THREE-DIMENSIONAL CALCULATION OF THE COUPLED VIBRATIONS OF A GROUP OF CIRCULAR TUBES IN AN UNCONFINED LIQUID

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ABSTRACT

This paper describes a computational method for determining the free vibrations of arbitrarily spaced parallel tubes immersed in an originally motionless fluid. The dynamic behaviour strongly depends on the presence of the surrounding liquid which causes a coupling effect between all the tubes and a decrease of the natural frequencies of the system. Because of the three-dimensional approach to this problem the influence of the boundary conditions of the tubes on the added fluid mass can be established.

In order to investigate fluid induced instabilities such as flutter and buckling the model might be adjusted to take account of the presence of a stationary flow parallel to the tubes.

1. Modified fluid-tube model.

Fig. 1 shows a configuration of ℓ arbitrarily spaced tubes surrounded by an unconfined fluid. Except for the tube length L_p all tube parameters may be different. In order to describe the fluid conditions at the surface of the tubes a local cartesian and cylindrical coordinate system for each tube have been chosen.

The fluid assumed to be inviscid, rotation-free, incompressible and non heat conducting.

The fluid motion due to the vibration of, say, the p^{th} tube can be expressed in a velocity potential ϕ_p, which is governed by Laplace's equation.

In order to assess the influence of different boundary conditions for the tube on the fluid motion a modified fluid-tube model is introduced which is shown in fig. 1. In accordance with a previous publication by two of the present authors in the field of fluid shell vibrations [1], the leading philosophy behind this model is that in an unbounded fluid area no disturbing influence due to tube vibrations can be observed at a sufficiently large distance from the tube. The system is extended by rigid tubes, which are added to each side of the real (flexible) tubes, resulting in a total dimensionless length $T_p = LL_p/R_p$. The dimensionless axial coordinate α_p is replaced by the coordinate α_p^* .

Assuming an unconfined fluid in the radial direction; the solution of Laplace's equation for ϕ_p can be written as

$$\phi_p = \sum_{n=1}^{\infty}\sum_{j=1}^{\infty} \sin \frac{j\pi\alpha_p^*}{T_p} \; K_n \left(\frac{j\pi\rho_p}{T_p}\right) [C_{jpn}\sin n\theta_p + D_{jpn}\cos n\theta_p]e^{i\omega t} \qquad (1)$$

where K_n is the modified Bessel function of order n, C_{jpn} and D_{jpn} are constants, and ω is a natural frequency of the system.

2. Coordinate Transformation.

In order to satisfy the boundary conditions at the tube surfaces the total velocity potential due to the vibration of all tubes should be expressed in the coordinates of each separate tube, which requires a coordinate transformation that can be written as follows

$$\phi^p = \sum_{k=1}^{\ell} \phi_k^p \qquad (2)$$

where ϕ^p is the total velocity potential expressed in coordinates of the p^{th} tube and ϕ_k^p is the velocity potential due to the k^{th} tube expressed in coordinates of the p^{th} tube.

Using the addition theorems for the modified Besselfunctions the following expressions can be derived

$$\begin{aligned} K_\nu(\rho_k)\cos\nu\theta_k &= \sum_{n=-\infty}^{\infty} (-1)^\nu K_{\nu+n}\left(\frac{Z_{pk}}{R_p}\right) I_n(\rho_p)\cos\{(n+\nu)\psi_{pk}-n\theta_p\} \\ K_\nu(\rho_k)\sin\nu\theta_k &= \sum_{n=-\infty}^{\infty} K_{\nu+n}\left(\frac{Z_{pk}}{R_p}\right) I_n(\rho_p)\sin\{(n+\nu)\psi_{pk}-n\theta_p\} \end{aligned} \quad \text{with } Z_{pk} > R_p\rho_p \qquad (3)$$

Because of the latter restriction eq. (2) has to be rewritten

$$\phi^p = \sum_{k=1}^{\ell}{}^* \phi_k^p + \phi_p \qquad (4)$$

Where $\sum^*$ denotes the summation for k = 1 to ℓ except k = p.
Substituting expressions (3) into eq. (1) gives

$$\phi^p = \sum_{j=1}^{\infty} \sin \frac{j\pi\alpha_p^*}{T_p} \{ \sum_{n=1}^{\infty} (K_{nj}^{ps} C_{jpn} + K_{nj}^{pc} D_{jpn}) + \sum_{k=1}^{\ell}{}^* \sum_{n=1}^{\infty} \sum_{\nu=-\infty}^{\infty} (H_{n\nu j}^{pk} C_{jkn} + L_{n\nu j}^{pk} D_{jkn}) \} e^{i\omega t} \tag{5}$$

where

$$K_{nj}^{ps} = K_n \left(\frac{j\pi\rho_p}{T_p}\right) \sin n\theta_p \; ; \; K_{nj}^{pc} = K_n \left(\frac{j\pi\rho_p}{T_p}\right) \cos n\theta_p ;$$

$$H_{n\nu j}^{pk} = (-1)^n K_{\nu+n} \left(\frac{j\pi Z_{pk}}{R_p T_p}\right) I_\nu \left(\frac{j\pi\rho_p}{T_p}\right) \{-\cos(n+\nu)\psi_{pk} \sin\nu\theta_p + \sin(n+\nu)\psi_{pk} \cos\nu\theta_p\}$$

$$L_{n\nu j}^{pk} = (-1)^n K_{\nu+n} \left(\frac{j\pi Z_{pk}}{R_p T_p}\right) I_\nu \left(\frac{j\pi\rho_p}{T_p}\right) \{ \sin(n+\nu)\psi_{pk} \sin\nu\theta_p + \cos(n+\nu)\psi_{pk} \cos\nu\theta_p\}$$

3. Boundary conditions at the tube surface.

The remaining constants in eq. (5) can be solved by using the following boundary conditions

$$\frac{1}{R_p} \frac{\partial\phi^p}{\partial\rho_p} \bigg|_{\rho_p=1} = \frac{\partial U_p}{\partial t} \cos\theta_p + \frac{\partial V_p}{\partial t} \sin\theta_p \tag{6}$$

The left hand side of equation (6) describes the fluid velocity normal to the tube surface, while V_p and U_p are the displacements in Y_p and Z_p direction, respectively.
The displacement functions are chosen as

$$U_p = \sum_{r=1}^{M} U_r^p X_r^p (\alpha_p) e^{i\omega t} \; ; V_p = \sum_{r=1}^{M} V_r^p X_r^p (\alpha_p) e^{i\omega t} \tag{7}$$

where U_r^p and V_r^p are amplitude coefficients of the p^{th} tube and X_r^p (α_p) is the r^{th} eigenfunction of a free transverse vibrating beam with the same length and boundary conditions as the tubes.
In order to get a suitable application of eq. (6), the eigenfunctions are developed in a Fourier sine series with respect to α_p^*

$$X_r^p = \sum_{j=1}^{\infty} \overset{*}{A}_j^{rp} \sin \frac{j\pi\alpha_p^*}{T_p} \text{ , where } \overset{*}{A}_j^{rp} = \frac{2}{T_p} \int_{\alpha_{p1}^*}^{\alpha_{p2}^*} X_r^p \sin \frac{j\pi\alpha_p^*}{T_p} d\alpha_p^* \tag{8}$$

Applying equations (5), (7) and (8) to eq. (6), a matrix equation for the remaining constants is obtained

$$\vec{C}_j = \frac{i\omega}{\pi} \bar{K}_j^{-1} \vec{V}_j^* \tag{9}$$

The coefficients of $\bar{K}_j$ are functions of ψ_{pk} and Z_{pk}, while $\vec{V}_j^*$ contains the amplitude coefficients U_r^p and V_r^p and the Fourier coefficients $\overset{*}{A}_j^{rp}$.
Using eq. (5) the solution of the velocity potential becomes

$$\vec{\phi} = \sum_{j=1}^{\infty} \frac{i\omega}{\pi} \bar{\alpha}_j \bar{A}_j \bar{K}_j^{-1} \vec{V}_j^* e^{i\omega t} \text{ with } \vec{\phi} = \begin{vmatrix} \phi^1 \\ \phi^2 \\ \\ \phi^\ell \end{vmatrix} \tag{10}$$

The coefficients of $\bar{\alpha}_j$ are functions of the axial coordinate α_p.
The matrix $\bar{A}_j$ describes the configuration of the system by means of the parameters ψ_{pk} and z_{pk}.

4. Added mass matrix.

After applying Bernoulli's linearized equation to equation (10), the fluid pressure at each tube surface becomes

$$\vec{p}_o = \rho_L \frac{\omega^2}{\pi} \sum_{j=1}^{\infty} \bar{\alpha}_j \bar{A}_j \Big|_{\rho_p=1} \bar{K}_j^{-1} \vec{V}_j^{*} e^{i\omega t} \qquad (11)$$

The influence of the fluid on the free vibrations of the tubes can be established by assessing the virtual work of this fluid pressure and adding the result to the right hand side of the virtual work equation for the tubes.
After lengthy manipulations the extended work equation can be written in matrix form as follows

$$[\,\bar{S} - \Omega^2 (\overset{*}{\bar{A}} - \overset{*}{\bar{\rho}}\, \overline{TM})\,]\, \vec{V} = \vec{0} \qquad (12)$$

where $\bar{S}$, $\overset{*}{\bar{A}}$ and $\overset{*}{\bar{\rho}}$ contain dimensionless system parameters, Ω^2 is the dimensionless natural frequency matrix, $\vec{V}$ is a vector with the unknown amplitude coefficients and $\overline{TM}$ the so-called added mass matrix, which discounts the influence of the fluid.

5. Results and discussion.

The rate of convergence for the added mass matrix depends on the radial configuration of the system (the tube distance) and the boundary conditions at the tube ends.
The radial convergence is given by the number of n-terms used in the calculations, while the axial convergence is described by the number of j-terms. In general $n = 7$ and $j = 10$ will give results with sufficient accuracy. Fig. 3 shows the influence of the tube length on the added mass coefficient for one tube. At increasing tube length, the solution based on the two-dimensional Laplace equation as derived by Chen [2] is found. Moreover the influence of the boundary conditions at the tube ends for clamped-clamped (CL-CL) and simply supported - simply supported (SS-SS) conditions vanishes in that case.
In order to get a quantitative comparison with Chen's results two indentical tubes were tested numerically. The results of the self-added and mutual-added mass coefficients are given in table 1 for different tube distances and tube lengths. In all cases Chen's solutions are enclosed by the three-dimensional solution. Again one can see that at increasing tube length a better agreement is found between both solutions.
The influence of the number of tubes on the natural frequencies is determined by extending a symmetrical system of 3 tubes up to 6, 15 and 25 tubes as indicated in Fig. 4.
For each configuration $2 \times \ell \times M$ frequencies and vibrating modes were found. As noticed by Païdoussis [3] the symmetrical configurations 3, 6 and 15 will give pairs of identical frequencies which can be associated with non-symmetrical modes, while the remaining distinct frequencies belong to symmetrical or skew-symmetrical modes.

For M =1 fig. 4 shows the vibrating mode for the lowest frequency of a system consisting of 15 and 25 tubes.

Fig. 5 shows the influence of the extension of the system on the lowest and highest frequency for M = 1. One can see that both solutions converge as the number of tubes increases. Generally it can be concluded that for a complex configuration only a limited number of tubes are needed for calculations to get a fair impression of the dynamic behaviour of the system.

Acknowledgement

The work reported in this paper has been carried out in cooperation with B.V. Neratoom, The Netherlands.

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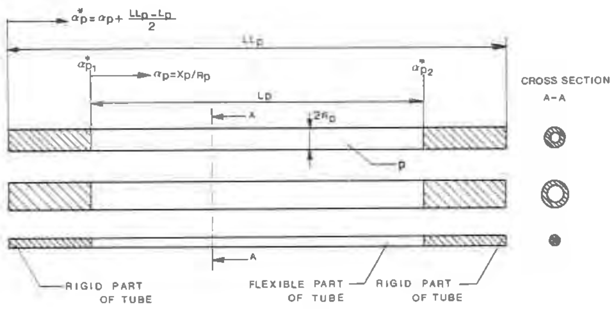

Fig 1 MODIFIED FLUID – TUBE MODEL

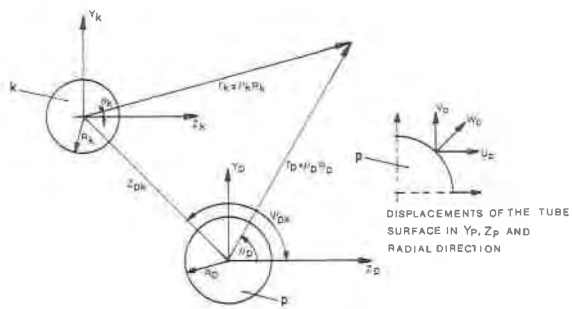

Fig 2 COORDINATE AND DISPLACEMENT NOTATIONS FOR TWO DIFFERENT TUBES p AND k

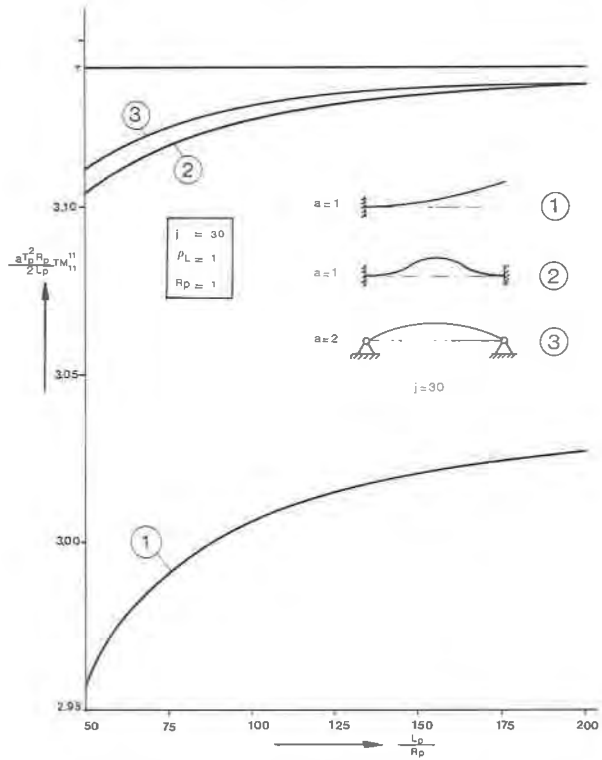

Fig 3 ADDED MASS COEFFICIENT FOR ONE TUBE AS A FUNCTION OF THE DIMENSIONLESS TUBE LENGTH

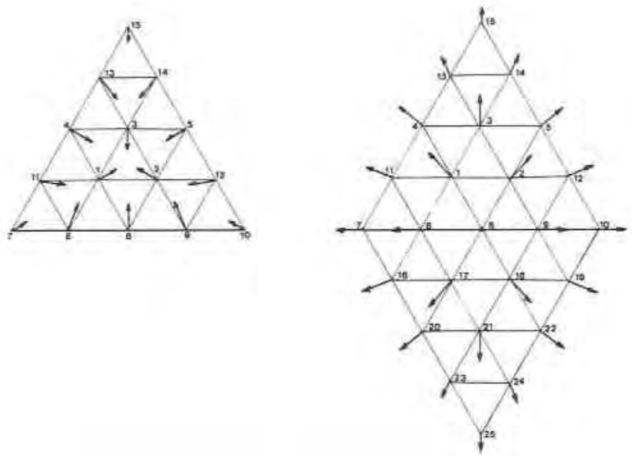

Fig. 4 VIBRATING MODES ASSOCIATED WITH THE LOWEST NATURAL FREQUENCY FOR 15 AND 25 TUBES

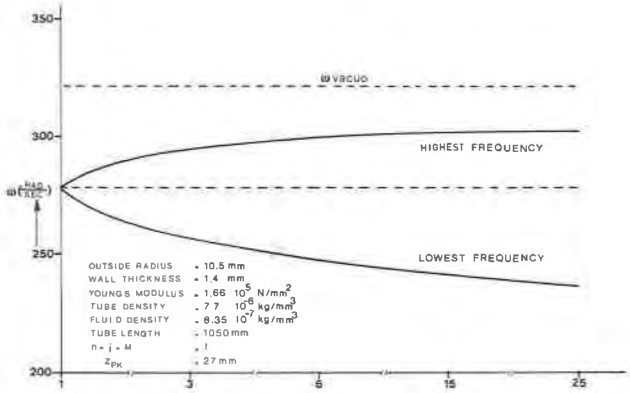

Fig. 5 HIGHEST AND LOWEST FREQUENCY AS A FUNCTION OF THE NUMBER OF TUBES (ALL ENDS SIMPLY SUPPORTED)

TABLE 1. Numerical comparison of 2-D and 3-D added mass coefficients.

	n	CHEN'S SOLUTION (ABSOLUTE VALUES)		3-D SOLUTION (ABSOLUTE VALUES)			
				TM_{11}^{11}	TM_{11}^{12}	TM_{11}^{33}	TM_{11}^{34}
SS-SS	1	1.0250	0.2250	1.0239	0.2180	1.0264	0.2294
L_p/R_p = 50	2	1.0308	0.2265	1.0296	0.2194	1.0322	0.2309
J = 15	5	1.0319	0.2269	1.0308	0.2198	1.0333	0.2313
GAP/R_p = 1.0	10	1.0319	0.2269	1.0308	0.2198	1.0333	0.2313
SS-SS	1	1.0078	0.1255	1.0076	0.1194	1.0087	0.1287
L_p/R_p = 50	2	1.0088	0.1256	1.0086	0.1195	1.0097	0.1288
J = 15	5	1.0089	0.1256	1.0087	0.1196	1.0098	0.1288
GAP/R_p = 2.0	10	1.0089	0.1256	1.0087	0.1196	1.0098	0.1288
SS-SS	1	1.0525	0.3284	1.0524	0.3282	1.0525	0.3285
L_p/R_p = 500	2	1.0709	0.3354	1.0708	0.3352	1.0709	0.3355
J = 15	5	1.0772	0.3388	1.0771	0.3386	1.0772	0.3389
GAP/R_p = 0.5	10	1.0773	0.3389	1.0771	0.3387	1.0772	0.3390
CL-CL	1	1.0525	0.3284	1.0514	0.3249	1.0532	0.3309
L_p/R_p = 100	2	1.0709	0.3354	1.0697	0.3319	1.0716	0.3379
J = 15	5	1.0772	0.3388	1.0759	0.3352	1.0779	0.3413
GAP/R_p = 0.5	10	1.0773	0.3389	1.0760	0.3353	1.0779	0.3414