumulation.

The situation is complex in cases of overall shakedown where it is no longer true even in creep free conditions that a residual stress develops which does not change during the cycle. The results obtained from the form of ADAPT employed upto now have proved conservative in comparison with cyclic inelastic calculations using the same plasticity model but have in some cases led to higher estimates of stresses and higher creep usage estimation.

2.1 Concepts Employed.

2.1.1 Original version.

Let the elastically calculated stress at a position in a structure and at a time t in a cycle be denoted by $\sigma^E (\underline{x},t)$ Then, with the yield condition written as

$$f(\sigma) = \sigma_y(\underline{x},t) \qquad (1)$$

(where f denotes the Mises yield function and σ_y is the yield stress, dependent upon $\underline{x}$ and t through the temperature) the condition for strict shakedown is that a time-independent residual stress $\rho(\underline{x})$ should exist such that for all $\underline{x}$ and t

$$f(\sigma^E(\underline{x},t) + \rho(\underline{x}))/ \sigma_y(\underline{x},t) < 1 \qquad (2)$$

where ρ also satisfies the condition of self-equilibrium,

$$\nabla^T\rho = 0 \qquad (3)$$

In a finite element representation, $\underline{\sigma}^E$ and ρ are represented by components at each integration point.

ADAPT seeks a simultaneous solution to both conditions in an iterative manner, alternatively finding sets of entries which satisfy one or the other condition. If a solution exists then the process converges and both conditions are finally satisfied.

There are two forms in which such convergence may fail to occur. The first is when strict, structural shakedown is impossible even though the elastically calculated <u>stress-range</u> is sufficiently small in a local sense. This situation occurs most commonly in cases of appreciable primary loads. Secondly and more common for LMFBR's however is when loads are predominantly thermal and no solution can be found because at some positions condition (2) cannot be satisfied owing to the elastically calculated stress-range being too great.

For the initial estimate made by ADAPT, (when (2) cannot be satisfied) the local $\rho\ (\underline{x})$ is chosen such as to minimise the shakedown ratio

$$r\ (\underline{x}) = \max_{t} \{f(\sigma^E\ (\underline{x},t) + \rho\ (\underline{x}))/\sigma_y(\underline{x},t)\} \qquad (4)$$

over the cycle. In the original version, the local estimate is allowed to change without restriction during subsequent iterations at the points where (2) cannot be satisfied. (At other points satisfaction is enforced in all iterations). The program converges to a solution where the equilibrium condition is satisfied throughout but there are regions where the local shakedown condition is not satisfied. These regions (generally larger than those in which (2) cannot even be satisfied in a pointwise manner), are identified with regions of cyclic plasticity. In some cases these turn out to give converged estimates of residual stress which were quite far from the initial local estimates.

2.2 Alternating plasticity version.

It is well known [1] for there to be a strong tendency in localised plasticity regions for the stress-trajectory during a cycle to be highly symmetrised with respect to the origin. Another way of putting it is that the residual stress in regions of alternating plasticity should be that which could occur if the local yield stress were increased just enough to allow all the shakedown conditions to be satisfied.

Incorporation of this principle at points where (2) cannot be satisfied implies that the average residual stress over the cycle at these points should not be very far from the condition which locally minimise the shakedown ratio i.e. the condition given by the initial estimate of ADAPT.

The modification of ADAPT to include alternating plasticity is therefore to replace the candidate local residual stress by that given by the initial estimate of ADAPT where alternating plasticity is suspected. The iteration proceeds as before except that in the alternating plasticity region no updating of the residual stress field is allowed or possible in the part of the iteration which seeks to satisfy (2).

The software is modified to allow the above procedure as an alternative.

3 PERFORMANCE TRIALS.

3.1 Bree Problem.

The case investigated is a Bree beam with a primary axial stress of 30MPa and an alternating thermal load corresponding to variation between a uniform temperature of 370° and a through-thickness gradient of 370 to 538°C. The latter loading gives an elastically calculated stress gradient (not counting the primary load) of +232 to -232MPa.

The yield stress is taken as 100MPa giving a point P on the Bree diagram approximately as shown on fig 1.

The beam was idealised with six plane-stress elements (of equal size through the thickness. The peak positions are therefore calculated as being at the outermost integration points rather than strictly on the surfaces. The problem was tackled using both the original and new forms of the algorithms.

In the original form the minimised shakedown ratio at the outermost points was 1.077 > 1, showing that strict shakedown was not possible. The iterative procedure converged, effectively after three iterations but with slight changes up to five iterations, indicating cyclic plasticity at the outer points with the calculated residual stress giving a shakedown ratio of 1.336. This would indicate an estimated stress (elastic plus residual) of 133.6MPa if a dwell period were imposed at the worst stage of the cycle.

In the revised version, the minimised shakedown ratio was again 1.077. However after three and five iterations the shakedown ratios at the most severe position were only 1.096 and 1.083 respectively, the latter corresponding to an elastic plus residual stress of 108.3MPa at the worst stage. Moreover cyclic plasticity was estimated to occur at the same points in both analyses.

There are indications that convergence in overall shakedown may have been a little slower, however the estimated elastic plus residual stresses are distinctly lower at the worst stage of the cycle and are much more highly symmetrised (the shakedown ratio 1.083 for equilibrated residuals being close to the value of 1.077 in the initial local estimate).

3.2 **Non-Uniform Tube**

This problem concerns a tube of non-uniform thickness varying from 4 to 6mm with an outside diameter of 100mm. It was modelled (with a half-model taking advantage of symmetry) using 120 plane stress elements with 5 through the thickness as in fig 2. The loading varies between a state of internal pressure and internal pressure plus through-thickness temperature gradient. The problem was originally devised as an extension of the Bree problem to show effects of biaxial stress and non-uniform distribution of stress. It has been selected for the present investigation for the same reasons and because a comparative inelastic analysis is also available. The particular case chosen is shown on the approximate load-interaction diagram in fig 3.

With the original form of ADAPT, convergence is effectively attained after 4 iterations with cyclic plasticity predicted over most of the inner surface and about half of the outer surface. The maximum shakedown ratio is 1.235 so that elastic plus residual stresses are predicted as 23.5% above yield at the worst stage. With the new version the convergence and predicted cyclic plasticity regions are much the same. However, the maximum shakedown ratio is reduced to 1.05, just 5% above yield at the worst stage. Symmetrisation of the predicted elastic + residual stress is also more nearly approached and, by the same token, results are closer to those of inelastic analysis (perfect plasticity with the same yield-stress). Fig 4 shows a comparison of standard ADAPT results with inelastic analysis for the x-x stress component on the inner surface.

The corresponding result using the alternating plasticity version is also shown for the position at which this stress component (effectively the dominant hoop component) is maximised and is much closer to the inelastic curve.

4 SUMMARY AND CONCLUSIONS.

A modification of the ADAPT algorithm within the shakedown design method has been described which is intended to provide a residual stress that gives a more nearly symmetrical stress-trajectory in cases of overall shakedown.

(i) The modified algorithm may imply a small penalty in slightly slower convergence

(ii) A higher degree of symmetrisation was attained and a lower estimated value of elastic plus residual stress was produced at the most severe stage of the cycle in each case.

(iii) Results were closer to those given by inelastic analysis.

It is concluded that a more realistic estimate of creep usage will be produced by the modified algorithm, with benefits in shakedown-based design.

Further trials on typical LMFBR cases are planned including comparisons of estimated creep/fatigue damage with those of the original version and using cyclic inelastic analysis.

5 ACKNOWLEDGEMENT

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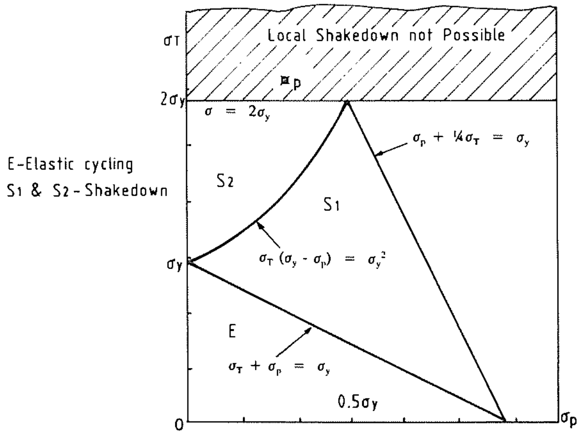
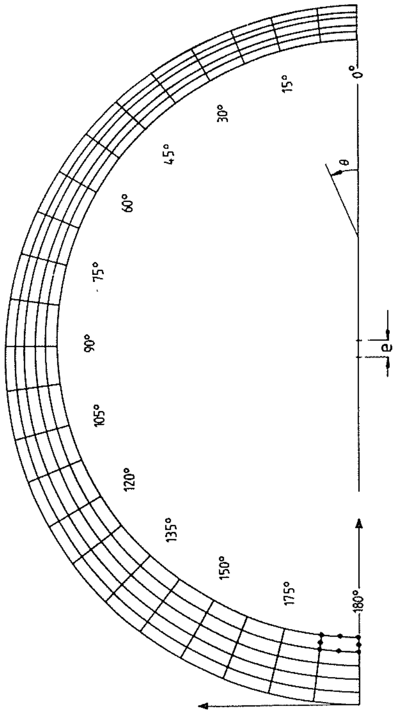


Fig.1 **Bree Diagram**



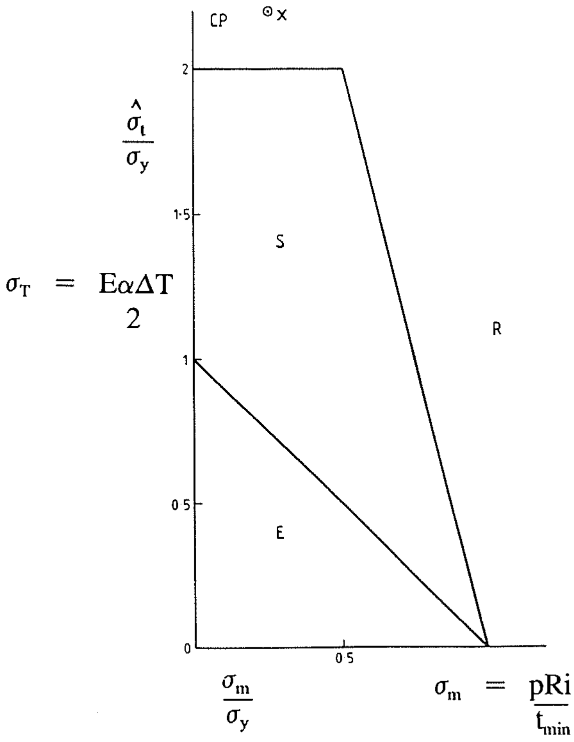


Fig.2 F.E. Model of Ring: Plane Stress Elements

Fig.3 Bree Diagram for Non-Uniform Thickness Ring

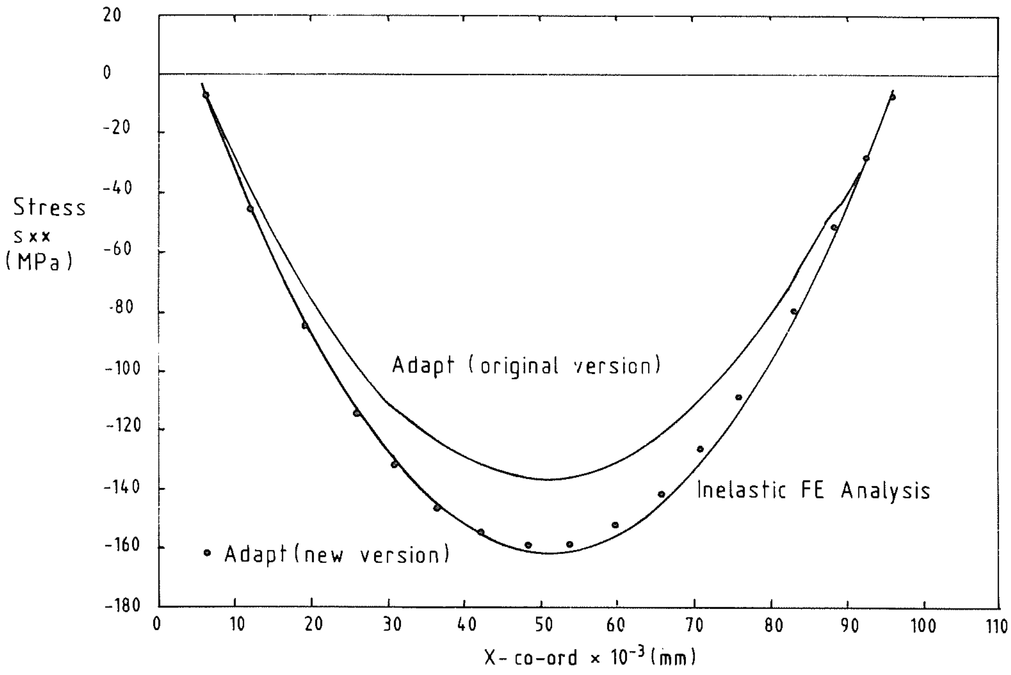


Fig.4 Axial Stress at Pressure + Thermal Gradient Interior Surface

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SHAKEDOWN METHOD WITH STRAIN BASED DAMAGE ESTIMATES

P.S. White[1] and D.E. Buckthorpe[2]

[1]European Gas Turbines, Engineering Research Centre, Whetstone, UK
[2]NNC Ltd., Risley, UK

Abstract. A brief account is given of currently proposed extentions of the Shakedown Design Method to account for stress-reduction during hold periods (including follow-up effects) and for the use of strain-based creep-damage estimates.

1. INTRODUCTION

In the development of assessment methods for combined creep-fatigue there are many complexities which, however, may be resolved into a few broad issues. First is the question of whether fracture mechanics must be used or whether treatment is possible in terms of states of stress and strain in an assumed undefected structure. In design assessment the latter is normal but with the proviso that life estimates must be rather conservative, corresponding to the development of sufficiently short cracks rather than complete component failure, with implications for the interpretation of failure data. Second, assuming that states of stress and strain can be predicted in adequate detail, there is the question of how such states determine the life estimate and, therefore, of what constitutes adequate detail. In practice the failure relations for combined creep-fatigue can be quite sensitive so that the necessary detail may be substantial. Given also that behaviour will probably involve both plasticity and creep, the third issue arises of whether it is always necessary to perform a complete inelastic analysis (which may well be difficult and expensive in the cyclic conditions typical for Fast Reactor (FR) components) or whether some kind of simplified analysis could suffice.

The Shakedown Design Method [1], which includes both strict shakedown and cases of localised cyclic plasticity, was introduced for precisely this purpose and has proved useful for wide classes of problems. However in its original form, with no allowance for reduction of stresses during hold periods, the creep damage estimates have sometimes been over-conservative. This paper outlines recent proposals for reduction of this conservatism both through inclusion of allowance for follow-up and through the use of strain-based damage criteria.

2. STRAIN-BASED DAMAGE

This methodology [2] has been developed in recent years, using ideas of strain-controlled cavity growth, to give predictions of creep damage and creep-fatigue interaction which are considered superior to, and can be much less conservative than, those of stress-based creep-damage in conditions of time-variable and multiaxial stress. Although there are some novel considerations for fatigue and for slow plastic strains (known as quasi-creep), only ordinary creep is considered here. In brief, the damage increment dD^c, corresponding to the stress component s and creep strain increment de^c at creep rate $\dot{e}^c$ in a fixed principal direction, is of the form

$$dD^c = bde^c/e_f \qquad (1)$$

where e_f is a ductility and b is a function of $\dot{e}^c/s$ with, in particular, zero values for large or negative arguments and sometimes leading to negative damage increments. Other rules have been introduced for compounding effects in different principal directions or for when such directions are not fixed.

Furthermore, the interaction of creep and fatigue damage is given not by a linear summation but by an L-shaped diagram such that sufficiently small creep or fatigue damage has no effect.

3. IMPLICATIONS FOR ANALYSIS

It is evident that this approach to creep damage requires, in principle, very full knowledge of all components of the stress and the creep strain-rate at all times. A precise evaluation might well need an inelastic analysis of high quality with, in particular, accurate constitutive equations. For FR problems, however, though there are cases of severe loading cycles which lead both to through-section plasticity and to widespread creep, many situations may be expected to involve significant inelastic behaviour only in fairly localised regions so that components have effectively elastic cores.

In order to deal with such situations in a more efficient manner, simplified methods based on the concept of Shakedown have been developed, as in [1]. In their original form these methods, as illustrated in [3], which are based upon estimating a residual stress state for shakedown, treat creep in terms of 'reference-stress' essentially representing the most significant equivalent stress at the start of a hold period, considered to remain constant during that period and treated by a stress-based damage criterion. In more recent developments account is being taken of partial relaxation to reduce the conservatism and procedures have also been proposed for estimating strain-based creep damage, when the criteria for shakedown are satisfied.

These developments depend upon the concept of elastic follow-up.

4. ELASTIC FOLLOW-UP

Elastic follow-up (in the sense of creep) is said to occur when an overall imposed displacement (or constrained thermal expansion), in a region of a structure, is shared between strongly and weakly creeping sub-regions. The latter then acts like a spring-washer under the head of a hot bolt, reducing the rate of stress-relaxation and increasing the accumulated creep strain in the strongly creeping part. In the most general situation the effects may be complicated, involving differing and interacting follow-up effects for different stress-components, as sketched out theoretically in [4] with the assumption of localised creep. But in the case of strict or overall shakedown in FR Components (see [1]), when significant creep is usually confined to small regions on surfaces of a structure and when mechanical loads are small, the effects may be expected to be simpler.

When mechanical loads are neglected it may be deduced from the general theory of [4] that the equivalent stress s_{eq} and the equivalent creep rate $\dot{e}^c_{eq}$ in the local strongly creeping region are related by

$$ds_{eq}/dt = -q^{-1}(3E/2(1+r))\dot{e}^c_{eq} \qquad (2)$$

$$\text{with } 1 \leq q \leq 1/V_{min} \qquad (3)$$

where $V_{min}(\leq 1)$ is a geometrical property depending upon the stiffness distribution while E and r denote Young's modulus and Poisson's ratio. The instantaneous follow-up factor $q(\geq 1)$ need not be constant if the stress-ratios vary with time but remains bounded as in (3). A number of cases of structures in overall shakedown have been investigated by inelastic analysis for typical FR situations, including highly non-uniform temperature states, and the instantaneous values of q determined. Although q was somewhat variable within individual analyses and certainly varied from one to another, in all cases the approximate bound

$$q \leq 3.0 \qquad (4)$$

was found to hold and the value of 3.0 has been initially proposed as a conservative follow-up factor for structures within shakedown. However it is easy to construct examples beyond shakedown where much greater values of q arise. One such example is a cantilever beam under prescribed transverse displacement with creep confined to a short hot region at the built-in end. (It is interesting to note that a similar value appears in French and Japanese treatments of follow-up within FR design rules but the restrictions of use are not then the shakedown criteria).

This value of q, in conjunction with a suitable creep law, is therefore proposed for use in (2) in order to give conservative estimates of stress-reduction during hold periods for the purpose of estimating stress-based creep damage. It is of course necessary also to estimate the initial value of s_{eq}. This value is obtained, as in the reference stress calculation [1], as the equivalent sum of the elastically calculated stress $\underline{s}^e$ and the estimated residual stress $\underline{r}$, multiplied by a correction factor K_2.

An example of the predictions, for a problem described more fully in [5], is as follows.

A T-shaped structure made of Type 316 stainless steel, with the horizontal legs essentially fixed at their ends, has a cyclic transverse displacement applied at the end of the vertical leg. The uniform temperature varies simultaneously, between 370 and 540°C, with a hold-period of 10^3h at 540°C. The structure is in generalised plane strain with stress concentrations at fillet radii at the junctions of the legs. At the peak position the shakedown calculation for steady cyclic conditions gave an estimated equivalent stress for the start of the hold as 172MPa. The follow-up calculation based on (2) with q = 3.0 gave a final stress of 150.6MPa using a standard UK creep law. The reduction of stress during the dwell reduced the creep damage estimate by a factor of about 4. An entirely independent inelastic analysis of the problem gave initial and final equivalent stress values of 141 and 122MPa, indicating that the simplified method was indeed conservative, essentially through the estimated initial stress. It is also noteworthy that this analysis (and another with somewhat different material properties) indicated a true follow-up factor of about 2.0, compared with the adopted value of 3.0.

5. COMPONENT ESTIMATES FOR STRAIN-BASED DAMAGE

In order to avoid excessive complications it is assumed in this present account, as is commonly the case in FR problems, that the strongly creeping region is localised at a small region of cyclic plasticity on the surface of an axisymmetric structure with negligible mechanical loads. The hoop, lengthwise tangential and normal directions are then fixed principal directions of stress with the third principal value being zero. With respect to corresponding local Cartesian axes the only non-zero stress components are s_{11} and s_{22} which are conveniently represented by

$$s_1 = s_{11} - s_{22}/2,\ s_2 = \sqrt{3}\, s_{22}/2,\ s_1^2 + s_2^2 = s_{eq}^2 \qquad (4)$$

while creep rates are similarly represented by

$$\mathring{e}_1{}^c = 3\mathring{e}_{11}{}^c/2,\ \mathring{e}_2{}^c = \sqrt{3}(\mathring{e}_{22}{}^c + \mathring{e}_{11}{}^c/2),\ (\mathring{e}_1{}^c)^2 + (\mathring{e}_2{}^c)^2 = 9(\mathring{e}_{eq}{}^c)^2/4 \qquad (5)$$

The general theory given in [4] may then be developed to give the biaxial follow-up relation

$$ds_a/dt = -(E/(1+r))V_{ab}\mathring{e}_b{}^c\ (a,b = 1,2) \qquad (6)$$

(summation convention) where V_{ab} is a matrix such that

$$0 \leqslant V_{ab}\, l_a\, l_b \leqslant l_b l_b \qquad (7)$$

for any l_1, l_2. The quantity V_{min} in (3) is then the minimum proper value of V_{ab}. Although the entries of V_{ab} can be evaluated (if the region of strong creep is known) this is quite complicated. However, with the empirical knowledge that $q \leqslant 3$ for the class of structures in shakedown, it is reasonable to interpret this through (3) in the form

$$V_{min} \geqslant 1/3 \qquad (8)$$

for the same class. It is then possible, after some rather complicated algebra, to obtain relatively simple bounds for component changes. Thus if p represents either s_1/s_{eq} or s_2/s_{eq} then

$$(V_{min}{}^{-1}-1)d\ln s_{eq}/dt \leqslant 2(1-p^2)^{-\frac{1}{2}}dp/dt \leqslant -(V_{min}{}^{-1}-1)d\ln s_{eq}/dt \qquad (9)$$

Further manipulation of these inequalities in conjunction with (2) then enables bounds to be obtained for the creep damage increment in (1), taking advantage of zero or even negative damage associated with compressive stress or creep-rate components. These estimates are the result of numerical solutions of ordinary differential equations such as (1). They also require an estimate of the initial stress state which is taken as $K_2(\underline{s}^e + \underline{r})$ as defined in the previous section. It should be noted that this estimate is usually quite good for localised regions which are somewhat beyond shakedown even though the estimated residual stress may be far from uniquely defined by a shakedown calculation at positions well away from such regions. The details of the damage procedure are rather complicated but can be encoded in convenient software.

6. CONCLUSION

It was originally hoped to include detailed examples of these methods but the limited space only allows a brief outline of the ideas. Nevertheless many examples are available [6] showing that strain-based damage with inelastic calculations can be advantageous in reducing the conservatism of damage estimates. The introduction of a well-founded follow-up factor gives a useful reduction for stress-based estimates within the shakedown method. The extension, indicated in section 5, to strain based damage is expected to reduce them still further, allowing the shakedown method to provide simple assessments without undue pessimism for an important class of problems.

7. ACKNOWLEDGEMENTS

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