

## ABSTRACT

HAMANN, FRANZ A. Sovereign Risk and Macroeconomic Fluctuations. (Under the direction of Professor John J. Seater).

This dissertation investigates the properties of macroeconomic fluctuations in a small open economy under the presence of sovereign default risk. It also studies its potential impact on monetary policy. The first chapter briefly reviews the literature. The second presents a simple model of sovereign risk that explains how default can be triggered by shocks that drive normal business cycles, albeit in the context of an endowment economy. The model is calibrated to the Argentine economy. It is able to reproduce counter-cyclical country risk spreads, large capital outflows during Sudden Stops, and default. Using numerical methods, this paper also proposes an algorithm for the solution of this family of models that allows to generalize the results of Eaton and Gersovitz (1981) into environments with varying degrees of persistence and volatility in the underlying stochastic income process.

In the third chapter the assumption of an endowment economy is relaxed. Once again, the model is calibrated to the Argentine economy. It reproduces the main macroeconomic volatilities and correlations as consumption, investment and output, but fails to reproduce those of trade balance, current account and interest rates. Further research is needed in this direction.

The fourth chapter focuses on monetary policy in small open economies. It presents a dynamic stochastic general equilibrium model of inflation targeting in a small open economy. The model is calibrated to the Colombian economy to study the response of some macroeconomic variables to different types of shocks that are relevant for emerging economies. The sensitivity of those responses to some key parameters is also analyzed. Furthermore, using simulated data from the model, the ability of the model to capture the spectra, the phase and the coherence of observed output and inflation are studied.

**Sovereign Risk and Macroeconomic Fluctuations**

by

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To Tomás, Mateo and Margarita.

## Biography

I was born in Cali (Colombia) on 1972, but was raised in Palmira, a small colombian town during my early years. I studied at the German School (Deutsche Schule) in Cali for 13 years (from 1977 to 1990). That was the best time of my life. After High School and in a violent and unrested environment in Colombia (the beginning of the 90's), I went to the Universidad Javeriana in Bogotá to study Economics, due to political motivations. At that time I thought economics a way of understanding a society, so I decided to continue with a Master at the Universidad de los Andes in Bogotá. At the same time, I decided to marry and have a family. At "los Andes" I met Martha Misas, a prominent colombian econometrician. She asked me to join the econometrics group at the Banco de la República (the Central Bank of Colombia). I got the Master degree in 1997 and a year later I entered the Ph.D. program at North Carolina State University. At the time of this writing I am heading the Macroeconomic Modeling Department of the Banco de la República, hoping that something of what I have written in this dissertation can give important insights to the policy makers about the conduct of monetary policy in emerging markets.

## Acknowledgements

I am indebted to professors John Seater, Paul Fackler and Enrique Mendoza for their comments and helpful orientation. This dissertation has benefited from the comments of the participants at the Macroeconomics Seminar at NCSU, the International Economics Seminar at Duke University, the Macroeconomics Seminar at the Banco de la República and the IMF as well as the Conference "Inflation Targeting in the Americas" at the Federal Reserve Bank of Atlanta. I am also grateful to Dr. John Lapp who introduced me to the topics in monetary economics and provided me helpful comments for future work. The third chapter of my dissertation is a joint work with Alvaro Riascos, Paulina Restrepo and Juan Manuel Julio. I am indebted to them. We had helpful conversations with Martin Uribe. All the errors in this document are exclusively mine. I am also very thankful to the Banco de la República in Colombia which supported me financially during the first four years. Last but not least, I want to thank my family for their understanding and support, both emotionally and financially. Specially Margarita, without her I would not have finished this dissertation.

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# Chapter 1

## Introduction

The financial crises in emerging markets during the 90's have revealed some empirical regularities in their business cycles. Periods of financial distress in emerging economies are characterized by large current account reversals and Sudden Stops in capital inflows (Calvo, 1998 [13]), soaring sovereign country risk reflected in hikes in the international interest rates faced by the economy and large output contractions (Oviedo, 2004 [51] and Neumeyer and Perri, 2001 [49]), accompanied by collapses in equity prices (Mendoza and Smith 2001, [45]) and prices of nontradable goods relative to tradable goods (Mendoza, 2000 [44]). In several cases, the magnitude of the crises led countries to default on their outstanding debt (Argentina 2001, Russia 1998, Ecuador 1999 and Indonesia 1998).

Despite the renewed interest in studying defaults, this phenomenon is not new. According to Standard & Poor's figures, in the 1830's 31% of the total number of governments that issued sovereign debt had defaulted on their obligations (Beers, 2000 [8]).<sup>1</sup> At the beginning of the twentieth century (1920) a boom in sovereign lending ended with a large number of defaults (21%) during the Great Depression (Cantor and Packer, 1995 [16]).<sup>2</sup> During the postwar period until the 1970's the trend was a decreasing one. In 1975 less than 5% of the governments were in default. In that year the rate started to climb

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<sup>1</sup>Standard & Poor's survey tracks the number of sovereign debt issuers year by year since 1824, as well as the number of defaulting countries in foreign currency debt in loans and bonds. Nowadays there are 201 countries in the survey.

<sup>2</sup>These authors have documented that 21 out of 58 nations defaulted on international bonds between 1930 and 1935.

dramatically, in 1982 it reached the 25% to reach another historical peak in 1990 at 31%. Since then the rate of defaults has declined to 13% in 2000. These figures as well as the recent episodes of the Argentinian default (November 2001) and the defaults of Russia (1998), Ecuador (1999) and Indonesia (1998), provide evidence that default is relatively a frequent event in international lending. Cole and English (1995 [18]) and Beim and Calomiris (2001, [9]) have documented the default episodes from 1800 to 1992.

The statistical significance of the interaction between default and emerging markets crises has been highlighted by Reinhart (2002, [53]). The study finds that 84% of the defaults occurred in emerging markets are associated with currency crises and about 50% of the currency crises are linked to defaults. This result seems to be a particular characteristic of emerging markets, as no statistical significance of the relationship between default and currency crises is found for developed economies.

In addition to the occurrence of default, the behavior of the sovereign country risk reflected on the interest rate that the economy faces in the international credit markets is closely related to the sharp movements in the current account, the collapse in private consumption and the currency crises. This phenomenon has been labeled by Calvo as a “Sudden Stop”. Sovereign credit ratings (used as an indicator of the likelihood of default) not only have significant impact on sovereign bond yield spreads, but also serve as good predictors of the occurrence of defaults (Larran et al., 1997 [36]). In consequence, it is not surprising that during periods of financial distress lower ratings are observed and countries face more difficulties borrowing from international credit markets as they have to pay higher interest rates for limited amount of funds.

The Argentinian default in December 2001 provides us with a social experiment that illustrates the nature and consequences of a sovereign default. During most part of the 90’s Argentina was an example among Latin American countries of economic prosperity. From 1990 to 1998 real GDP grew at an average rate of 4.6% per year (even under the effects of the Mexican Crisis in 1995), the inflation rate was close to zero and the economy was enjoying the benefits of the access to international financial markets. At the beginning of 1998 the EMBI+ spread was about 400 basis points and the current account deficit was about 5% of GDP. Three years later this country end up in the largest default ever and a deep depression with large welfare losses.

The dynamics of the Argentinian crisis (and other emerging markets) and the empirical findings about default described above challenge the study of the business cycles

in small open economies. The smooth movements in the current account and the level of foreign debt, as well as the neutrality of the business cycle to the external interest rates shocks predicted by conventional models of business cycles in a small open economy<sup>3</sup> are inconsistent with the dynamics of the emerging markets crises and the Sudden Stops of capital inflows. One important reason of this inconsistency is the role assigned to international creditors. In conventional business cycle models, international credit markets are assumed to be perfect, a small open economy can borrow funds at a fixed risk-free rate up to a point limited only by the extent of their wealth. So, the challenge is to build a model in which the typical macroeconomic fluctuations co-exist with sharp and sudden movements of the current account, trade flows and interest rates. As highlighted by Arellano and Mendoza (2002, [6]), the common starting point of much of the literature on emerging markets crises has been the introduction of some type of financial-market imperfection that distinguishes emerging economies from industrial countries. However, the majority of the studies focuses on partial equilibrium models that *qualitatively* predict results consistent with the dynamics of the crises. Little is known about the *quantitative* properties of this type of models.

This dissertation is attempt to understand the dynamics of business cycle fluctuations in emerging markets. The second chapter presents a simple model of sovereign risk that explains how default can be triggered by shocks that drive normal business cycles, albeit in the context of an endowment economy. The model features incomplete external financial markets and the inability of the sovereign country to commit to repay debts. These two features coupled with risk neutral international lenders generate an endogenous risk premium and an endogenous borrowing constraint that drive the dynamics of default. The model is calibrated to the Argentinian economy. It is able to reproduce counter-cyclical country risk spreads, large capital outflows during Sudden Stops, and default. In a simple experiment conducted here, it is shown that by increasing trade sanctions to the artificial economy, it is possible to deter default but it is not possible to isolate the economy from the occurrence of Sudden Stops. Despite this, the welfare gains from eliminating default are very large: 7% of steady state consumption. Using numerical methods, this paper also proposes an algorithm for the solution of this family of models that allows to generalize the results of Eaton and Gersovitz (1981) into environments with varying degrees of persistence and volatility in the underlying stochastic income process.

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<sup>3</sup>See Mendoza (1991, [46]) and Correia et al. (1995, [19]).

In the third chapter the assumption of an endowment economy is relaxed and the welfare implications of default are studied. Allowing for capital accumulation has two implications on affecting incentives to repay. The first is that capital increases the likelihood of default because the country has an asset to save if defaults. The second is that, as markets are incomplete and the country engages in precautionary savings, capital serves as a buffer to mitigate consumption falls during recessions. The model is calibrated to the Argentinian economy and reproduces a limited number of the sample second moments of the macroeconomic time series. In particular, the model reproduces the main macroeconomic volatilities and correlations as consumption, investment and output, but fails to reproduce those of trade balance, current account and interest rates. Further research is needed in this direction to improve the ability of the model to reproduce these facts.

The fourth chapter focuses on monetary policy in small open economies. This work is part of a joint project with Alvaro Riascos, Paulina Restrepo and Juan Manuel Julio in developing a general equilibrium model to perform monetary policy analysis for the Central Bank of Colombia. The chapter presents a dynamic stochastic general equilibrium model of inflation targeting in a small open economy. The model is calibrated to the Colombian economy to study the response of some macroeconomic variables to different types of shocks that are relevant for emerging economies. The sensitivity of those responses to some key parameters is also analyzed. Furthermore, using simulated data from the model, the ability of the model to capture the spectra, the phase and the coherence of observed output and inflation are studied. A frequency domain comparison methodology proposed by Diebold, Ohanian and Berkowitz (1998,[50]) is followed. The Colombian data is characterized by: first, cyclical inflation and output gap (as measured by Hodrick-Prescott filter) are dominated by periodic movements between 2 and 25 quarters with a peak between 10 and 12 quarters. The cross spectrum and coherence show results in the same direction. Second, the coherence does not show any significant dominance of frequencies for the cross movements but the correlation jumps to 0,6 for periodic movements around 5 quarters. These facts are compared to the data simulated from the model. The simulated data spectra and cross spectra do not differ statistically from the respective population quantities up to at least 10 quarters. The model spectra presents more persistence than the observed data and the population coherence is captured for most frequencies but the ones around the peak of the model's theoretical coherence and very long run periodic movements. Subsequent research will address these issues.

## Chapter 2

# The Basic Model of Sovereign Risk

This chapter presents a simple quantitative model of a small open sovereign economy that cannot commit to repay its external debt to study the basic determinants of endogenous sovereign risk along the business cycle and the basic mechanics of a sovereign default. In the model, international financial markets are assumed incomplete. A sovereign country can only borrow or lend by holding a non-contingent asset. The country can default in any period at the cost of permanent exclusion from the international financial markets and a “trade sanction” in terms of output. This punishment has a fundamental role in determining the business cycle properties of this economy. The international financial market is characterized by risk neutral competitive lenders. They borrow funds at the risk-free rate and lend to the sovereign at the risk-free rate adjusted by a default premium. In equilibrium, this premium is determined by the sovereign’s inability to commit and the international lender’s zero profit condition. Since the markets are incomplete, default may also occur in equilibrium. In addition, the equilibrium exhibits an endogenous borrowing limit. This limit plays an important role in the dynamics towards default.

The model is calibrated to the Argentine economy to study its ability to replicate the coexistence of “normal” business cycle fluctuations with the sharp movements in capital flows, interest rates and consumption that characterize the process of default. The baseline calibration is able to capture most of the sample second moments of the Argentinian business cycle. However, this requires relatively large default penalties to match the historical Argentinian default rate. Departing from the baseline calibration, it is shown that

increasing the trade sanction serves as a “precommitment device” to avoid optimal default. This point was stressed by Bulow and Rogoff (1989). A caveat is that this “policy” is not enough to eliminate completely the Sudden Stops problem. The model allows to quantify these issues.

This work departs from the seminal work of Eaton and Gersovitz (1981, [22]) on sovereign debt repudiation, but extends the results to a more general set of conditions and offers a quantitative evaluation of this family of models. It is also related to the type of real business cycle models in small open economies developed by Mendoza (1995, [46]) albeit in the context of an endowment economy. The class of research performed here is related to diverse areas in international macroeconomics and finance. On the international macro side, there has been significant efforts for explaining the negative correlation between output and external interest rates at business cycle frequencies. Oviedo (2004) builds a small open economy real business cycle model with domestic neoclassical banks that intermediate funds between the domestic and foreign agents. The study finds that his model is unable to break the neutrality of the business cycle to external interest rates shocks. Neumeyer and Perri (2001) move in a similar direction but add an *exogenous* risk premium. They find that these shocks can account for about 50% of total output volatility. Motivated by this result, Uribe and Yue (2003) estimate a similar model using a VAR with the structure given by the theoretical model. They find that, taking into account that output fluctuations can also affect country interest rates only 12% of output volatility is explained by interest rates shocks. A common feature of this models is that the “risk premium” does not arise from the interaction of optimizing domestic and foreign agents.

On the finance side, this work is closely related to the work of Chatterjee, Corbae, Nakajima and Rios-Rull (2001) that studies the effect of default on aggregate debt market in a closed economy. The study reverts the results of Kocherlakota (1996, [33]) and Kehoe and Levine (1993, [31]), who find that state contingent bilateral contracts, where the punishment is the threat of permanent exclusion from the financial market, are enough to prevent default. The main difference is the degree of completeness in financial markets. Unlike Chatterjee et al. (2001), Kehoe and Levine (1993) and Kocherlakota (1996) models have a complete set of earning-contingent financial claims. When markets are complete, Alvarez and Jermann (2000, [3]) show that in high state earnings the incentives to default are higher, but since in equilibrium the individual rationality constraint is binding, it prevents equilibrium default. In Chatterjee et al. (2001) the contract between a bank and a household

is incomplete in the following sense: the contract can depend on the level of debt and the credit situation but cannot depend on current earnings. They find that this kind of model closely matches some relevant features of the total debt distribution in the U.S. economy.

The rest of the chapter proceeds as follows. Section 2.1 presents the structure of the model and characterizes the equilibrium. In section 2.2 the model is calibrated to the Argentine economy and the results are shown. The model is able to capture most of the sample second moments of the Argentine economy along with the dynamics of default. Section 2.3 presents a counterfactual exercise in which the default punishment is increased so as to deter equilibrium default. The results show that the economy is more stable, but still is not free of the Sudden Stop problem. Section 5 concludes and outlines the direction for future research.

## 2.1 The Model

There are two types of agents in the model. A small open economy representative agent (sovereign country) borrows funds from foreign risk-neutral competitive lenders.<sup>1</sup> The sovereign country can default on its debt but at the cost of permanent exclusion from future borrowing and a punishment in terms of output. Therefore its default decision is the result of optimally balancing the total cost of default, given by the forgone benefits of consumption smoothing and the output loss, against the costs of repayment, given by the short run disutility of repaying the loan in units of consumption. Risk-neutral lenders are able to assess the probability of default of the country and will restrict the amount of funds available for lending to limit optimally their degree of exposure to default risk. In this fashion, an endogenous probability of default and an endogenous borrowing constraint arise.

### 2.1.1 The Sovereign Country

Time is discrete, and each period the sovereign receives a stochastic endowment of a perishable consumption good that assumes discrete values defined over the set  $Y =$

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<sup>1</sup>There are many ways to decentralize this problem. One trivial decentralized version of this problem is to have three types of agents: domestic households, a domestic benevolent government and international lenders. The government borrows in the international financial market to finance lump-sum transfers to households and cannot commit to repay to international lenders.



$\{y_L, y_H\}$  with  $y_L < y_H$ . The country's endowment follows a Markov process with stationary transition probability  $\pi(y'|y) = \Pr(y_{t+1} = y' | y_t = y) > 0$  for  $y, y' \in Y$ . The preferences defined over consumption are given by:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

where  $\beta \in (0, 1)$  and  $u$  is a continuous, strictly increasing and strictly concave utility function. In this setting, the sovereign can smooth consumption by accessing the international credit market. The country can borrow funds in the form of one-period consumption loans available in the form of a set of discrete values,  $A = a_1, \dots, a_n$ . When  $a > 0$  the country is lending (at the risk free rate), and when  $a < 0$  the country is borrowing. The country can choose to repay or default on its outstanding debt. Let  $d \in \{0, 1\}$  denote a "credit rating" for a country, where  $d = 1$  indicates a default record and  $d = 0$  denotes no default record. The state of the economy at any point in time is described by a state vector  $s \in S$  where the state  $s = (y, a, d)$  indicates a country's the level of output  $y$ , net foreign assets  $a$ , and the default state  $d$ . The state space is  $S = Y \times A \times \{0, 1\}$ . Associated with each element of  $A$  there is a nonnegative loan contract price  $q_a \geq 0$ .

Consider the case in which the country starts the current period with some debt and has no default record. If the country decides to repay, then the resource constraint is given by the set:

$$\Gamma_0(s; q) = \{c \in \mathbb{R}_+, a' \in A : c + q_{a'} a' \leq y + a\} \quad (2.1)$$

where the subindex 0 on  $\Gamma$  denotes that the set is conditional on not defaulting. The price  $q$  is related to the interest rate that the country faces in the international credit market as  $q = \frac{1}{1+r}$  and depends on the endowment and debt level. A "purchase" of a one period loan with a negative value  $a$  means that the country has entered into a contract in which it promises to deliver, conditional on not defaulting,  $-a > 0$  units of the consumption good next period.

Consider now the case in which the country that starts the current period with no default record and some debt. If the country decides to default for the next period, then the following things happen:

1. The beginning of period debt level is set to zero, so that debt is discharged.
2. The credit rating in the next period will be set to a "default credit rating".

3. A country with a default credit rating cannot obtain any new loans and loses a fraction  $0 < \epsilon < 1$  of real output.
4. The country can never return to the international financial market.

Notice that if the country decides to default, it is not allowed to borrow new loans and has to consume the stochastic endowment thereafter so default is an absorbing state. The resource constraint in this case is:

$$\Gamma_1(s; q) = \{c \in \mathbb{R}_+, a' = 0 : c \leq y(1 - \epsilon)\}$$

The problem of the sovereign at time  $t$  can be described as exercising an option to default. Define  $Q = [0, \bar{q}]$  as the entire set of prices of a loan contract available to the country and  $\bar{q}$  is the price of a risk-free loan. Let  $v(s; q) : S \times Q \rightarrow R$  be a function that assigns a value, for each  $q$ , to  $s$ . For a given  $q$ , this value can be thought to represent the expected lifetime utility of the country in the case in which the current state is  $s$ . So,  $\beta Ev(s'; q)$  is the expected value this period of next period's lifetime utility if tomorrow's state is  $s'$ . Now, define an operator that yields the maximum lifetime utility achievable when the country's current endowment is  $y$  and its future lifetime utility is assessed according to a given function  $v$  for a given  $q$  as follows:

1. For  $a < 0$  and  $d = 0$ :

$$T(v) = \max \left\{ \max_{(c, a') \in \Gamma_0(s; q)} \{u(c) + \beta Ev(y', a', 0; q)\}, u(y(1 - \epsilon)) + \beta Ev(y', 0, 1; q) \right\} \quad (2.2)$$

2. For  $a \geq 0$  and  $d = 0$ :

$$T(v) = \max_{(c, a') \in \Gamma_0(s; q)} \{u(c) + \beta Ev(y', a', 0; q)\} \quad (2.3)$$

3. For  $d = 1$ :

$$T(v) = \max_{c \in \Gamma_1(s; q)} \{u(c) + \beta Ev(y', 0, 1; q)\} \quad (2.4)$$

The first part of this definition says that if the country has debt and has not defaulted, it has the option to default. If the value of the program of repaying the loans exceeds the value of the program to return to autarky, then the country will repay the loans. Otherwise, it will default. The second part of the definition states that when the country has no debt

or is a net lender, default is not an option. The third part says that, once the country is in default (i.e. in autarky) there is no possibility of borrowing or lending.

Equations (2.2) to (2.4) can be compactly expressed as  $T(v)(q) = v$ . Since  $S$  contains only a finite number of elements, there always exists a solution to these maximization problems. If  $v^*$  is the optimal value function, then the policy functions  $c : S \rightarrow R_+$ ,  $a : S \rightarrow A$  and  $d : S \rightarrow \{0, 1\}$  are the optimal decision rules provided  $c(s)$ ,  $a(s)$  and  $d(s)$  are measurable and satisfy (2.2)-(2.4). These sovereign's policy rules map the current state of the economy, represented in the level of output, loans and default record, into the consumption decision, the next period's loans level and the next period's default decision. In the next subsection the international lenders behavior is described.

### 2.1.2 International Lenders

International lenders are assumed to be risk neutral. They can borrow funds from an independent market at the risk free price,  $\bar{q}$ , and will lend as long as they are paid a return that is at least as high as the risk free rate,  $\bar{r}$ . For  $a < 0$ , if the country has no default record it can buy loan contracts in which a lender provides  $-q_{a'}a'$  units of the consumption good today in exchange in exchange for the country's promise to deliver, conditional on not defaulting,  $-a'$  units of the consumption good next period. For  $a \geq 0$  the country can buy, regardless of the default record, contracts in which a lender promises to provide for sure,  $a'$  units of the consumption good next period in exchange for  $q_{a'}a'$  units of the consumption good today. So, the lenders choose loans  $a'$  to maximize expected profits, taking  $q$  as given:

$$\Omega(a'; q) = -q_{a'}a' + \bar{q}(1 - \lambda)a'$$

where  $\lambda$  is the probability of default and  $\Omega$  are the profits. For  $a' \geq 0$ , the probability of default is zero and  $q = \bar{q}$ . For  $a' < 0$ , the price of the bond takes into account the possibility that the country may not repay the loans and so the expected return to the lenders must be at least as high as the risk free rate:

$$q_{a'} \geq [1 - \lambda_{a'}] \bar{q}. \quad (2.5)$$

In this setup there is a "natural debt limit" that lenders may impose on the sovereign. This limit is given by allowing the sovereign to borrow up to the annualized value of the highest possible realization of the stochastic endowment (in every future period)

discounted at the lowest possible interest rate. So,  $-\frac{y_H}{1-q}$  is the highest amount of debt that could be paid by the sovereign in the luckiest situation. It can be shown that a loan of this size (or larger) will have a price of zero in any equilibrium.<sup>2</sup> Therefore, for those loans the probability of default is one, regardless of the initial state of the economy. Setting the borrowing limit as described, will not have any effect on the equilibrium price of the loans. However, as it will be shown in the next section, the *equilibrium* borrowing limit in the model is also an optimal outcome that is closely linked to the probability of default.

### 2.1.3 Equilibrium

In this section we characterize the equilibrium concept used to solve the model. The equilibrium concept used is that one of a stochastic, stationary rational expectations equilibrium. Let  $d'(s)$  be a default decision function that take the value of 1 if the utility from defaulting strictly exceeds the utility from repaying and 0 otherwise.<sup>3</sup> Let  $\Lambda(s; q)$  be a set of default probabilities consistent with the optimizing behavior of the country in state  $s$  and given the loan's price  $q$ .<sup>4</sup> Now, by imposing the restrictions dictated by the optimizing behavior and the zero profit condition, define the loan's price correspondence is:

$$\varphi(q, a') = \begin{cases} \bar{q}(1 - \lambda) & \text{if } a < 0 \\ \bar{q} & \text{if } a \geq 0 \end{cases}$$

which determines the set of prices for a loan  $a'$  that are consistent with zero profits at a given price vector  $q$ .

The equilibrium is represented by a time invariant probability of default function,  $\lambda^* \in \Lambda(s; q)$ , a time invariant loan's price function  $q \in \varphi(q, a')$ , a time invariant optimal set of loans,  $A^*$ , and a time invariant optimal probability distribution over the state space of the small open economy,  $\psi$ . The stationary equilibrium is defined recursively. Given the

<sup>2</sup>Chatterjee et al. (2002) have shown the conditions under which this is true. In particular, they show that it is enough to require that consumption should never be below  $y_L(1 - \epsilon)$  and above  $y_H + a_{n_2} - a_1$ . These conditions are imposed in the numerical exercise performed in this paper. Also Aiyagari (1994,[1]) has shown that as consumption goes to zero its marginal utility goes to infinity, then an agent will never hold an amount of assets that may induce non-positive consumption. Thus a borrowing limit is implied by this fact. In his model the interest rate is a constant risk free rate, so the value of the borrowing constraint is the annuity of the worst possible income realization discounted at the risk free rate.

<sup>3</sup>If there are endowment levels at which the country is indifferent between defaulting and repaying, this is a default decision correspondence.

<sup>4</sup>Formally,  $\Lambda(s; q)$  is a set of nonnegative numbers such that if  $\lambda \in \Lambda(s; q)$  then there is a  $d'(s)$  such that  $\sum_{y'} \pi(y'|y) d'(s) = \lambda$ .

transition rules  $a(s)$  and  $d(s)$  and the transition probabilities  $\pi(y'|y)$ , the optimal transition probability  $P(s, B)$  can be computed.  $P(s, B)$  is the probability that a country in state  $s$  will reach a state vector that lies in  $B$ .<sup>5</sup>

More formally, the stationary stochastic rational expectations equilibrium of the model is characterized by:

1. an equilibrium loan's price function  $q^* \in \varphi(q^*, a'(s))$  and optimal set of default probabilities  $\Lambda(s; q^*)$
2. an optimal set of loans  $A^*$
3. an optimal borrowing policy  $a'(s)$  and default policy  $d'(s)$

such that:

1. The sovereign country solves (2.2)-(2.4).
2. Expected profits for international lenders are zero, i.e. (2.5) holds with equality.
3. The sovereign demand for loans is in the available set of loans,  $a'(s) \in A^*$

The first condition states that the sovereign country optimize to find a set of decision rules for consumption, loan allocation and default that depend on the current state of the economy. The second condition states that the profits for international lenders are zero, because the international loan market is competitive. The third condition is the market clearing condition. So,  $q^*$  is an equilibrium price vector if for all  $a' \in A$ ,  $q^* \in \varphi(q^*, a')$ .

There is nothing in the definition of the equilibrium that precludes the possibility that  $q^*$  may be zero for some  $a < 0$ . As mentioned earlier, this happens when the probability of default is one, regardless of the initial state of the economy. Then  $\Lambda^*(s; q^*) = \{1\}$  and  $q^* = 0$ . On one side, international lenders are indifferent whether to provide loans  $a'$  at a price of zero since the expected payoff is zero but so does the cost. On the other side, if the sovereign country takes one of these free loans, it gets nothing in the current period, but generates a liability for the next period. So, these loans will not be demanded either. This intuition forms the background for the computation of the borrowing limit.

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<sup>5</sup>The optimal transition probability  $P(s, B)$  is very useful to compute the probability of default. Since there are transient and absorbent states, the probability of default is given by the probability of reaching the set of absorbent states.

Intuitively, the lender evaluates the current states of nature for which it is not possible for the sovereign to repay in the following period. The lender fixes the borrowing limit at the lowest debt level for which default is a certain outcome. It may be possible that the sovereign would like to borrow a higher amount, but the lender will not be willing to engage in such a contract. In this sense, the model exhibits “credit rationing”. When such a situation arrives, it is not possible to smooth consumption and the economy may experiment large welfare losses.

## 2.2 Calibrating the Model to the Argentine Economy

In this section the model is calibrated to the Argentine economy to study its ability to reproduce the salient properties of the Argentinian business cycle and at the same to account for the behavior of the main macroeconomic variables during a default episode.<sup>6</sup> Calibrating the model amounts to select the parameters so that the statistics of the model match as closely as possible some relevant statistics of the Argentinian business cycle and its default episode in 2001. In particular, it is studied the model’s ability to reproduce some of the sample second moments, like volatility and correlations, of the observed Argentinian data between 1980 and 2003. First, in 2.2.1 some facts about the Argentinian business cycle and the 2001 default episode are presented. Then in 2.2.2 the benchmark calibration is described.

### 2.2.1 The Argentinian Business Cycle and the 2001 Default

This section presents the salient features of the Argentinian business cycle and its 2001 default. Two main sources of national accounts quarterly data are used: the Ministry of Finance data (MECON) for the 1980-2003 sample period and the IMF for the 1994-2003 sample period.<sup>7</sup> In addition, to obtain a proxy for the external interest rate, the procedure proposed by Neumeier and Perri (2001) is followed. All data are transformed to be model

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<sup>6</sup>There is nothing that prevents the use of this model to evaluate other country’s default experiences. For instance, there are quite striking similarities between the 2001 Argentinian default and the 1934 Colombian default experiences.

<sup>7</sup>Before the first quarter of 1993 the total consumption includes inventories and statistical discrepancy. The database used in this study is available at [http://www.webpondo.org/fhamann/data/arg\\_default.xls](http://www.webpondo.org/fhamann/data/arg_default.xls)

consistent.<sup>8</sup> To obtain measures of the business cycle fluctuations, consumption and output data are in logs and deseasonalized using the Census X12 filter. Then the Hodrick-Prescott filter is applied and a measure of the cyclical component of the time series is obtained. By using the HP filter we focus on business cycle frequencies corresponding to 8 years. The trade balance and the current account correspond to their shares in the GDP. No transformations are made to the interest rate.

Figure 2.1 illustrates the main properties of the Argentinian business cycle from 1994 to 2003. There are some relevant facts to note: first, after the Tequila crisis in 1995 the Argentinian economy recovered from a -6.5% gdp gap in the last quarter of 1995<sup>9</sup> to a 5% gap by the end of 1998. From then on Argentina struggled to return to sustainable growth rates. Second, the 2001 default episode increased the volatility of the main macroeconomic variables. The aggregate demand increases the cyclical volatility from about 5% during normal times to 15% during the default episode. A third fact is a large current account reversal accompanied by unprecedented high levels of external interest rates during the preceding quarters to the default episode.<sup>10</sup> The economy moved from a 4.7% current account deficit in the first quarter of 2001 to a 5% current account surplus in the last quarter of 2001. Fourth, a deep depression is registered later. In the first quarter of 2002, after defaulting on external debt in December 2001, the output gap was -10% and consumption was 15% below trend. These figures reflect the magnitude of the capital outflows that Argentina experienced during 2001 and the sharp depression that followed in 2002.

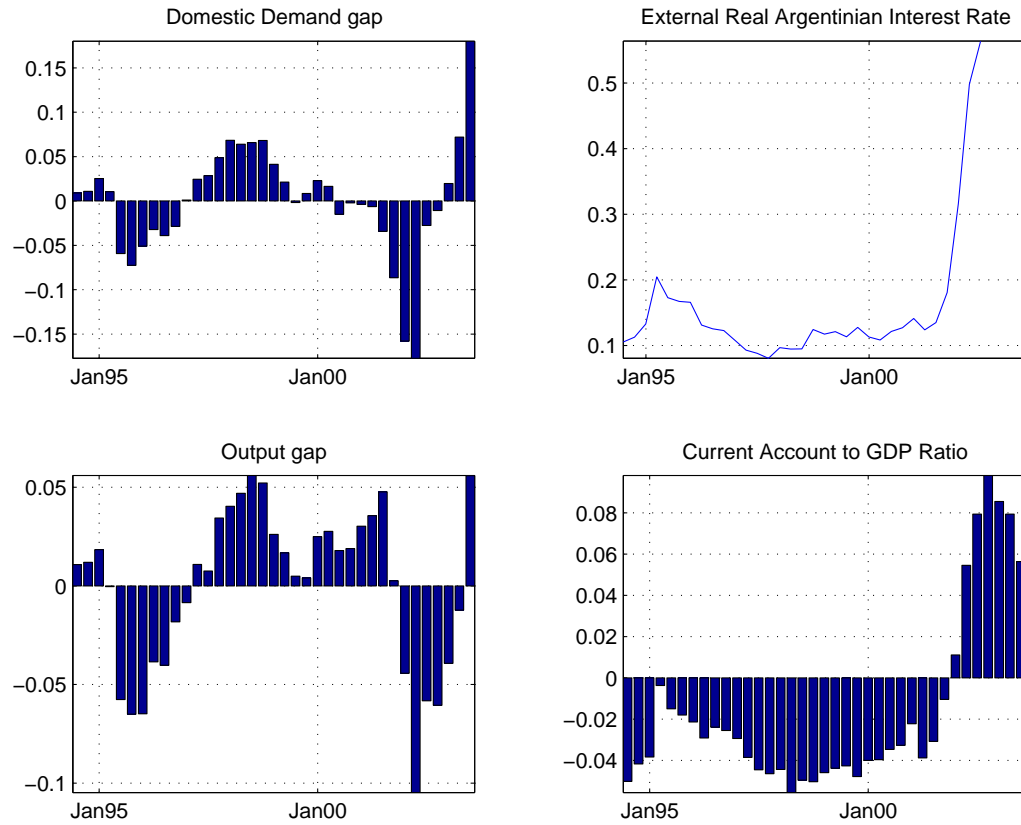
Table 2.1 shows the main correlations between the variables for the sample period 1980-2003, while Table 2.2 does it for 1994-2003. Some features of the data are worth noticing. First, with respect to macroeconomic instability: regardless of the sample period, output fluctuates about 4% from trend. Consumption is more volatile than output and deviates about 6% from trend. That consumption fluctuates more than output is not a standard result. This is the consequence of computing consumption as total absorption to make the data model-consistent. The trade balance volatility is between 3% and 6% depending on the sample period. The current account volatility is similar to that of output, about 4%. The external interest rate variability is between 8% and 12%.

<sup>8</sup>In particular, real consumption is measured as real absorption, since in this model there is no difference between private and public consumption and also there is no investment.

<sup>9</sup>The “gap” means the log deviation of the deseasonalized time series with respect to the Hodrick-Prescott filter.

<sup>10</sup>The EMBI+ spread started to climb steadily: by the end of 1999 the EMBI+ spread was 622 basis

Figure 2.1: The Argentinian Business Cycle 1994-2003



Source: International Financial Statistics (IMF), Neumeyer and Perri (2001) and author's calculations.

Table 2.1: Argentinian Business Cycle Statistics: 1980-2003

Variable	$\sigma_x$	$\rho(x, y)$	$\rho(x_t, x_{t-1})$
GDP	4.33	1.00	0.72
Consumption	6.13	0.99	0.76
Trade Balance	2.69	-0.91	0.87
Current Account	4.34	-0.91	0.66
Interest Rate	8.65	-0.92	0.78

Source: Neumeyer and Perri (2000) and author's calculations.



Table 2.2: Argentinian Business Cycle Statistics: 1994-2003

Variable	$\sigma_x$	$\rho(x, y)$	$\rho(x_t, x_{t-1})$
GDP	4.00	1.00	0.77
Consumption	6.25	0.82	0.62
Trade Balance	5.88	-0.87	0.91
Current Account	4.34	-0.90	0.90
Interest Rate	10.53	-0.88	0.67

Source: International Financial Statistics - IMF and author's calculations.

Second, with respect to the comovements of the macroeconomic time series: first, consumption is procyclical. Second, the trade balance, current account and the external interest rate are countercyclical. The countercyclicity of the trade balance and the external interest rates has also been widely documented in Oviedo (2004), Uribe and Yue (2003) and Neumeyer and Perri (2001).

### 2.2.2 The Baseline Model

This subsection discuss the calibration of the baseline model and evaluates the ability of the model to reproduce the Argentinian business cycle facts.

#### Calibration and Solution

Some parameters of the model are set to replicate some features of the data while others are taken from other studies. The functional form used to represent the preferences is a standard CES utility function,  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$  where  $\gamma$  is the coefficient of relative risk aversion. That parameter is set to 5, which is a standard value used in other studies for small open economies in emerging markets. The discount factor,  $\beta$ , is set at 0.9615, to match the annual average real interest rate of 17% observed in the data. This relatively large discount factor reflects the high degree of impatience that characterizes emerging market economies and is also used in other studies in emerging markets. The output process is assumed to be a two state discrete Markov Chain. Its standard deviation and the autocorrelation were set to match as closely as possible the observed cyclical properties of output. The default penalty,  $\epsilon$ , is fixed at 0.1, to match as closely as possible the historical default rates that points, 800 by the end of 2000 and 1761 in the third quarter of 2001.

Table 2.3: Calibration Values and Targets

Parameter	Symbol	Value	Target
Coefficient of Risk Aversion	$\gamma$	5	Other EM's studies
Discount Factor	$\beta$	0.9615	Observed real interest rate
Risk-free Interest Rate	$r$	0.01	U.S. quarterly interest rate of T-Bills
Output Volatility	$\sigma_y$	0.04	Argentinian GDP data
Output Autocorrelation	$\rho_y$	0.7	Argentinian GDP data
Default Penalty	$\epsilon$	0.1	Beim and Calomiris (2000)

Argentina has exhibited during the last two hundred years. Argentina defaulted one time during the nineteenth century and three times during the twentieth century. Beim and Calomiris (2000) report three default episodes: the first from 1890 to 1893, the second from 1956 to 1965 and the third from 1982 to 1992. The recent 2001 default episode is the fourth in history. This means that in about 200 years, Argentina has defaulted 4 times, and has remained in default during 26 years (about 10% of the time). The quarterly probability of default is rate is between 0.75% (if one considers only the 20th century) and 0.5% (if one considers the two centuries).

The required default penalty of 10% of GDP is a large fraction of output. Using a sample of 200 countries from 1948 to 1997, Rose (2002, [54]) presents evidence of a reduction in bilateral trade flows of 8% once the countries engage in a debt restructuring process. The value of the obtained sanction is 2% of GDP higher than this number, but in the numerical computations this figure is required to match the observed default rates described above.

The solution of the model is performed by state-space discretization. The state space  $S$  is partitioned in an equally spaced grid of 2000 points.  $A$  is initialized using an equally spaced grid in the interval  $[-\frac{y_H}{1-q}, 0]$ . The solution algorithm also exploits the recursive nature of the problem. It starts by guessing a loan's price function  $q$  defined on a given  $A$ , and given those two objects, the sovereign solves the problem (2.2)-(2.4). This yields a probability of default function which permits to compute a new  $q$  and  $A$  by using the lender's profit maximization condition, equation (2.5). The algorithm's details are described in Appendix A.

## Results

Figure 2.2 presents the equilibrium price of the loans as a function of the net foreign assets to output ratio. For the baseline calibration, low debt to output ratios (less than 20% of output) would show sovereign bond prices close to the risk free bond prices (near 1). As the debt to output ratios approach the borrowing limit (about 60% in the baseline calibration) the price of the bond approaches 0.

A particular feature of the model is that once the economy enters in the red zone (between 20% to 40% of debt to output ratio) the sovereign bond price can vary substantially. This result is important because a negative shock can easily drive the economy into this zone. If the shock is persistent, as it is in the baseline calibration, typically the country enters in a spiral of higher debt requirements and high interest payments until the point in which consumption is so low that the optimal outcome is to default.

Intuitively the slope of the equilibrium sovereign bond price can be interpreted as the marginal cost of borrowing. This marginal cost takes into account the total costs of defaulting (financial autarky and the trade sanction) and it is balanced against the marginal benefit: the discount rate. For a given discount factor, as the price of the loans falls it induces the country to borrow more (given the interest rate). But as borrowing increases so does the probability of default, which is reflected in the interest payments. Once the country is in the red zone the interest payments can drive consumption so low that induces default. In this context, a non-contingent one-period loan is a weak insurance instrument against idiosyncratic shocks.

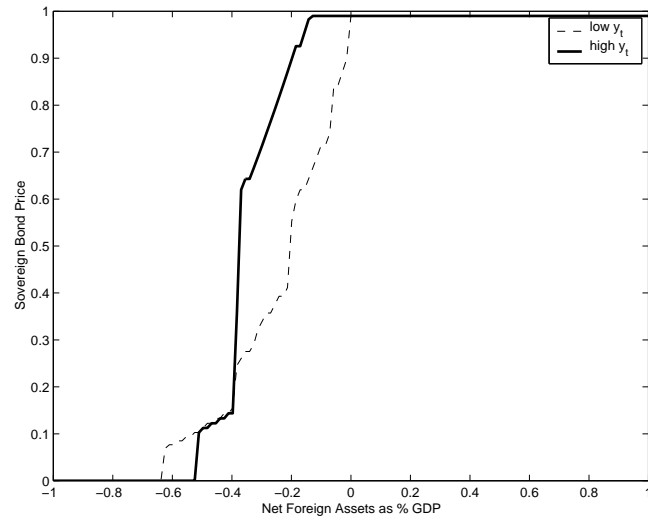
The calibrated model is used to simulate artificial data. These data are qualitatively contrasted against the observed data. The same transformations that were made to the observed data are made to the simulated data.<sup>11</sup> The comparison is made by qualitatively observing the sample second moments of both types of data. The simulation is initialized at  $s_0 = (y_H, 0, \bar{a})$ , that is at a high level of output, good default rating and the average debt level. The initial realizations of the simulation are discarded to avoid any dependence on the initial values. The last portion of the simulated data (38 data) that corresponds to the observed sample (1994:1-2003:2) is kept.

Table 2.4 reports the results. The calibrated model is able to reproduce some of the sample second moments of the observed Argentine time series. In particular, the baseline

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<sup>11</sup>Output was not filtered since it is a two-state Markov Chain.

Figure 2.2: Equilibrium Sovereign Bond Prices



Student Version of MATLAB

Table 2.4: Business Cycle Statistics of Benchmark Model

Variable	$\sigma_x$	$\rho(x, y)$	$\rho(x_t, x_{t-1})$
GDP	3.98	1.00	0.79
Consumption	6.45	0.92	0.64
Trade Balance	9.11	-0.61	-0.83
Current Account	11.78	0.68	0.08
Interest Rate	12.20	-0.46	-0.79

calibration is able to mimic the consumption and interest rates volatilities, but overestimates the trade balance and current account ones. A particular result of this model is that consumption is more volatile than output. This is explained by the presence of sovereign risk and the borrowing constraint coupled with equilibrium default. First, sovereign risk itself limits the ability of the sovereign to smooth consumption by imposing a borrowing constraint during bad times. A sufficiently low realization of output may lead the country to borrow above the equilibrium debt limit. When this limit binds, consumption falls more than output. Second, if the fall in consumption is sufficiently large it may trigger default. After default, the economy goes to financial autarky.

Figure 2.3 shows the simulation of two “successive” default episodes. The time paths of the variables are built under the assumption of an instant return to the financial markets after default. This assumption only serves to illustrate the basic mechanics of default in the model, not to simulate the model. Notice how, during the previous quarters to default the country smooths consumption and the risk premium fluctuates around average levels. Then a low output realization occurs, and the country runs into relatively large trade deficit trying to keep consumption stable. As the shock is very persistent, consumption keeps falling, sovereign bond prices plummet as country risk increases rapidly. At some point the country cannot borrow above the limit. A Sudden stop is reflected in a large reversion of the trade deficit into a trade surplus (a large capital outflow), such a movement occurs in tandem with the default decision.

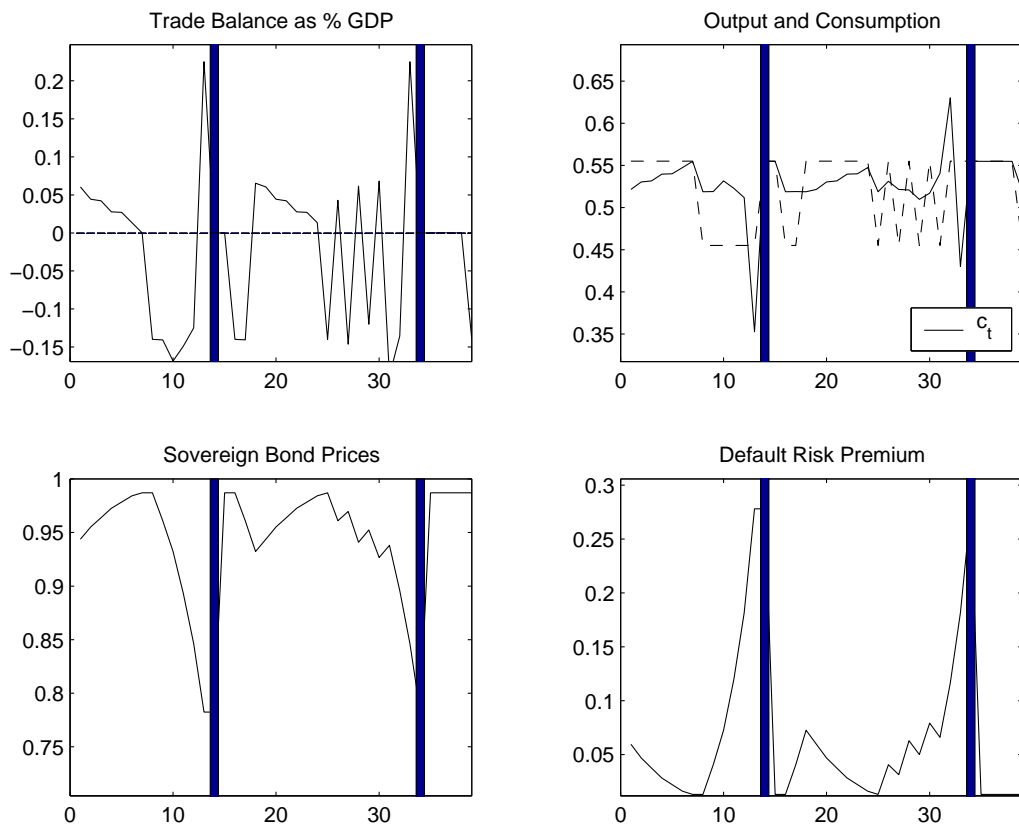
In the context of this model, it is not surprising that an economy that exhibits a relatively large rate of default exhibits a very volatile absorption. It will be shown in section 2.3 that as the trade sanctions increase, the rate of default falls and so the consumption variability falls below output variability and the properties of the model approach those of a small open endowment economy with incomplete markets.

The model also captures the fact that consumption is procyclical and the trade balance and the interest rate are countercyclical. The first two results are a consequence of market incompleteness and the type of preferences used in the model.<sup>12</sup> When markets are incomplete the consumption output correlation depends on the sign of the third derivative of the utility function. The CRRA utility function in this paper has a positive third derivative. This implies that, for a given interest rate, the uncertain future endowment induces

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<sup>12</sup>Markets are “incomplete” in the sense that the economy can not borrow contingent on the realization of output.

Figure 2.3: Two Simulated Default Episodes



the sovereign to be “more prudent” and to save some output in case of a low realization tomorrow. As the country accumulates assets in good times but accumulates debt in bad times to smooth consumption, the trade balance is countercyclical.

The interest rate that the country faces internationally is countercyclical because it depends on the probability of default, which in turn increases positively with the level of debt. As the sovereign borrows more during low output periods, the probability of default is higher and so does the interest rate.<sup>13</sup>

The model fails to capture a countercyclical current account. This is also a well known feature of small open economy models with a stochastic endowment and a single non-contingent asset used to smooth consumption. The sovereign has a precautionary savings motive to accumulate assets in good times and accumulate debt in bad times. This generates the procyclical pattern of the the current account.

### 2.3 Is There a Case for Trade Sanctions?

This section evaluates the sensitivity of the results to varying degrees of trade sanctions to the economy. It is shown how as the size of the punishment increase the default rate falls. This mechanism serves as a “precommitment device”. In the model, the total cost of default is not only the cost of financial autarky, but also the trade sanction. If the trade sanction is too low, the country will default “too fast”. For some values of the parameters of the model, financial autarky may not be enough to deter default. If the trade sanction is high enough, it may help to deter default.<sup>14</sup>

Table 2.5 presents the sample second moments of baseline model when the punishment is such that it induces the country not to default in any state of nature. This is done by raising  $\epsilon$  from 0.1 to 0.35. This is obviously an extremely large sanction, but it serves to test the consistency of the results. Notice how consumption volatility falls from 6% to about 4%. It is still is high a figure (close to output’s volatility) but lower than in the case in which the economy defaults. The volatilities of the external accounts and the external interest rate are cut to about a third of the original figures. The model is able to capture the well known fact that economies that do not experience default exhibit more

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<sup>13</sup>This result was first derived by Eaton and Gersovitz (1981).

<sup>14</sup>The empirical significance and the theoretical details of trade sanctions are discussed in Bulow and Rogoff (1989, [11, 12]).

Table 2.5: Business Cycle Statistics of “Default-Free” Economy

Variable	$\sigma_x$	$\rho(x, y)$	$\rho(x_t, x_{t-1})$
GDP	4.20	1.00	0.79
Consumption	3.95	0.97	0.40
Trade Balance	3.71	0.36	-0.13
Current Account	3.88	0.51	-0.48
Interest Rate	3.53	-0.16	-0.51

Source: author’s calculations

stable balance of payments and interest rates.

Figure 2.4 shows the ergodic properties of this “default-free” Argentinian economy. The first two upper panels show the optimal debt and consumption policies,  $a(s)$  and  $c(s)$  respectively. The mid panels show the value function,  $v(s)$  and the equilibrium price of the loans,  $q^*$ . In this case the equilibrium price is not that steep as in the baseline calibration. Note also the the borrowing limit is looser (85% in this case). The lower panels show the ergodic density and distribution, respectively. Notice that their properties resemble those of a small open economy with incomplete markets and borrowing constraints. The spike at the borrowing limit is a defining feature of this type of economies.

There are important policy implications of raising the trade sanction to mitigate the lack of commitment problem on the side of the debtors. First, it may deter default, but it doesn’t eliminate the possibility of Sudden Stops of capital inflows. Although the borrowing limit is looser than in the baseline model, the economy may still experiment Sudden Stops. Notice how the baseline economy with high sanctions spends a large fraction of time (about 2.5%) in a debt constrained state of nature. Furthermore, the bulk of the distribution of debt corresponds to high levels of indebtedness. So, even if trade sanctions are successful preventing default, the economy will exhibit high consumption volatility due to the presence of borrowing limits, and large levels of indebtedness. Second, since the occurrence of default limits the ability of the small open economy to smooth consumption over time, there may be large welfare gains of deterring default. In the model, default can be avoided by imposing large trade sanctions. A trade sanction reduces the likelihood of a sovereign default and may induce some welfare gains, since the costs of the sanctions only materialize in the event the economy defaults.

A first approximation to these gains can be computed performing a simple com-



Figure 2.4: Equilibrium Properties with Default-detering Punishment

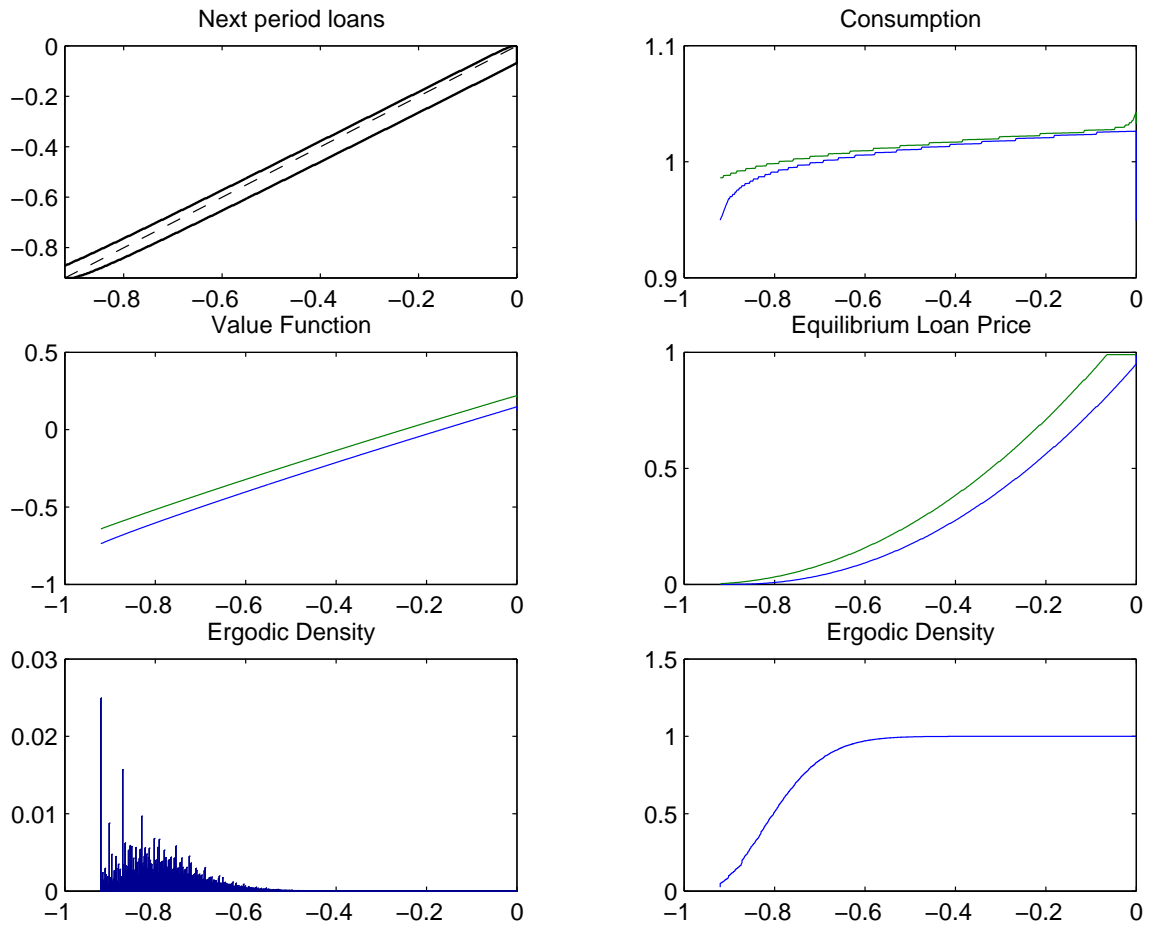


Table 2.6: Welfare Gains of Deterring Default

Economy	$\bar{c}$	$\sigma_c$	$\bar{a}$	default rate
Benchmark	0.988	0.06	-0.44	0.006
Default-free	1.002	0.04	-0.62	0

putation a la Lucas (1987, [40]) as follows:

$$v(s; q^*) = \phi(s, q^*, \tilde{q})^{1-\gamma} w(s; \tilde{q})$$

where  $v(s; q)$  is the value function of the benchmark default-prone economy in state  $s$  and loan price  $q^*$ ,  $w(s; \tilde{q})$  is the value function of the high-punishment and default-free economy in state  $s$  and loan price  $\tilde{q}$  and  $1 - \phi(s, q^*, \tilde{q})$  is the fraction of consumption the sovereign is willing to give up if confronted with a default-free economy. Using the invariant measure of the default-free economy and the benchmark economy, the average steady state welfare gain is 7% of consumption. This cost is extremely large, meaning that Argentines would be willing to give up a large fraction of consumption to avoid default. Where do these gains come from? Table 2.6 presents the steady state properties of both economies. Note that, first average consumption is larger by 1.4% in the default-free economy; second, consumption volatility is smaller in 2%; third, average debt is smaller in the benchmark economy. The steady state welfare gain comes from a higher and more stable consumption.

These computations are preliminary and depend crucially on the fact that the economy has no other asset to insure against idiosyncratic risk. Allowing the economy to accumulate capital may reduce significantly these welfare gains. A more complete analysis in this direction is presented in the next chapter.

## 2.4 Conclusion

This chapter has presented a simple stochastic small open economy model that captures the basic dynamics of the macroeconomic fluctuations and default. In particular, the model is calibrated for Argentina and it is able to reproduce the following facts: countercyclical country risk spreads, large capital outflows during Sudden Stops, and default. In the model, the interaction between a sovereign country and international lenders determines endogenously the country risk premium and a limit to indebtedness. As inter-

national financial markets are assumed incomplete, a sovereign country that tries to insure against idiosyncratic movements in output may find optimal to default. The punishment is the permanent exclusion from the international financial markets and a trade sanction. Fully-informed international lenders limit their exposure to default risk by restricting the amount of funds available to the country.

Given that the model matches most of the sample second moments of the Argentine time series, it is used to perform a simple quantitative experiment and answer two questions: how large should be trade sanctions to deter default? What are the consequences of such policy? The results show that by increasing trade sanctions to 35% of output, it is possible to deter default. The logic of this result is not new. The novelty is that this policy does not isolate the economy from the Sudden Stop problem. Although the economy would not default, it would spend 2.5% of time borrowing constrained. Despite this, the welfare gains from eliminating default are substantial (7% of steady state consumption). These gains come from a higher and more stable steady state consumption. The robustness of this result to more general environments is still an open question.

An additional contribution of the paper is that it proposes an algorithm for the solution of this family of models that allow to generalize the results of Eaton and Gersovitz (1981) into more environments with varying degrees of persistence and volatility in the underlying stochastic income process.

The model has several drawbacks. The main limitation is that the environment in which agents interact is a pure exchange economy. The agents have no other alternative than borrowing and lending in the international credit market to insure against stochastic shocks. A deeper understanding of the dynamics of the emerging market crises requires a model that allows domestic capital accumulation as an additional savings channel. This natural extension permits the study of the effect of sovereign country risk on the interaction of savings and investment. The endogeneity of the default risk premium would break the type of Fisherian separation in savings and investment decisions that characterizes conventional models of business cycles in small open economies. The international interest rate may play a more important role as a determinant of the macroeconomic fluctuations.

Another limitation is that the model lacks a mechanism to capture the domestic liability dollarization. Recently Calvo, Iquierdo and Mejia (2004, [14]) have shown that this channel seems to be important in reproducing the real exchange rate collapses that occur during Sudden Stops. Arellano (2004, [5]), following the algorithm proposed here, has made

progress in that direction.

In addition, the type of debt contract studied here rules out the possibility of re-entering the international credit market. There is no possibility of re-contracting. If re-contracting is allowed, one can conjecture that the cost of exclusion will be lower and so the probability of default may be higher, all other things equal. An extension in that direction would bring the model closer to reality.

An interesting line of research would be to study what is the impact of monetary and fiscal policy on the occurrence of default. Recently many emerging markets are conducting monetary policy by modifying the nominal interest rate to target inflation. As monetary policy has real effects in the short-run, it may be possible that monetary policy has destabilizing effects and magnify the effects of exogenous shocks. Or it may be possible that it helps to stabilize and contribute in the opposite direction. Models of this type are well suited to study these issues.

## Chapter 3

# On the Welfare Gains of Reducing the Likelihood of Default in Emerging Markets

In 2001 the Argentinian real GDP fell by 20 percent and investment by more than 20 percent. At same time, the Sudden Stop of capital inflows moved the current account from a sizeable deficit (4% of GDP) to a huge surplus (10% of GDP) and the Argentine external debt over GDP ratio increased so much that it forced the Argentinian government to default. The Argentinian crises, as other financial crises in emerging markets during the 90's, revealed some empirical regularities in their business cycles. Periods of financial distress in emerging economies are characterized by large current account reversals and sudden stops in capital inflows (Calvo, 1998 [13]), soaring sovereign country risk reflected in hikes in the international interest rates faced by the economy and large output contractions (Oviedo, 2004 [51] and Neumeyer and Perri, 2001 [49]), accompanied by collapses in equity prices (Mendoza and Smith 2001, [45]) and prices of nontradable goods relative to tradable goods (Mendoza, 2000 [44] and Arellano 2004, [5]). In addition to the economic figures, the large degree of political unrest and social discomfort suggest that the welfare losses

associated to the crises and default might have been large.

The Argentinian default in December 2001 provides us with a social experiment that illustrates the nature and consequences of a sovereign default. During most part of the 90's Argentina was an example among Latin American countries of economic prosperity. From 1990 to 1998 real GDP grew at an average rate of 4.6% per year (even under the effects of the Mexican Crisis in 1995), the inflation rate was close to zero and the economy was enjoying the benefits of the access to international financial markets. At the beginning of 1998 the Argentine EMBI+ spread was about 400 basis points and the current account deficit was about 5% of GDP. Three years later this country end up in the largest default ever and a deep depression. How large was the welfare loss?

This chapter offers a first answer to that question. In particular, this work quantifies the welfare gains of reducing the likelihood of a sovereign default? Second, what are the determinants of those welfare gains? To answer these questions, this paper uses a quantitative model of a small open economy that cannot to commit to repay its external debt. In the model, the cost of default is the permanent exclusion from international financial markets and a punishment in terms of real output. The international financial market is characterized by risk neutral competitive lenders. They borrow funds at the risk-free rate and lend to the sovereign at the risk-free rate adjusted by a default premium. In equilibrium, this premium is determined by the sovereign's inability to commit and the international lender's zero profit condition. Since the markets are incomplete, default may also occur in equilibrium. In addition, the equilibrium exhibits an endogenous borrowing limit. This limit plays an important role in the dynamics towards default. Here, market incompleteness may drive the economy to optimal default but, unlike Hamann (2004) the country has an additional asset to smooth consumption: domestic capital.

The capital stock has a dual role in a model in which the country is not able to commit to repay its debt. The first role is that capital is an additional asset that allows the country to save and insure against bad shocks. A country that smooths consumption, may either increase debt or reduce capital when a low productivity shock hits the economy. So, through this channel capital helps to reduce the incentives to default during recessions. The second channel works in the opposite direction: as capital grows, all other things equal, autarky becomes a more attractive option. Since domestic capital cannot be expropriated, it increases the likelihood of default. These two channels change the incentives to default relative to an endowment economy.

Another reason to include capital is that it may generate a countercyclical current account and procyclical capital flows, as it is observed in emerging economies data. In an economy in which business cycles are driven by productivity shocks, a positive temporary shock leads to an increase in saving for consumption smoothing and to an increase in investment, as the expected return on capital increases. If the investment effects dominates the savings effect, then external borrowing is procyclical and the current account is countercyclical. This fact fails to be reproduced in standard models of incomplete markets.

The model is calibrated to the Argentine economy to study the determinants of the welfare gains of reducing the likelihood of sovereign defaults. In studying the determinants of the welfare gains from reducing default, the focus is on one type of policy: direct penalties. These penalties try to capture the idea that default may entail trade sanctions from other countries. These trade sanctions can be very costly. Using a sample of 200 countries from 1948 to 1997, Rose (2002, [54]) presents evidence of a reduction in bilateral trade flows of 8% once the countries engage in a debt restructuring process.

In the model these direct penalties can be approximated by a punishment in terms of output. There is a large body of literature that has studied the role of such penalties. It has been shown that increasing the penalty serves as a “precommitment device” to avoid optimal default. This point was stressed in Bulow and Rogoff (1989). A caveat, shown in Hamann (2004), is that this “policy” is not enough to eliminate completely the Sudden Stop problem. This is important because the occurrence of Sudden Stops (without default) may still deliver large welfare losses, although smaller than in the case of default. In that paper, although raising sanctions does not eliminate Sudden Stops, it still entails large welfare gains equivalent to a 7% increase in steady state consumption. These gains may be exaggerated by the endowment economy assumption. This paper tests the robustness of those results in an environment in which output is endogenous and the economy can save in a domestic asset.

This work extends the seminal work of Eaton and Gersovitz (1981, [22]) on sovereign debt repudiation, and relates it to the type of real business cycle models in small open economies developed by Mendoza (1995, [46]). Also, the class of research performed here is related to diverse areas in macroeconomics, international macroeconomics and finance. On the macroeconomics literature the welfare calculations performed here follow the methodology proposed by Lucas (1987, [40]). Lucas argued that the welfare gains from eliminating postwar consumption volatility in the U.S. were negligible (0.001% of annual consumption).

This result originated a large amount of research evaluating the sensitivity of Lucas conclusion. Broadly speaking, two aspects have been tested: one group of studies relaxes the type of preferences, maintaining the complete markets assumption while the other keeps the standard preferences assumption and studies the implications of market incompleteness.<sup>1</sup> Nowadays, Lucas' computation has reached the status of conventional wisdom: the welfare gains from stabilization policies in the United States are insignificant.

This result contrasts with similar calculations performed for emerging markets. For example, Obstfeld and Rogoff (1998) show that the welfare cost of permanent exclusion from international financial markets ranges between % and % of real GDP for several emerging economies. That computation is significantly higher than Lucas'. Also, it is made under the assumption of an endowment economy under full-commitment to repay external debt. As mentioned above, in this chapter these two assumptions are relaxed to assess the welfare gains of reducing the likelihood of default.

The rest of the paper proceeds as follows. Section 3.1 presents the structure of the model and characterizes the equilibrium. In section 3.2 the model is calibrated to the Argentinian economy and the results are shown. The welfare analysis is presented in section 3.3. Section 3.4 performs a sensitivity analysis of the results. The last section concludes.

### 3.1 The Model

There are two types of agents in the model. A small open economy representative agent (sovereign country) borrows funds from foreign risk-neutral competitive lenders and accumulates capital to smooth consumption.<sup>2</sup> The sovereign country can default on its debt but at the cost of permanent exclusion from future borrowing and a punishment in terms of output. Therefore its default decision is the result of optimally balancing the total cost of default, given by the forgone benefits of consumption smoothing and the output loss, against the costs of repayment, given by the short run disutility of repaying the loan in units of consumption. Risk-neutral lenders are able to assess the probability of default of

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<sup>1</sup>Some studies that belong to first group are Pemberton (1996, [52]), Dolmas (1998, [20]), Alvarez and Jermann (1999, [2]) and Tallarini (2000, [57]). To the second belongs: Imrohroglu (1989, [30]), Atkeson and Phelan (1994, [7]) and Krusell and Smith (1999, [34]).

<sup>2</sup>There are many ways to decentralize this problem. One trivial decentralized version of this problem is to have three types of agents: domestic households, a domestic benevolent government and international lenders. The government borrows in the international financial market to finance lump-sum transfers to households and cannot commit to repay to international lenders.



the country and will restrict the amount of funds available for lending to limit optimally their degree of exposure to default risk. In this fashion, an endogenous probability of default and an endogenous borrowing constraint arise.

### 3.1.1 The Sovereign Country

#### Production Technology

Time is discrete, and each period the sovereign produces an internationally tradable good using the technology:

$$F(\epsilon_t, k_t) = \epsilon_t k_t^\alpha - \frac{\phi}{2}(k_{t+1} - k_t)^2$$

where  $k_t$  is the capital stock at the beginning of period  $t$ ,  $\epsilon_t$  is the level of productivity at the beginning of period  $t$ ,  $\frac{\phi}{2}(k_{t+1} - k_t)^2$  is the cost of adjusting capital as a function of net investment and  $0 < \alpha < 1$ . The productivity process follows a discrete stochastic process defined over the set  $E = \{\epsilon_L, \epsilon_M, \epsilon_H\}$  with  $\epsilon_L < \epsilon_M < \epsilon_H$ . More specifically,  $\epsilon$  follows a Markov process with stationary transition probability  $\pi(\epsilon'|\epsilon) = \Pr(\epsilon_{t+1} = \epsilon' | \epsilon_t = \epsilon) > 0$  for  $\epsilon, \epsilon' \in E$ . The capital stock evolves according to:

$$k_{t+1} = (1 - \delta)k_t + i_t$$

where  $i_t$  is gross investment during period  $t$  and  $\delta$  is the depreciation rate.

#### Preferences and Resource Constraint

The sovereign's preferences defined over consumption are given by:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

where  $\beta \in (0, 1)$  and  $u$  is a continuous, strictly increasing and strictly concave utility function. In this setting, the sovereign can smooth consumption by accessing the international credit market. The country can borrow funds in the form of one-period consumption loans,  $a \in A$ . If  $a > 0$  the country is lending (at the risk free rate), but if  $a < 0$  the country is borrowing. The country can choose to repay or default on its outstanding debt. Let

$d \in \{0, 1\}$  denote a “credit rating” for a country, where  $d = 1$  indicates a default record and  $d = 0$  denotes no default record.

The country’s state position at a point in time is described by a state vector  $s \in S$  where the state  $s = (\epsilon, k, a, d)$  indicates a country’s net foreign assets,  $a$ , the capital stock,  $k$ , the level of productivity,  $\epsilon$ , and the default rating,  $d$ . The state space is  $S = E \times K \times A \times \{0, 1\}$ . Associated with each element of  $A$  there is a nonnegative loan contract price  $q \geq 0$ . To simplify the notation, the dependence of  $q$  on  $a$  and  $k$  has been suppressed.

Consider the case in which the country starts the current period with some debt and has no default record. If the country decides to repay, then the resource constraint is given by the set:

$$\Gamma_0(s; q) = \{c \in \mathbb{R}_+, a' \in A, k' \in K : c + qa' \leq F(\epsilon_t, k_t) + a + (1 - \delta)k - k'\} \quad (3.1)$$

where the subindex 0 on  $\Gamma$  denotes that the set is conditional on not defaulting. The price  $q$  is related to the interest rate that the country faces in the international credit market as  $q = \frac{1}{1+r}$  and depends on the endowment level and the level of debt. A “purchase” of a one period loan with a negative value  $a$  means that the country has entered into a contract in which it promises to deliver, conditional on not defaulting,  $-a > 0$  units of the consumption good next period.

Consider now the case in which the country that starts the current period with no default record and some debt. If the country decides to default for the next period, then the following things happen:

1. The beginning of period debt level is set to zero, so that debt is discharged.
2. The credit rating in the next period will be set to a “default credit rating”.
3. A country with a default credit rating cannot obtain any new loans and loses a fraction  $0 < \theta < 1$  of real output.
4. The country can never return to the international financial market.

The penalty when the country defaults aim to capture direct trade sanctions imposed by other countries to the defaulting country. Using a sample of 200 countries from 1948 to 1997, Rose (2002, [54]) presents evidence of a reduction in bilateral trade flows of 8% once the countries engage in a debt restructuring process.

Notice that if the country decides to default, it is not allowed to borrow new loans and has to consume the stochastic endowment thereafter so default is an absorbing state. The resources constraint in this case is:

$$\Gamma_1(s; q) = \{c \in \mathbb{R}_+, a' = 0, k' \in K : c \leq (1 - \theta)F(\epsilon_t, k_t) + (1 - \delta)k - k'\}$$

The problem of the sovereign at time  $t$  can be described as exercising an option to default. Define  $Q = [0, \bar{q}]$  as the entire set of prices of a loan contract available to the country and  $\bar{q}$  is the price of a risk-free loan. Let  $v(s; q) : S \times Q$  be a function that assigns a value, for each  $q$ , to  $s$ . For a given  $q$ , this value can be thought to represent the expected lifetime utility of the country in the case in which the current state is  $s$ . So,  $\beta E v(s'; q)$  is the expected value this period of next period's lifetime utility if tomorrow's state is  $s'$ . Now, define  $T$  as the operator that yields the maximum lifetime utility achievable when the country's current state is  $s$  and its future lifetime utility is assessed according to a given function  $v$  for a given  $q$  as follows:

1. For  $a < 0$  and  $d = 0$ :

$$T(v) = \max \{w^r, w^d\} \quad (3.2)$$

where

$$w^r = \max_{(c, a', k') \in \Gamma_0(s; q)} \{u(c) + \beta E v(\epsilon', k', a', 0; q)\} \quad (3.3)$$

and

$$w^d = u((1 - \theta)F(\epsilon, k)) + \beta E v(\epsilon', k', 0, 1; q) \quad (3.4)$$

2. For  $a \geq 0$  and  $d = 0$ :

$$T(v) = \max_{(c, a', k') \in \Gamma_0(s; q)} \{u(c) + \beta E v(\epsilon', k', a', 0; q)\} \quad (3.5)$$

3. For  $d = 1$ :

$$T(v) = \max_{(c, k') \in \Gamma_1(s; q)} \{u(c) + \beta E v(\epsilon', k', 0, 1; q)\} \quad (3.6)$$

The first part of this definition says that if the country has debt and has not defaulted, it has the option to default. If the value of the program of repaying the loans exceeds the value of the program to return to autarky, then the country will repay the loans. Otherwise, it will default. The second part of the definition states that when the country has no debt

or is a net lender, default is not an option. The third part says that, once the country is in default (i.e. in autarky) there is no possibility of borrowing or lending.

Equations (3.2) to (3.6) can be compactly expressed as  $T(v)(q) = v$ . Since  $S$  contains only a finite number of elements, there always exists a solution to these maximization problems. If  $v^*$  is the optimal value function, then the policy functions  $c : S \rightarrow R_+$ ,  $a : S \rightarrow A$ ,  $k : S \rightarrow K$  and  $d : S \rightarrow \{0, 1\}$  are the optimal decision rules provided  $c(s)$ ,  $a(s)$ ,  $k(s)$  and  $d(s)$  are measurable and satisfy (3.2)-(3.6). These policy rules map the current state of the economy, represented in the level of output, loans and default record, into the consumption decision, the next period's loans level and the next period's default decision. In the next subsection the international lenders behavior is described.

### 3.1.2 International Lenders

The international lenders are assumed risk neutral. They can borrow funds from an independent market at the risk free price,  $\bar{q}$ , and will lend as long as they are paid a return that is at least as high as the risk free rate,  $\bar{r}$ . For  $a < 0$ , if the country has no default record it can buy loan contracts in which a lender provides  $-qa'$  units of the consumption good today in exchange in exchange for the country's promise to deliver, conditional on not defaulting,  $-a'$  units of the consumption good next period. For  $a \geq 0$  the country can buy, regardless of the default record, contracts in which a lender promises to provide for sure,  $a'$  units of the consumption good next period in exchange for  $qa'a'$  units of the consumption good today. So, the lenders choose loans  $a'$  to maximize expected profits, taking  $q$  as given:

$$\Omega(a'; q) = -qa' + \bar{q}(1 - \lambda)a'$$

where  $\lambda$  is the probability of default. For  $a' \geq 0$ , the probability of default is zero and  $q = \bar{q}$ . For  $a' < 0$ , the price of the bond takes into account the possibility that the country may not repay the loans and so the expected return to the lenders must be at least as high as the risk free rate:

$$q \geq [1 - \lambda] \bar{q}. \tag{3.7}$$

In this setup there is a "natural debt limit" that lenders may impose on the sovereign. This limit is given by allowing the sovereign to borrow up to the annualized value of the highest possible realization of the stochastic productivity shock (in every future period) discounted at the lowest possible interest rate. So,  $-\frac{\epsilon_H k_H^\alpha}{1 - \bar{q}}$  is the highest amount of

debt that could be paid by the sovereign in the luckiest situation. It can be shown that a loan of this size (or larger) will have a price of zero in any equilibrium. Therefore, for those loans the probability of default is one, regardless of the initial state of the economy. Setting the borrowing limit as described, will not have any effect on the equilibrium price of the loans. However, as it will be shown in the next section, the *equilibrium* borrowing limit in the model is also an optimal outcome that is closely linked to the probability of default.

### 3.1.3 Equilibrium

In this section we characterize the equilibrium concept used to solve the model. The equilibrium concept used is that one of a stochastic, stationary rational expectations equilibrium. Let  $d'(s)$  be a default decision function that take the value of 1 if the utility from defaulting strictly exceeds the utility from repaying and 0 otherwise.<sup>3</sup> Let  $\Lambda(s; q)$  be a set of default probabilities consistent with the optimizing behavior of the country in state  $s$  and given the loan's price  $q$ .<sup>4</sup> Now, by imposing the restrictions dictated by the optimizing behavior and the zero profit condition, define the loan's price correspondence is:

$$\varphi(q, a', k') = \begin{cases} \bar{q}(1 - \lambda) & \text{if } a < 0 \\ \bar{q} & \text{if } a \geq 0 \end{cases}$$

which determines the set of prices for a loan  $a'$  that are consistent with zero profits at a given price vector  $q$ .

Then the equilibrium is represented by a time invariant probability of default function,  $\lambda^* \in \Lambda(s; q)$ , a time invariant loan's price function  $q \in \varphi(q, a', k')$ , a time invariant optimal set of loans,  $A^*$ , and a time invariant optimal probability distribution over the state space of the small open economy,  $\psi$ . The stationary equilibrium is defined recursively. Given the transition rules  $k(s)$ ,  $a(s)$  and  $d(s)$  and the transition probabilities  $\pi(\epsilon'|\epsilon)$ , the optimal transition probability  $P(s, B)$  can be computed.  $P(s, B)$  is the probability that a country in state  $s$  will reach a state vector that lies in  $B$ .<sup>5</sup>

<sup>3</sup>If there are endowment levels at which the country is indifferent between defaulting and repaying, this is a default decision correspondence.

<sup>4</sup>Formally,  $\Lambda(s; q)$  is a set of nonnegative numbers such that if  $\lambda \in \Lambda(s; q)$  then there is a  $d'(s)$  such that  $\sum_{y'} \pi(y'|y) d'(s) = \lambda$ .

<sup>5</sup>The optimal transition probability  $P(s, B)$  is very useful to compute the probability of default. Since there are transient and absorbing states, the probability of default is given by the probability of reaching the set of absorbent states.

More formally, the stationary stochastic rational expectations equilibrium of the model is characterized by:

1. an equilibrium loan's price function  $q^* \in \varphi(q^*, a'(s))$  and optimal default set probabilities  $\Lambda(s; q^*)$
2. an optimal set of loans  $A^*$
3. the optimal policies for capital accumulation,  $k'(s)$ , borrowing,  $a'(s)$ , and default,  $d'(s)$

such that:

1. The sovereign country solves (3.2)-(3.6).
2. Expected profits for international lenders are zero, i.e. (3.7) holds with equality.
3. The sovereign demand for loans is in the available set of loans,  $a'(s) \in A^*$

The first condition states that the sovereign country optimize to find a set of decision rules for consumption, loan allocation and default that depend on the current state of the economy. The second condition states that the profits for international lenders are zero, because the international loan market is competitive. The third condition is the market clearing condition. So,  $q^*$  is an equilibrium price vector if for all  $a' \in A$ ,  $q^* \in \varphi(q^*, a', k')$ .

There is nothing in the definition of the equilibrium that precludes the possibility that  $q^*$  may be zero for some  $a < 0$ . As mentioned earlier, this happens when the probability of default is one, regardless of the initial state of the economy. Then  $\Lambda^*(s; q^*) = \{1\}$  and  $q^* = 0$ . On one side, international lenders are indifferent whether to provide loans  $a'$  at a price of zero since the expected payoff is zero but so does the cost. On the other side, if the sovereign country take one of these free loans gets nothing in the current period, but generates a liability for the next period. So, these loans will not be demanded either. This intuition forms the background for the computation of the borrowing limit.

Intuitively, the lender evaluates the current states of nature for which it is not possible for the sovereign to repay in the following period. The lender fixes the borrowing limit at the lowest debt level for which default is a certain outcome. It may be possible that the sovereign would like to borrow a higher amount, but the lender will not be willing to

engage in such a contract. In this sense, the model exhibits “credit rationing”. When such a situation arrives, it is not possible to smooth consumption completely and the economy may experiment large welfare losses.

## 3.2 Calibrating the Model to the Argentine Economy

In this section the model is calibrated to the Argentine economy to study its ability to reproduce the salient properties of the Argentinian business cycle and at the same to account for the behavior of the main macroeconomic variables during a default episode.<sup>6</sup> Calibrating the model amounts to select the parameters so that the statistics of the model match as closely as possible some relevant statistics of the Argentinian business cycle and its default episode in 2001. In particular, parameters are chosen so that the model can replicate some of the sample second moments, such as volatility and correlations, of the observed Argentinian data between 1980 and 2003. First, in 3.2.1 some facts about the Argentinian business cycle and the 2001 default episode are presented. Then in 3.2.2 the benchmark calibration is described.

### 3.2.1 The Argentinian Business Cycle and the 2001 Default

This section presents the salient features of the Argentinian business cycle and its 2001 default. Two main sources of national accounts quarterly data are used: the Ministry of Finance data (MECON) for the 1980-2003 sample period and the IMF for the 1994-2003 sample period.<sup>7</sup> In addition, to obtain a proxy for the external interest rate, the procedure proposed by Neumeyer and Perri (2001) is followed. All data are transformed to be model consistent.<sup>8</sup> To obtain measures of the business cycle fluctuations, consumption and output data are in logs and deseasonalized using the Census X12 filter. Then the Hodrick-Prescott filter is applied and a measure of the cyclical component of the time series is obtained.

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<sup>6</sup>There is nothing that prevents the use of this model to evaluate other country’s default experiences. For instance, there are quite striking similarities between the 2001 Argentinian default and the 1934 Colombian default experiences.

<sup>7</sup>Before the first quarter of 1993 the total consumption includes inventories and statistical discrepancy. The database used in this study is available at [http://www.webpondo.org/fhamann/data/arg\\_default.xls](http://www.webpondo.org/fhamann/data/arg_default.xls)

<sup>8</sup>In particular, real consumption is measured as real absorption, since in this model there is no difference between private and public consumption and also there is no investment.

By using the HP filter we focus on business cycle frequencies corresponding to 8 years. The trade balance and the current account correspond to their shares in the GDP. No transformations are made to the interest rate.

Figure 3.1 illustrates the main properties of the Argentinian business cycle from 1994 to 2003. There are some relevant facts to note: first, after the Tequila crisis in 1995 the Argentinian economy recovered from a -6.5% GDP gap in the last quarter of 1995<sup>9</sup> to a 5% gap by the end of 1998. From then on Argentina struggled to return to sustainable growth rates. Second, the 2001 default episode increases the volatility of the main macroeconomic variables. The aggregate demand increases the cyclical volatility from about 5% during normal times to 15% during the default episode. A third fact is a large current account reversal accompanied by unprecedented high levels of external interest rates during the preceding quarters to the default episode.<sup>10</sup> The economy moved from a 4.7% current account deficit in the first quarter of 2001 to a 5% current account surplus in the last quarter of 2001. Fourth, a deep depression is registered later. In the first quarter of 2002, after defaulting on external debt in December 2001, the output gap was -10% and consumption was 15% below trend. These figures reflect the magnitude of the capital outflows that Argentina experimented during 2001 and the sharp depression that followed in 2002.

Table 3.1 shows the main correlations between the variables for the sample period 1980-2003. Some features of the data are worth noticing. First, with respect to macroeconomic instability: regardless of the sample period, output fluctuates about 4% from trend. Consumption is more volatile than output and deviates about 5% from trend. Investment is more than two times the volatility of consumption. That consumption fluctuates more than output is not a standard result. This is the consequence of computing consumption as including public consumption to make the data model-consistent. The trade balance volatility is between 3% and 6% depending on the sample period. The current account volatility is similar to that of output, about 4%. The external interest rate variability is about 9%.

Second, with respect to the comovements of the macroeconomic time series: first, consumption and investment are procyclical. Second, the trade balance, current account

<sup>9</sup>The “gap” means the log deviation of the deseasonalized time series with respect to the Hodrick-Prescott filter.

<sup>10</sup>The EMBI+ spread started to climb steadily: by the end of 1999 the EMBI+ spread was 622 basis points, 800 by the end of 2000 and 1761 in the third quarter of 2001.



Figure 3.1: The Argentinean Business Cycle 1994-2003

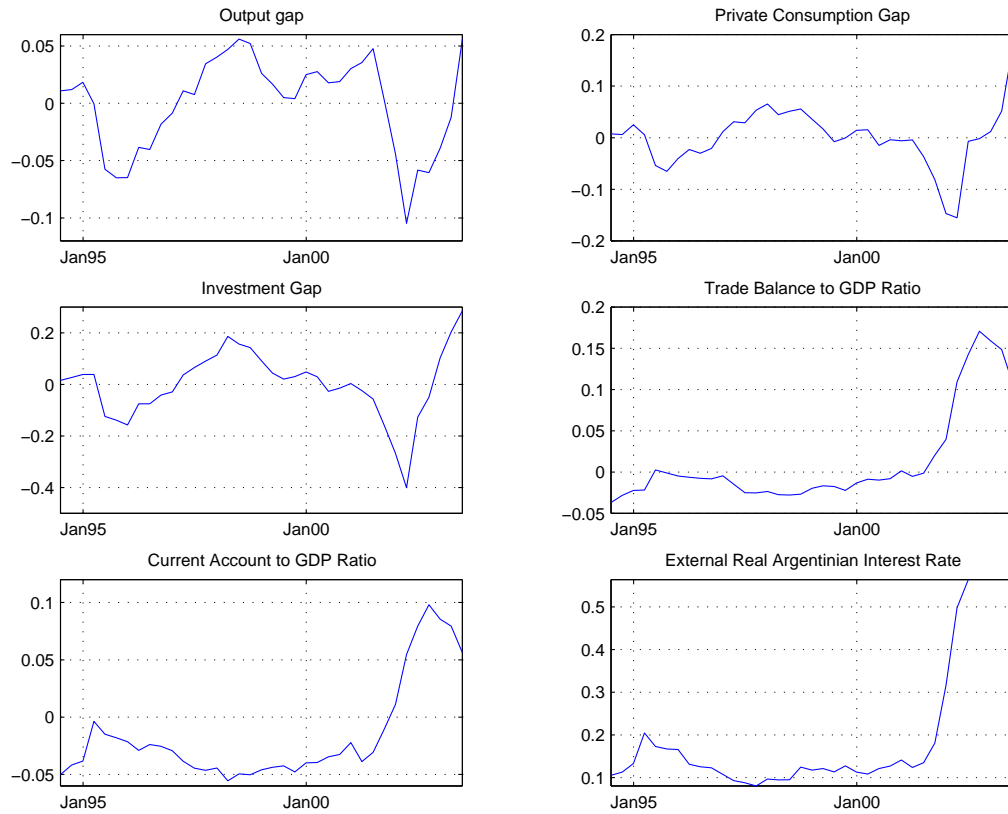


Table 3.1: Argentinean Business Cycle Statistics: 1980-2003

Variable	$\sigma_i$	$\rho(i, y)$	$\rho(i_t, i_{t-1})$
GDP	4.33	1.00	0.72
Consumption	5.10	0.98	0.73
Investment	12.54	0.94	0.73
Trade Balance	2.69	-0.91	0.87
Current Account	4.34	-0.91	0.66
Interest Rate	8.65	-0.92	0.78

Source: International Financial Statistics - IMF, Neumeyer and Perri (2001) and author's calculations.

and the external interest rate are strongly countercyclical. The countercyclicity of the trade balance and the external interest rates has also been widely documented in Oviedo (2004), Uribe and Yue (2003) and Neumeyer and Perri (2001).

### 3.2.2 The Baseline Model

This subsection discuss the calibration of the baseline model and evaluates the ability of the model to reproduce the Argentinian business cycle facts.

#### Calibration and Solution

Some parameters of the model are set to replicate some features of the data while others are taken from other studies. The functional form used to represent the preferences is a standard CES utility function,  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$  where  $\gamma$  is the coefficient of relative risk aversion. That parameter is set to 5, which is a standard value used in other studies for small open economies in emerging markets. The capital share,  $\alpha$ , according to Kydland and Zarazaga (2002, [35]) is 0.4 for Argentina. The discount factor,  $\beta$ , is set at 0.9615, to match the annual average real interest rate of 17% observed in the data. This relatively large discount factor reflects the high degree of impatience that characterizes emerging market economies and is also used in other studies in emerging markets. The productivity process is assumed to be a three state discrete Markov Chain. This is done so to capture normal, high and low levels of productivity. Its standard deviation and the autocorrelation were set to match as closely as possible the observed cyclical properties of output. The default penalty,  $\theta$ , and the investment adjustment cost,  $\phi$ , were fixed at 0.4 and 0.025 respectively, to match as closely as possible the historical default rates that Argentina has exhibited during the last two hundred years. Argentina defaulted one time during the nineteenth century and three times during the twentieth century. Beim and Calomiris (2000) report three default episodes: the first from 1890 to 1893, the second from 1956 to 1965 and the third from 1982 to 1992. The recent 2001 default episode is the fourth in history. This means that in about 200 years, Argentina has defaulted 4 times, and has remained in default during 26 years (about 10% of the time). The quarterly probability of default is rate is between 0.75% (if one considers only the 20th century) and 0.5% (if one considers the two centuries).

The model needs an extremely large default penalty (in addition to financial au-

Table 3.2: Calibration Values and Targets

Parameter	Symbol	Value	Target
Coefficient of Risk Aversion	$\gamma$	5	Other EM's studies
Discount Factor	$\beta$	0.9615	Observed real interest rate
Capital Share	$\alpha$	0.4	Kydland and Zarazaga (2002)
Risk-free Interest Rate	$r$	0.01	U.S. quarterly interest rate of T-Bills
Productivity Volatility	$\sigma_\epsilon$	0.4	Argentinian GDP data
Productivity Autocorrelation	$\rho_\epsilon$	0.7	Argentinian GDP data
Capital Adjustment Cost	$\phi$	0.025	Default Rate in Beim and Calomiris (2000)
Default Penalty	$\theta$	0.4	Default Rate in Beim and Calomiris (2000)

tarky) to be able to reproduce the observed default rates and average levels of external debt to GDP ratios. Notice that in this model the punishment is 4 times larger than in a model without capital accumulation like in Hamann (2004). Capital not only provides insurance against default, but also a large capital stock increases the incentives to default. Default is more painful in a model in which the sovereign has only one asset to smooth consumption, therefore the required punishment to deter the country from defaulting is lower.

The solution of the model is performed by state-space discretization. The state space  $S$  is partitioned in an equally spaced grid of 2400 points. The capital stock assumes values in the set  $K = k_1, \dots, k_{n_1}$  and the stock of debt assumes values in the set  $A = a_1, \dots, a_{n_2}$ .  $A$  is initialized using an equally spaced grid in the interval  $[-\frac{\epsilon_H k_H^\alpha}{1-q}, 0]$ . The capital grid used is between 0.1 and 6. The solution algorithm also exploits the recursive nature of the problem. It starts by guessing a loan's price function  $q$  defined on a given  $A$ , and given those two objects, the sovereign solves the problem (3.2)-(3.6). This yields a probability of default function which permits to compute a new  $q$  and  $A$  by using the lender's profit maximization condition, equation (3.7). The algorithm's details are described in Hamann (2004, [26]).

## Results

The calibrated model is used to simulate artificial data. These data are qualitatively contrasted against the observed data. The same transformations that were made to the observed data are made to the simulated data.<sup>11</sup> The comparison is made by quali-

<sup>11</sup>Obviously the simulated data is not deseasonalized.

Table 3.3: Business Cycle Statistics: Benchmark Model

Variable	$\sigma_i$	$\rho(i, y)$	$\rho(i_t, i_{t-1})$
GDP	4.15	1.00	0.80
Consumption	5.75	0.76	0.47
Investment	10.04	0.72	0.63
Trade Balance	1.27	0.32	0.47
Current Account	1.23	0.12	0.69
Interest Rate	1.51	-0.28	0.49

tatively observing the sample second moments of both types of data. The simulation is initialized at  $s_0 = (\epsilon_H, 0, \bar{a}, \bar{k})$ , that is at a high level of output, good default rating, the average debt level and the average mean capital. The initial realizations of the simulation are discarded to avoid any dependence on the initial values. The last portion of the simulated data (90 data) that corresponds to the observed sample (1980:1-2003:2) is kept.

Table 3.3 reports the results. The calibrated model is able to reproduce only a few sample second moments of the observed Argentine time series. In particular, the baseline calibration is able to mimic the consumption and investment volatilities, but underestimates the trade balance, the current account and the interest rates ones. A particular result of this model is that consumption is more volatile than output. This is explained by the presence of sovereign risk and the borrowing constraint coupled with equilibrium default and capital accumulation. In a similar model in which there is only one asset to smooth consumption, sovereign risk itself limits the ability of the sovereign to smooth consumption by imposing a borrowing constraint during bad times. A sufficiently low realization of productivity may lead the country to be willing to borrow above the equilibrium debt limit. When the limit binds consumption falls more than output. Second, if the fall in consumption is sufficiently low it may trigger default. After default, the economy goes to financial autarky. When there is capital accumulation, the fall in consumption can be mitigated by reducing capital. The fall in consumption is smoother in this model. So, an economy that exhibits a relatively large default rate exhibits a very volatile consumption, although less volatile than in an endowment economy model.

The model also captures the fact that consumption and investment are procyclical and the interest rate is countercyclical. The first two results are a consequence of market

incompleteness and the type of preferences used in the model.<sup>12</sup> When markets are incomplete the consumption output correlation depends on the sign of the third derivative of the utility function. The CRRA utility function in this paper has a positive third derivative. This implies that, for a given interest rate, the uncertain future productivity induces the sovereign to be “more prudent” and to save some output in case of a low realization tomorrow (precautionary savings).

The interest rate that the country faces internationally is countercyclical because it depends on the probability of default, which in turn increases positively with the level of debt. As the sovereign borrows more during low output periods, the probability of default is higher and so does the interest rate.<sup>13</sup> However, the calibrated model fails to capture a countercyclical current account. In a model with a single asset to smooth consumption this is a standard result. It is less obvious in a model in which there is domestic saving through capital. When the interest rate is endogenous as in this model, the cyclical pattern of the current account depends on the conditional covariances between the marginal cost of investment, the intertemporal rate of substitution and the marginal cost of borrowing abroad. Let’s assume for a moment that the interest rate is given. In this case, productivity shocks affect the current account via two channels. First, at a given external interest rate, a positive shock increases investment tending to worsen the current account as the sovereign borrows abroad to finance the additional capital accumulation. This is a pro-borrowing effect. Second, the productivity shock has a pro-saving effect. At a given rate, the sovereign has a precautionary savings motive to accumulate assets and capital in good times and accumulate debt reduce capital and in bad times. This tends to generate a procyclical pattern of the the current account. The dominance of any of this channels depends on the persistence of the shocks and on the interest rate. More persistent shocks tend to generate a countercyclical current account, since future output raises more than current output (because capital is given at  $t$  and will be higher in the near future). At the same time, current investment increases more than future investment. So, savings falls and investment increases and the current account should be countercyclical. If the shocks are not persistent enough, the current account is procyclical. In this model, the default option provides an additional motive for precautionary savings, tending to strengthen the

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<sup>12</sup>Markets are “incomplete” in the sense that the economy can not borrow contingent on the realization of productivity. Notice that there are three possible productivity states and only two assets to smooth consumption.

<sup>13</sup>This result was first derived by Eaton and Gersovitz (1981).

Table 3.4: Sources of Welfare Gains from Reducing the Likelihood of Default

Economy	$\bar{y}$	$\bar{c}$	$\sigma_c$	$\bar{k}$	$\bar{a}$	default rate
Benchmark	2.01	1.96	5.75%	4.77	-1.04	0.006
Default-free	2.15	1.98	4.67%	5.62	-0.94	0.0001

pro-savings effect.

### 3.3 Welfare Comparisons

The calibrated model fails to reproduce many of the sample second moments of the Argentine economy, but still captures the main volatilities of consumption, investment and output. Therefore, it is still interesting to evaluate the welfare gains of reducing the likelihood of default in the benchmark economy. In the model the occurrence of default limits the ability of the small open economy to smooth consumption over time. There may be large welfare gains of deterring default. Here, default can be avoided by imposing large trade sanctions. A trade sanction reduces the likelihood of a sovereign default and may induce some welfare gains, since the costs of the sanctions only materialize in the event the economy defaults.

A first approximation to these gains can be computed performing a simple computation a la Lucas (1987, [40]) as follows:

$$v(s; q^*) = \phi(s, q^*, \tilde{q})^{1-\gamma} w(s; \tilde{q})$$

where  $v(s; q)$  is the value function of the benchmark default-prone economy in state  $s$  and loan price  $q^*$ ,  $w(s; \tilde{q})$  is the value function of the high-punishment and default-free economy in state  $s$  and loan price  $\tilde{q}$  and  $1 - \phi(s, q^*, \tilde{q})$  is the fraction of consumption the sovereign is willing to give up if confronted with a default-free economy. Using the invariant measure of the default-free economy and the benchmark economy, the average steady state welfare gain is 0.56% of consumption. This cost is not that large when compared to the case of an endowment economy (7% in Hamann (2004)), meaning that the country in the benchmark model would be willing to give up only a small fraction of steady state consumption to avoid default.

To analyze the result, some long-run statistics of the simulated data are computed. Table 3.4 presents the long-run properties of both economies. Note that, first average consumption and output are larger by about 1% in the default-free economy; second, consumption volatility is smaller in 1%; third, average debt is higher and capital is lower in the benchmark economy. The steady state welfare gain comes from a higher and more stable consumption in the default-free economy. Notice that the default free economy also accumulates more capital and has less debt. A higher capital stock relaxes the dependence on foreign debt. So, economies that find default very costly exhibit a higher level of capital and consumption, a lower level of debt and a more stable consumption.

### 3.4 Sensitivity Analysis

This section performs the sensitivity analysis of the welfare calculations to changes in the main parameters of the model. The focus is on the effect of these changes on average consumption, debt and capital as well as the volatility of consumption and the default rate. The aim is to gain some insight on the forces that are driving the equilibrium of the model, in particular the default incentives.

The top panel of Table 3.5 summarizes the effect of an increase in the coefficient of relative risk aversion. A change in  $\gamma$  has two opposing effects on the incentives to default. A higher degree of risk aversion increases the curvature of the instantaneous utility function, making default more likely during bad times. As consumption approaches very low levels, repayment of debt hurts more in terms of the current marginal utility and default is more likely. On the contrary, a higher degree of risk aversion increases the precautionary savings motive. A more risk averse agent tends to accumulate more assets to avoid the unhappy event of default. In this model, it is the second effect that dominates. Notice that as  $\gamma$  increases, consumption becomes more volatile, the economy accumulates more capital and assets and the default rate falls.

The second panel illustrates the effect of a change in the discount factor. A higher  $\beta$  means that the country puts more weight on future consumption and utility than current consumption and utility. A more patient economy should save more and default less than a low discount economy. As a result average consumption and output are higher and also consumption is more stable.

The third panel presents the effect of changing the ability of the economy to adjust capital. Higher costs of adjusting capital increase the default rate, but average output, consumption and assets do not vary much. When capital is more costly to adjust, the current account adjusts more slowly regardless of its cyclical pattern. This means that trade imbalances and interest payments tend to persist over time. Consider a negative persistent productivity shock. An economy with higher installation costs bears longer the effect of the shock. Since default is more likely to occur during bad times, the rate of default increases.

The fourth panel summarizes the effects of a higher capital share. This result is also standard, a higher capital share allows the economy to have a higher long-run capital, higher output and consumption and a lower level of debt. These results in addition to a more stable consumption, induce a lower default rate.

The last panel presents the effect of less persistent shocks. The default rate is higher the higher the persistence of the shocks. A higher  $\rho$  has also two opposing effects. First, as  $\rho$  increases, the unconditional variance of productivity shocks is higher, inducing more volatility and increasing the value of staying in the credit market. Second, as the unconditional variance of output can be higher, it is feasible for the economy to fall lower when shocks are more persistent. therefore a given negative shock, is more pervasive than a temporary one inducing higher default rates. In the model, the second effect dominates.

The last column of Table 3.5 presents the welfare gain of eliminating default by increasing the trade sanction. Results show that the welfare gain of reducing the likelihood of default ranges between 0.01% and 0.7% of steady state consumption. These figures are strictly higher than Lucas' computation of the welfare gain of eliminating postwar consumption volatility, but significantly lower than the 7% welfare gains of eliminating default in the endowment economy calculated in Hamann (2004). One possible interpretation of this result is that capital helps to make markets more "complete" and helps the country to insure against idiosyncratic risk. The results also show that as default rates fall, so do the welfare gains of reducing the likelihood of default by increasing the punishment. Intuitively, economies with higher macroeconomic stability, exhibit lower default rates and the punishment, as a commitment-inducing device, becomes less effective.



Table 3.5: Sensitivity Analysis

	$\bar{y}$	$\bar{c}$	$\sigma_c$	$\bar{k}$	$\bar{a}$	Default Rate	Welfare Gain
Risk Aversion							
$\gamma = 10$	2.02	2.00	6.66%	5.36	-0.75	0.003	0.16%
$\gamma = 5$	2.01	1.96	5.75%	4.77	-1.04	0.006	0.56%
$\gamma = 2.5$	1.97	1.94	5.48%	4.53	-1.12	0.005	0.61%
Discount Factor							
$\beta = 0.99$	2.09	1.99	5.1%	5.24	-0.83	0.003	0.03%
$\beta = 0.96$	2.01	1.96	5.75%	4.77	-1.04	0.006	0.56%
$\beta = 0.9$	1.87	1.90	6.4%	3.96	-1.13	0.008	0.70%
Adjustment Cost							
$\phi = 0.05$	2.02	1.94	5.89%	4.80	-1.03	0.004	0.025%
$\phi = 0.025$	2.01	1.96	5.75%	4.77	-1.04	0.006	0.56%
$\phi = 0.01$	2.00	1.96	5.67%	4.72	-1.02	0.007	0.65%
Capital Share							
$\alpha = 0.35$	1.77	1.79	7.3%	4.11	-1.08	0.008	0.69%
$\alpha = 0.4$	2.01	1.96	5.75%	4.77	-1.04	0.006	0.56%
$\alpha = 0.45$	2.29	2.17	4.39%	5.34	-0.87	0.002	0.08%
Autocorrelation							
$\rho = 0.8$	2.01	1.96	5.75%	4.77	-1.04	0.006	0.56%
$\rho = 0.4$	2.04	1.95	4.54%	5.01	-1.18	0.004	0.03%
$\rho = 0.0$	2.10	1.95	3.61%	5.28	-1.29	0.003	0.01%

### 3.5 Conclusion

This paper studies the welfare gains of reducing the likelihood of sovereign default in Emerging Markets using a quantitative model of incomplete markets and lack of commitment to repay sovereign debt. The model allows for capital accumulation which has a dual role on affecting incentives to repay. The first is that capital increases the likelihood of default because the country has an asset to save if defaults. The second is that, as markets are incomplete and the country engages in precautionary savings, capital serves as a buffer to mitigate consumption falls during recessions. The model is calibrated to the Argentine economy and reproduces a limited number of the sample second moments of the macroeconomic time series. In particular, the model reproduces the main macroeconomic volatilities and correlations as consumption, investment and output, but fails to reproduce those of trade balance, current account and interest rates. Further research is needed in this direction to improve the ability of the model to reproduce these facts.

Despite this result, the model is used to assess the sensitivity of the welfare gains of reducing the likelihood of default in the artificial economy. The results show that the welfare gain of reducing the likelihood of default are low. The figures range between 0.01% and 0.7% of annual steady state consumption. These figures are higher than Lucas' computation of the welfare gain of eliminating postwar consumption volatility, but significantly lower than the 7% welfare gains of eliminating default in an endowment economy. This result is interpreted as capital being helpful in completing the market to insure against idiosyncratic productivity shocks. The computations also show that as default rates fall, so do the welfare gains of reducing the likelihood of default by increasing the punishment. Economies with higher macroeconomic stability, exhibit lower default rates and the punishment, as a commitment-inducing device, becomes less effective.

## Chapter 4

# Inflation Targeting in a Small Open Economy

Nowadays Colombia's Central Bank, uses the so called "Model of Transmission Mechanism" (MTM) as the main model for monetary policy analysis and forecast<sup>1</sup>. The model, consists of a monetary policy rule, a Phillips curve (augmented by expectations) for "core" inflation, an equation for food inflation, another for imported goods, the output gap and the exchange rate. This setup aims to capture the main channels through which monetary policy is transmitted to the real sector as well as the sluggish response of inflation to monetary policy shocks. The MTM has proven to be a useful tool for monetary policy analysis and forecast. However, it is also recognized that this kind of models have important limitations<sup>2</sup>. The fact that we only use a subset of semi-structural economic relationships, with no specifications about agents, markets and their interaction, limits the scope, the consistency and the interpretability of the model. One consequence is that the language used to support the monetary policy decisions is usually vague and sometimes even inconsistent.

At a theoretical level, it has been argued that nominal rigidities and departures from perfect competition may be an important channel through which monetary policy

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<sup>1</sup>See Gomez, Vargas and Uribe (2001, [25]) and Gmez and Julio (2003, [24]).

<sup>2</sup>More precisely, most of the equations of the model are subject to the Lucas critique. For example, the Phillips curve implicitly assumes the presence of some rigidity, but since agents don't optimize and there is not a well defined economic structure in the model, it is impossible to know where the rigidity comes from.

has real effects. The macroeconomic implications of these types of models are surveyed by Walsh (2003, [60]). Recent explorations by Chari, Kehoe and McGrattan (1996, [17]) show that a high labor-supply elasticity is required in order to explain a significant fraction of price dynamics in the U.S. data. However, Nelson (1998, [48]) has shown that the type of models developed by Chari, Kehoe and McGrattan cannot account for the sluggish response of inflation that seems to be present in the U.S. data. Melo and Riascos (2004, [43]) have found a sluggish response in output, employment and inflation to policy shocks using Colombian data and built a structural model with two types of rigidities: a real rigidity in the labor market and a limited participation constraint in the financial market. Their model can reproduce the sluggish response of inflation, but fails to reproduce the labor market dynamics; the issue of the role of nominal rigidities remains unexplored in Colombia.

This paper evaluates *quantitatively* how much of the observed dynamics of the Colombian macroeconomic data can be explained by the presence of nominal rigidities in the context of a small open economy. It also explores its implications for monetary policy when operating under an Inflation Targeting framework and using the nominal interest rate as the policy instrument<sup>3</sup>. We develop and calibrate a DSGE model for monetary policy analysis in Colombia to study the effects of different types of shocks and their transmission mechanisms through the economy. When able to identify the sources of the shocks, one wants to be able to conclude to what types of shocks must the monetary authority react and how to do it. In other words, the model can help to identify the shock and its nature. Then one should be able to define what change should take place on the policy instruments (the interest rate in this case) to take inflation back to its target and the product to its potential<sup>4</sup>.

In particular, we introduce a nominal rigidity and a market imperfection to a small open economy model. The model economy has two sectors: a standard competitive sector populated by firms that hire capital and labor from households to produce homogeneous output, and a monopolistically competitive sector in which firms buy the homogeneous product at a competitive price, differentiate their product by putting a tag (a process we call “branding”) and sell it to households. The consumption good that enters the agent’s utility function is a Dixit-Stiglitz aggregate of all the differentiated goods. The

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<sup>3</sup>Efforts in this direction are also pursued in other Central Banks. See Smets and Wouters, (2003, [56]), Murchinson, Rennison and Zhu (2003, [47]), Scott (2003, [58]).

<sup>4</sup>In this framework potential output is understood as the resulting output in an environment characterized by the absence of frictions and/or shocks. See Gal (2002, [23]).

first sector can be thought as a wholesale sector and the second sector as a retail sector. The introduction of monopolistic competition in the retail sector provides a rationale for price-setting behavior. What is important, is to introduce some sort of nominal rigidity. In this respect, we follow Calvo(1983, [15]) assuming that a fraction of retailers adjust their price infrequently. Monetary policy is conducted to target inflation and potential output. The monetary authority adjusts the nominal interest rate to meet these targets. Since this is a small open economy, monetary policy decisions will impact not only domestic savings households decisions but also their external financing.

We test the empirical power of this simple model in explaining the dynamics of output and inflation in Colombia for the period 1980:1-2004:1. Our criteria to evaluate the model is that its calibrated version should be able to reproduce salient and/or interesting features of Colombian data on output gap and inflation. We compare, at the frequency domain, the observed data with the data simulated from the theoretical model.

We deviate from the standard practice in the Real Business Cycle literature of comparing a subset of second moments of observed and artificial data. We analyze the model's ability to reproduce the data by comparing the complete set of second moments. The method is to compare the model and data spectra. For a given set of parameters, they can be used to assess the performance of our model. As we will show later, this tool is very useful to determine the advantages and limitations of our model.

The methodology for comparing the theoretical spectrum and the data spectrum consists of four steps. First, we estimate the sample data spectrum and compute its uncertainty using bootstrap techniques. Second, from the estimated spectrum and its uncertainty we determine the salient and/or interesting features which we expect the theoretical model to reply. Third, we compute the model's theoretical spectrum. Finally, we compare the theoretical and observed estimated spectrums at the required frequencies.

The rest of the paper proceeds as follows: in the next section we lay out the model. Section 3 shows the calibration procedure. Section 4 shows the response of the model to different shocks and its sensitivity to some key parameters, as the degree of price stickyness and the degree of response of the risk premium to the external debt to output ratio. In section 5 we determine to what extent the model replicates the salient features of Colombian data using the methodology proposed by Diebold et al. (1998). The last section summarizes our findings.

## 4.1 The Model

We consider a small open economy with a representative household, two types of firms and a government that makes unproductive expenditure, issues national currency and behaves according to an interest rate policy rule. The first type of firms hire labor and capital from households and produce an homogeneous good. The second type of firms buy the homogeneous good, put a label at no cost, and end up with a differentiated good. One way to think about the second type of firms is as “branding” firms<sup>5</sup>. They buy wheat, pack it and put a label on it. This is just a device to introduce price-stickiness into the model<sup>6</sup>. From now on we will refer to the first type of firms as “producers” and to the second as “retailers”. Households consume differentiated consumption goods and pay a liquidity cost, they also supply homogeneous indivisible labor, accumulate capital and supply it to producers. They receive lump sum transfers from the government and hold wealth as cash. Producers hire labor and capital from households as factor inputs and produce homogeneous goods. These homogeneous goods are demanded by retailers, which transform homogeneous goods into differentiated consumption goods and sell these to households. The consolidated monetary and fiscal authority issues money, makes net lump sum transfers to households, makes some unproductive expenditure and collects the liquidity costs from households<sup>7</sup>. All quantities are in per-capita terms if not stated otherwise.

### 4.1.1 The Representative Household

Households are the owners of the firms that produce the homogeneous good as well as of the retail sector firms and are consumers. Their income at period  $t$  is given by the nominal wage, nominal returns to capital, the benefits from retailers and the net lump sum transfers obtained from the government in this same period. Apart from their income they also count with a real money stock given at the beginning of the period as well as with a stock of real domestic private bonds and foreign assets<sup>8</sup>. Expenditure is determined by consumption, the liquidity costs and investment. At period  $t$ , they also decide the level

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<sup>5</sup>This type of setup is not new in the literature, to our knowledge it was first implemented by Bernanke, Gertler and Gilchrist (1999, [10]).

<sup>6</sup>See Schmitt-Grohe and Uribe (2004,[55]).

<sup>7</sup>By doing so we intend to eliminate the wealth effect.

<sup>8</sup>Stock variables are given at the beginning of the period and flows are known at the end, i.e.  $M_t$  is known at the start of period  $t$ ,  $P_{t-1}$  is given at the end of period  $t-1$  so it's known at the beginning of period  $t$ , as  $m_t = \frac{M_t}{P_{t-1}}$ , real money holdings are known at the start of period  $t$ .

of expected real money holdings, real domestic private bond holdings and foreign asset holdings for period  $t + 1$ . Then the budget constraint is given by:

$$c_t + \Phi + m_{t+1}^d + \frac{P_t x_t}{P_t^c} + b_{t+1} + \frac{e_t F_{t+1}}{P_t^c} = \frac{W_t}{P_t^c} h_t^s + \frac{R_t}{P_t^c} k_t^s + \frac{\Pi_t^R}{P_t^c} + \frac{\Pi_t}{P_t^c} + m_t^d \frac{P_{t-1}^c}{P_t^c} + b_t \frac{P_{t-1}^c}{P_t^c} (1 + i_t) + \frac{e_t F_t}{P_t^c} (1 + i_t^f) \quad (4.1)$$

where:  $c_t$  is real consumption,  $m_t^d$  is real money demand,  $x_t$  is real investment,  $W_t$  is the nominal wage,  $h_t^s$  is the number of hours worked per-capita,  $R_t$  is the nominal return to capital,  $k_t^s$  is capital supply,  $\Pi_t$  are the benefits from the homogeneous good producers,  $\Pi_t^R$  are the benefits from the retailers,  $\tau_t$  are government lump sum transfers to the households,  $P_t^c$  is the price index of consumption goods and  $P_t$  is the price index of homogeneous goods,  $b_t$  are net real private domestic bonds,  $F_t$  are net foreign assets (or liabilities depending on the sign) denominated in units of the tradable homogeneous good,  $e_t$  is the nominal exchange rate (COP/USD),  $i_t$  is the domestic nominal interest rate and  $i_t^f$  is the foreign nominal interest rate denominated in dollars.  $M_0$ ,  $k_0$ ,  $b_0$  and  $F_0$  are known. As  $m_t = \frac{M_t}{P_{t-1}^c}$ , hence  $m_0$  is known and the same follows for  $b_0$ .  $\Phi$  is a function which determines the transaction costs, and is given by

$$\Phi(c_t, m_{t+1}, x_t) = \kappa \left( \frac{c_t + \nu \frac{P_t}{P_t^c} x_t}{m_{t+1}} \right)^a \quad (4.2)$$

where all variables are in real terms (relative to the consumption good) and  $\nu$  is a parameter that determines the fraction of investment that affects the optimal choice of real money holdings. According to this expression, as the household consumes or invests more, its liquidity costs increase, and they decrease with the real money holdings they save for next period. Money is introduced like this for simplicity.

The external nominal interest rate is defined as

$$(1 + i_t^f) = (1 + i_t^*) \left( 1 + \vartheta \left( \frac{F_t}{y_t} \right) \right) \quad (4.3)$$

where  $i_t^*$  is the international risk free nominal interest rate and  $\vartheta$  is the risk premium function<sup>9</sup>. Notice that if the net foreign assets ( $F_t$ ) are negative, then the country is a

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<sup>9</sup>The risk premium function is defined as  $\vartheta \left( \frac{F_t}{y_t} \right) = \omega_{ss} + \omega_1 + \omega_2 * Exp \left[ \omega_3 \left( \frac{\frac{F_t}{y_t}}{\frac{F_{ss}}{y_{ss}}} \right) * \mu_t^\vartheta \right]$  where the subscript *ss* stands for the steady state value of the variable and  $\mu_t^\vartheta$  is an exogenous variable which logarithm follows a standard autoregressive process of order one of the form  $\log(\mu_{t+1}^\vartheta) = \rho_4 \log(\mu_t^\vartheta) + (1 - \rho_4) \log(\bar{\mu}^\vartheta) + \epsilon_{t+1}$

net debtor and otherwise it is a net lender. It is also assumed that the purchase power parity (PPP) is satisfied, so  $P_t = e_t P_t^*$ . This means that the price for the homogeneous good equals the foreign price for the homogeneous good times the exchange rate. We set  $P_t^* = 1$  for all  $t$ , therefore  $P_t = e_t$  and so the depreciation rate equals the inflation rate of homogeneous goods,  $\pi_t = d_t$ . If we define  $q_t = \frac{P_t}{P_t^c}$  as the relative price of homogeneous goods to heterogeneous goods, and  $\frac{P_{t-1}^c}{P_t^c} = \frac{1}{(1+\pi_t^c)}$ , then the budget constraint (4.1) can be rewritten as

$$c_t + \Phi + m_{t+1}^d + q_t x_t + b_{t+1} + q_t F_{t+1} = \frac{W_t}{P_t^c} h_t^s + \frac{R_t}{P_t^c} k_t^s + \frac{\Pi_t^R}{P_t^c} + \frac{\Pi_t}{P_t^c} + \frac{m_t^d}{(1+\pi_t^c)} + \frac{b_t}{(1+\pi_t^c)}(1+i_t) + F_t q_t (1+i_t^f) \quad (4.4)$$

Households accumulate capital according to the following expression:

$$k_{t+1} - (1+\delta)k_t - f\left(\frac{x_t}{k_t}\right)k_t = 0 \quad (4.5)$$

where  $f$  is a twice continuously differentiable and concave function, which reflects investment adjustment costs in capital, and  $\delta$  is the depreciation rate. The specification of the function  $f$ , is such that when the economy is in steady state, there are no adjustment costs<sup>10</sup>.

Consumption and leisure generate utility to households, but they have a habit stock which generates disutility, this is

$$u(c_t, H_t, h_t, \mu_t^u) = \mu_t^u \log(c_t) - \gamma \log(H_t) - B h_t \quad (4.6)$$

where  $H_t$  is the habit stock,  $B$  is a parameter,  $\mu_t^u$  is an exogenous variable that represents an intertemporal preference shock<sup>11</sup>, and

$$c_t = \left[ \int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \quad (4.7)$$

where  $c(z)$  is the consumption of a specific good  $z$  coming from the retailer  $z$ , and  $\theta$  is the elasticity of consumption of each good  $z$  with respect to the whole bundle.

<sup>10</sup>We assume that  $f$  is a quadratic function  $f\left(\frac{x_t}{k_t}\right) = c_2 \left(\frac{x_t}{k_t}\right)^2 + c_1 \left(\frac{x_t}{k_t}\right) + c_0$ .  $c_2$  determines the concavity of the function, that is, how expensive it is on the margin to adjust the capital outside the steady state and is fixed in order to replicate investment's volatility. Parameters  $c_1$  and  $c_2$  are determined by the fact that there are no adjustment costs on the steady state.

<sup>11</sup>The log of this exogenous variable follows a standard autoregressive process of order one,  $\log(\mu_{t+1}^u) = \rho_3 \log(\mu_t^u) + (1-\rho_3) \log(\bar{\mu}^u) + \epsilon_{t+1}$ .



The functional form of the utility function deserves some explanation. First, the linear specification of utility involving  $h$  follows Hansen (1985, [28]) where labor is indivisible. Workers can either work some given number of hours or not at all (i.e. they can't work part time). Second, the utility function is separable in consumption and leisure. Third, agents trade employment lotteries instead of hours of work. This implies that hours worked are proportional to employment<sup>12</sup>.

On the other hand,  $H$  represents the consumption habits of each individual:

$$H_{t+1} - H_t - \rho(c_t - H_t) = 0 \quad (4.8)$$

where  $H_0$  is given. Consumption habit today depends on last period's consumption and habit<sup>13</sup>. The higher habit is, the more disutility it is going to generate. In the present period the individual is going to have to consume more to be as satisfied as last period<sup>14</sup>.

Then the representative household's dynamic problem is

$$\max_{\{c, h, x, k, m, H\}} \sum_{t=0}^{\infty} \beta^t u(c_t, H_t, h_t, \mu_t^u)$$

subject to (4.2), (4.3), (4.4), (4.5), (4.7), and (4.8) .

According to this, the first order conditions of the household's problem are the following:

$$u_{c_t}(c_t, H_t, h_t, \mu_t^u) + \eta_t \rho = \lambda_t (1 + \Phi_{c_t}(c_t, m_{t+1}, x_t)) \quad (4.9)$$

$$u_{h_t}(c_t, H_t, h_t, \mu_t^u) + \lambda_t \frac{W_t}{P_t^c} = 0 \quad (4.10)$$

---

<sup>12</sup>Each period instead of choosing hours households choose a probability of working  $\alpha$ . The new commodity being introduced is a contract between the firm and the household that commits to work  $h_0$  hours with a probability  $\alpha$ . The contract is what is being traded, so the household gets paid whether it works or not. Since households are identical all are going to choose the same  $\alpha$ . So all households are going to offer  $\alpha h_0$  which is a fixed quantity. As the utility function is linear in leisure it implies an infinite elasticity of substitution between leisure in different periods. This follows no matter how small this elasticity is for the individuals in the economy. Therefore the elasticity of substitution between leisure in different periods for the aggregate economy is infinite and independent of the willingness of the individuals to substitute leisure across time.

If  $\alpha$  increases then it means that people are willing to work more, this means that a higher portion of people are working. Therefore the sum of hours worked is higher and with the same population (assuming there is no population growth) the number of hours worked per-capita is going to be higher.

<sup>13</sup>Commonly known as inward looking habit.

<sup>14</sup>This friction is introduced in order to obtain the persistence in consumption which is observed in the data.

$$\lambda_t (\Phi_{x_t}(c_t, m_{t+1}, x_t) + q_t) = \gamma_t f_{x_t} \left( \frac{x_t}{k_t} \right) k_t \quad (4.11)$$

$$\beta E_t \left( \lambda_{t+1} \frac{R_{t+1}}{P_{t+1}^c} + \gamma_{t+1} \left( f \left( \frac{x_{t+1}}{k_{t+1}} \right) + \frac{\partial \left( f \left( \frac{x_{t+1}}{k_{t+1}} \right) k_{t+1} \right)}{\partial (k_{t+1})} \right) + \gamma_{t+1} (1 - \delta) \right) = \gamma_t \quad (4.12)$$

$$\beta E_t \left( \frac{\lambda_{t+1}}{(1 + \pi_{t+1}^c) (1 + \Phi_{m_{t+1}}(c_t, m_{t+1}, x_t))} \right) = \lambda_t \quad (4.13)$$

$$\beta E_t \left( \frac{\lambda_{t+1} (1 + i_{t+1})}{(1 + \pi_{t+1}^c)} \right) = \lambda_t \quad (4.14)$$

$$\beta E_t \left( \lambda_{t+1} (1 + i_{t+1}^f) q_{t+1} \right) = \lambda_t q_t \quad (4.15)$$

$$\beta E_t (\eta_{t+1} + U_{H_{t+1}}(c_{t+1}, H_{t+1}, h_{t+1}, \mu_{t+1}^u) - \eta_{t+1} \rho) = \eta_t \quad (4.16)$$

and equations (4.4), (4.5) and (4.8). Where  $\lambda$ ,  $\gamma$  and  $\eta$  are the lagrange multipliers associated with the budget constraint, the evolution of capital and the evolution of the stock of habit, respectively.

#### 4.1.2 The Producers

This sector is competitive and the producers seek to maximize their profits by choosing the level of capital and labor, given the rental rate of capital, the nominal wage and a technology to produce output, which is sold at price  $P_t$ . The technology is assumed to be a standard Cobb-Douglas production function. Hence the problem faced by producers is to solve

$$\max_{\{k, h\}} \Pi_t = P_t A_t (k_t^d)^\alpha (h_t^d)^{(1-\alpha)} - R_t k_t^d - W_t h_t^d \quad (4.17)$$

where  $A_t$  is the level of productivity, the subscript  $d$  represents the specific input's demand and  $\log(A_t)$  will follow a standard autoregressive process of order one<sup>15</sup>. The first order conditions for the producers of the homogeneous good are the standard ones.

<sup>15</sup> $\log(A_{t+1}) = \rho_1 \log(A_t) + (1 - \rho_1) \log(\bar{A}) + \epsilon_{t+1}$  where  $\bar{A}$  represents the average value taken by  $A$  across time.

### 4.1.3 The Retailers

The retailers, purchase homogeneous output from producers at a price  $P_t$ , and turn it into their specific brand of consumption good at zero additional cost. However, on each period retailers face a constant probability,  $1 - \varepsilon$ , of receiving a signal, that tells them that they can re-optimize their price, this probability behaves as in Calvo (1983, [15]). The other  $\varepsilon$  retailers follow a backward indexation rule, see Christiano, Eichenbaum, Evans (2001, [37])<sup>16</sup>. This probability is independent across firms and time. We assume that if a retailer doesn't receive the signal, it fixes his price according to<sup>17</sup>:

$$p_t^{rule}(z) = p_{t-1}^c(z)(1 + \pi_{t-1}^c) \quad (4.18)$$

where  $p_{t-1}^c$  is retailer's last periods price and  $\pi_{t-1}^c$  is the period  $t - 1$  rate of inflation of the aggregate consumption price index.

With probability  $1 - \varepsilon$  a retailer is going to optimize and set  $p_t^{opt}$ . If this is the case the retailer's problem is the following:

Each retailer<sup>18</sup> ( $z$ ) expected profits at period  $t + j$  are given by:

$$E_t(\Pi^R(z)_{t+j}) = E_t(c(z)_{t+j}(p^c(z)_{t+j} - P_{t+j})) \quad (4.19)$$

The real profits of each retailer are  $\Pi^R(z)_{t+j}/P_{t+j}^c$ , so those firms who are allowed to adjust their price in period  $t$  will choose  $p^c(z)_{t+j}$  to:

$$\max_{\{p^c(z)_t\}} E_t \sum_{j=0}^{\infty} (1 - \varepsilon) \varepsilon^j \Delta_{t+j} \frac{\Pi^R(z)_{t+j}}{P_{t+j}^c}$$

where the discount factor  $\Delta_{t+j} = \beta^j \frac{u'(C_{t+j}, h_{t+j}, H_{t+j})}{u'(C_t, h_t, H_t)}$  is an appropriate discount factor according to the market's real interest rate, and households take it as given for their maximization problem. Notice that in period  $t$  the firm chooses a price from now on,

<sup>16</sup>This indexation rule makes it possible for the model to have inflation different from zero. It also implies that in the steady state prices are going to have zero dispersion, i.e. the price that follows the backward indexation rule is equal to the optimal price. Other pricing rules are  $p_t^{rule}(z) = p_{t-1}^c(z)$  or  $p_t^{rule}(z) = p_{t-1}^c(z)(1 + \bar{\pi})$  where  $\bar{\pi}$  is the long run inflation. These rules are studied by Dotsey, King and Wolman (1999, [21]).

<sup>17</sup>One way to interpret this pricing rule is to assume that on each period retailers face a constant probability  $1 - \varepsilon$ , of wanting to gather information about the state of the economy in order to re-optimize their price (see Mankiw and Reis, 2002, [41]). So those  $1 - \varepsilon$  who gather the information, re-optimize their price according to it. In contrast the other  $\varepsilon$  retailers follow a backward indexation rule, they keep changing their prices according to past information. So in a sense this is not exactly a case of sticky prices, because as one can see everyone is changing prices but not re-optimizing. This is more a case of sticky information.

<sup>18</sup>Retailers are indexed by  $z$ .

$p^c(z)_{t+j} = p^c(z)_t$  because of the uncertainty on future price changes, in other words, the firm does the maximization taking into account that today they can re-optimize prices (with probability  $(1 - \varepsilon)$ ) and that for  $j$  periods they are not going to re-optimize them (with probability  $\varepsilon^j$ ).

From the households problem it can be shown (see appendix 1) that the demand for the consumption good  $c(z)_t$  is:

$$c(z)_{t+j} = \left( \frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} c_{t+j} \quad (4.20)$$

so the maximization problem ends up being:

$$\max_{\{p^c(z)_t\}} E_t \sum_{j=0}^{\infty} (1 - \varepsilon) \varepsilon^j \Delta_{t+j} c_{t+j} \left[ \left( \frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{1-\theta} - \varphi_{t+j} \left( \frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} \right]$$

where  $\varphi_{t+j} = \frac{P_{t+j}}{P_{t+j}^c}$ .

After solving for  $p^c(z)_t$ , the solution becomes (see appendix 2 for derivation):

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta - 1} E_t \left[ \frac{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \varphi_{t+j} \left( \frac{P_{t+j}^c}{P_t^c} \right)^{\theta}}{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left( \frac{P_{t+j}^c}{P_t^c} \right)^{\theta-1}} \right] \quad (4.21)$$

or what is the same

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta - 1} E_t \left( \frac{\Theta_t}{\Psi_t} \right)$$

where

$$\Theta_t = \Delta_t c_t \varphi_t + \varepsilon E_t \left( (1 + \pi_{t+1}^c)^\theta \Theta_{t+1} \right)$$

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left( (1 + \pi_{t+1}^c)^{\theta-1} \Psi_{t+1} \right)$$

and  $p_t^{opt}$  denotes the price of the good  $c(z)_t$  set by the retailer  $z$  in the case in which he decides to optimize. Since (4.20) implies that the price index is also a CES aggregator, it can also be shown that the price index  $P_t^c$  is given by<sup>19</sup>

$$P_t^c = \left[ \varepsilon (p_t^{rule})^{1-\theta} + (1 - \varepsilon) (p_t^{opt})^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (4.22)$$

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<sup>19</sup>As we know the consumption index is  $c_t = \left[ \int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$  which implies that the demand for the  $z$ -th good is  $c(z)_{t+j} = \left( \frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} c_{t+j}$ , where  $P_{t+j}^c$  is an index of the cost of buying a unit of  $c(z)_t$  :  $P_t^c = \left[ \int_0^1 (p_t^c(z))^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$ . This integral can be divided into two. So, retailers can be separated into two groups, a fraction  $(1 - \varepsilon)$  that optimizes their price, and a fraction  $\varepsilon$  that doesn't.

and then the aggregate inflation dynamics is given by

$$(1 + \pi_t^c) = \left( \varepsilon(1 + \pi_{t-1}^c)^{(1-\theta)} + (1 - \varepsilon) \left( \frac{p_t^{opt}}{P_t^c} \right)^{(1-\theta)} (1 + \pi_t^c)^{(1-\theta)} \right)^{\frac{1}{1-\theta}} \quad (4.23)$$

#### 4.1.4 Consolidated Monetary and Fiscal Authority

On each period  $t$ , the government issues money, transfers a net lump sum to households and makes unproductive expenditures. It is also assumed that the government collects the liquidity costs paid by households. Seigniorage as well as the liquidity costs represent income for the government so their budget constraint is the following:

$$m_{t+1}^s - \frac{m_t^s}{1 + \pi_t^c} + \Phi(c_t, m_{t+1}, I_t) = \tau_t + \left( \frac{g_t}{y_t} \right) y_t \quad (4.24)$$

where the letters with subscript  $s$  represent a supply, and  $g_t$  is real government expenditure.  $\log\left(\frac{g_t}{y_t}\right)$  follows a standard autoregressive process of order one<sup>20</sup>.

It is also assumed that monetary policy is conducted with an interest rate policy rule, of the form:

$$i_t = i + \zeta(\pi_t^c - \bar{\pi}^c) + \xi(y_t - y^{ss}) \quad (4.25)$$

where  $i$  is the steady state nominal interest rate level,  $\bar{\pi}^c$  is the inflation target<sup>21</sup>, and  $y^{ss}$  corresponds to the steady state level of output (this is the level of output in absence of shocks)<sup>22</sup>.  $y_t$  is determined by the production technology described in the last subsection.  $\zeta$  and  $\xi$  are parameters that determine the importance that the monetary authority gives to inflation and output respectively when using the nominal interest rate as the policy instrument.

#### 4.1.5 Competitive Equilibrium

To characterize the competitive equilibrium, the following definitions are used:

**Definition:** A price system is a positive sequence  $\{W_t, R_t, p_t^{rule}, p_t^{opt}, P_t^c, P_t, e_t, i_t, i_t^f\}_{t=0}^\infty$ .

<sup>20</sup> $\log\left(\frac{g_{t+1}}{y_{t+1}}\right) = \rho_2 \log\left(\frac{g_t}{y_t}\right) + (1 - \rho_2) \log\left(\frac{\bar{g}}{\bar{y}}\right) + \epsilon_{t+1}$  where  $\left(\frac{\bar{g}}{\bar{y}}\right)$  represents the average value taken by  $\frac{g}{y}$  across time.

<sup>21</sup>Notice that this target is in terms of the inflation of the prices of heterogeneous goods.

<sup>22</sup>It's not the level of output in the absence of frictions because transaction costs are still present in the steady state.

**Definition:**  $\{A_t, \mu_t^y, \mu_t^u, \frac{g_t}{y_t}, P_t^*\}_{t=0}^\infty$  are taken as exogenous sequences.  $m_0, k_0, b_0, F_0, H_0 > 0$  are also taken as given. An equilibrium is a price system, a sequence of consumption  $\{c_t\}_{t=0}^\infty$ , investment  $\{x_t\}_{t=0}^\infty$ , capital  $\{k_t\}_{t=1}^\infty$ , number of hours worked per-capita  $\{h_t\}_{t=0}^\infty$ , habit stock  $\{H_t\}_{t=1}^\infty$ , domestic real private bonds  $\{b_t\}_{t=1}^\infty$ , net foreign assets  $\{F_t\}_{t=1}^\infty$  and a positive sequence of real money  $\{m_t\}_{t=1}^\infty$  in order that:

1. Given the price system and net lump sum transfers, household's optimal control problem is solved with  $\{m_t^d = m_t^s = m_t\}_{t=1}^\infty$ ,  $\{k_t^d = k_t^s = k_t\}_{t=1}^\infty$ ,  $\{b_t = 0\}_{t=1}^\infty$ ,  $\{h_t^d = h_t^s = h_t\}_{t=0}^\infty$ ,  $\{c_t\}_{t=0}^\infty$  and a level of  $\{F_t\}_{t=1}^\infty$  such that  $(1 + i_t) = (1 + i_t^f)(1 + d_t)$  is satisfied.
2. The government's budget constraint (4.24) and policy rule (4.25) are satisfied for all  $t \geq 0$ .
3.  $Y_t = C_t + I_t + G_t + F_{t+1} - (1 + i_t^f)F_t$  for all  $t$ .

This last condition is the standard resource restriction in a small open economy.

## 4.2 Calibration

We now proceed to calibrate the model. There are some parameters that are uncontroversial, while others deserve some explanation. Parameter  $B$  is calibrated to obtain  $h = \frac{1}{3}$  in steady-state. The capital share within the production function is set at  $\alpha = \frac{1}{3}$  which approximately corresponds to the capital share in income. The capital stock time series in Colombia is a constructed one, which assumes a quarterly depreciation rate of 0.012, so we set  $\delta = 0.012$ . The parameter  $\theta$  that determines the degree of competition in the differentiated goods market, is set to 5 in order to obtain a markup of 25% according to the most recent research on market structure available in Colombia<sup>23</sup>. The parameter  $\varepsilon$  that determines the degree of price stickiness is set to 0.75 in order to have prices changing every one year.  $\beta$ , which in equilibrium is equal to  $\frac{1}{1+r}$  is fixed at 0.984 according to Vasquez (2003,[59]) who estimated the annual long term interest rate for Colombia in 6.81% which corresponds to 1.6% quarterly. The inflation target  $\bar{\pi}$  is fixed at 5.5% (annual rate) according to the target set for this year by the Central Bank.  $i$  was fixed according to  $\bar{\pi}$  and  $r$ . We

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<sup>23</sup>See Arango et al. (1991,[4])

set the international interest rate  $i_t^* = 0.03$ . The parameters  $\zeta$  and  $\xi$  corresponding to the weight given by the monetary authority to the inflation and output gap respectively were fixed in 1.7 and 0 according to Melo and Riascos (2004,[43]), although they estimated the rule with a lag on the interest rate.

The parameter  $\omega_{ss}$  of the risk premium function was calibrated according to the spread between  $i$  and  $i_t^*$ . We calibrate the rest of parameters of the risk premium function,  $\vartheta$ , to match the long term total external debt to GDP ratio, which for Colombia is about 30%.

Investment adjustment costs were calibrated so that in the steady state there are no adjustment costs,  $f\left(\frac{x}{k}\right) = c_2\left(\frac{x}{k}\right)^2 + c_1\left(\frac{x}{k}\right) + c_0 = \left(\frac{x}{k}\right)$  and  $f'\left(\frac{x}{k}\right) = 1$ . For a given  $c_2$ , this two conditions determine  $c_1$  and  $c_0$ . So,  $c_2$  is fixed to replicate investment's volatility which according to the Hodrick and Prescott filter is 18.8% for Colombia.

Since there is no information about the parameters that determine the evolution of habit over time, we calibrate them to replicate some stochastic properties of the consumption time series in Colombia:  $\varphi$  is set to replicate its volatility as close as possible (which is of 1.4% for Colombia according to data filtered with Hodrick and Prescott) and  $\rho$  is fixed to obtain the observed persistence of consumption's cyclical component.

We pay special attention to the parameter  $a$  in the transaction cost function, which determines the elasticity of the quantity of money demanded to consumption and interest rate. The first order conditions of the model allow us to obtain an approximation to the money demand of this economy. So we decided to estimate the values of  $a$  and  $\kappa$ . Using equations (4.14) and (4.13) we solve deterministically for  $m_{t+1}$  and obtain:

$$m_{t+1}^{1+a} = \frac{a\kappa(c_t + \nu q_t x_t)^a}{\frac{i_{t+1}}{1+i_{t+1}}} \quad (4.26)$$

Applying logs to equation (4.26) we obtain:

$$\log(m_{t+1}) = \frac{1}{1+a}\log(a\kappa) + \frac{a}{1+a}\log(c_t + \nu q_t x_t) - \frac{1}{1+a}\log(i_{t+1}) + \frac{1}{1+a}\log(1 + i_{t+1})$$

and we estimate it in order to solve for the coefficients  $a$ ,  $\kappa$  and  $\nu$ . We used non-linear ordinary least squares with the following three restrictions:  $a > 1$ ,  $0 < \nu < 1$  and  $\kappa > 0.0645$ . The restriction on  $a$  is to avoid the case of a linear function, the one on  $\nu$  is straight forward and in principle  $\kappa$  should be  $\kappa > 0$  but 0.0645 is the minimum value for which we were able to obtain the solution. What we found was a corner solution on  $\kappa$ , so our results were

$a = 1.858$ ,  $\kappa = 0.0645$  and  $\nu = 0.025$ . M1 was used for  $m$ , for  $q$  which can be defined as the real exchange rate (recall that in the model  $q = \frac{e_t}{P_t^c} = \frac{P_t}{P_t^c}$ ) we used the spot's market nominal exchange rate times the U.S. core CPI (CPI minus food and energy) divided by Colombia's CPI, and for  $i$  we used the CD's 90 days interest rate.

We finally describe the parameters related to the exogenous shocks. We focus only on the productivity shock since it is the only one used in our simulations. For the productivity shock,  $A$ , we performed a standard Solow residual computation to obtain an autocorrelation coefficient of  $\rho_1 = 0.83$ . The standard deviation is calibrated to reproduce as closely as possible the observed output's volatility (using a Hodrick and Prescott filter it is  $1.62\%$ )<sup>24</sup>. Finally, the standard deviation of the forcing variable  $A$  is set to reproduce as closely as possible the observed output's volatility which was found to be  $1.62\%$  according to Hodrick and Prescott's filter.

The autocorrelation of the remaining shocks, government expenditures, preferences and risk premium were found to be  $0.773$ ,  $0.71$  and  $0.69$  respectively<sup>25</sup>. As we mentioned above this parameters are not considered in our simulation exercise.

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<sup>24</sup>Using labor, capital and product quarterly data from 1984:1 until 2003:4, and expressing the production function in logarithms one can solve for  $\log(A_t)$  in order to obtain a time series for  $A$ . From this new data we found an average value  $\bar{A} = 1.19$  (in levels). The parameter  $\rho_1$  was found by running the following regression  $\log(A_t) = \rho_1 \log(A_{t-1}) + (1 - \rho_1) \log(\bar{A}) + \epsilon_t$  where  $\epsilon$  is an error term. We performed a Wald's test to prove the null hypothesis  $\rho_1 + (1 - \rho_1) = 1$  and we obtained a F-statistic value of  $0.2156$  and a P-value of  $0.6437$ , so our null hypothesis is accepted, and  $A$  can actually follow a standard autoregressive process of order one as stated before.

<sup>25</sup>The autocorrelation  $\rho_2$  of the variable  $\frac{g_t}{y_t}$  was found by doing the following: we took the ratio between real total government expenditure and real GDP, we found the mean of this series and found  $\frac{\bar{g}}{\bar{y}} = 0.15$ , then we ran a regression of the autoregressive process and found  $\rho_2 = 0.773$  and that the standard deviation of the error is  $0.0063$ .

For the preference shock we took the consumer sentiment survey made by "Fedesarrollo" and specifically used the consumer confidence index. We assumed that by construction the index has media zero, this is because consumers are asked if they feel positive or negative about something and the negative answers are subtracted from the positive ones, so in steady state opinions should be divided in half. As the media of the process was assumed to be zero, then we ran the regression of the autoregressive process without intercept and we found the autocorrelation  $\rho_3$  of the variable  $\mu_t^u$  to be  $\rho_3 = 0.71$  and the standard deviation of the error  $0.07$ .

The autocorrelation  $\rho_4$  of the variable  $\mu_t^{\vartheta}$  was found by doing the the following: a daily series of the EMBI was used as a proxy of the variable  $\mu_t^{\vartheta}$ . As our model is quarterly then we found the quarterly geometric average of the series. We know that we are assuming that this variable has  $\bar{\mu}^{\vartheta} = 1$ , and so the intercept of the autoregressive process is zero, so we found the logarithm of our quarterly series and subtracted its mean from it. Then we ran a regression of the autoregressive process and found  $\rho_4 = 0.69$  and that the standard deviation of the error is  $0.0245$ .



## 4.3 Model Dynamics

### 4.3.1 Solving the Model

In order to solve the model, we first state the first order nonlinear dynamic system that characterizes the competitive equilibrium. In order to calculate the steady state we transform the system equations into their deterministic steady state representation and solve using numerical methods. Then we log-linearize around the deterministic steady state. At this stage the system is expressed in terms of relative deviations from the steady state.

After solving the model using the method of King, Plosser and Rebelo (2001,[32]) we obtain matrices  $\mathbf{M}$  and  $\mathbf{H}$  which generate the dynamic solution by iterating on the following two equations:

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{H}\mathbf{x}_t \\ \mathbf{x}_{t+1} &= \mathbf{M}\mathbf{x}_t + \mathbf{R}\eta_{t+1} \end{aligned} \tag{4.27}$$

where  $\mathbf{Y}$  is a vector composed by control, co-state and flow variables,  $\mathbf{x}$  is a vector of endogenous and exogenous states,  $\mathbf{H}$  characterizes the policy function and  $\mathbf{M}$  the state transition matrix.  $\eta_{t+1}$  is an innovation vector and  $\mathbf{R}$  is a matrix composed of zeros, ones or a parameter instead of a one. This matrix determines which variables are hit by the shock and in what magnitude. This state space representation will help us to compute the spectrum of the data.

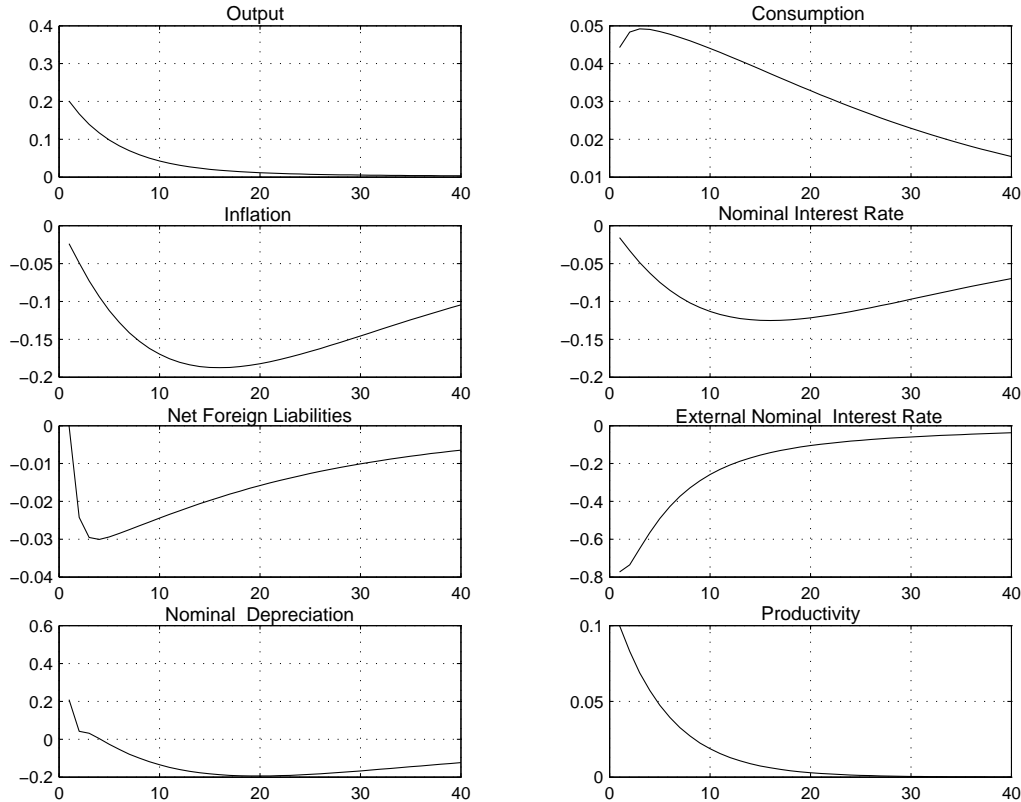
### 4.3.2 Impulse Responses

We report the response of the model to a 10% shock to productivity, preferences, real government expenditures and risk premium. Figure 4.1 shows the impulse response to a productivity shock. Higher productivity today and in the future increase consumption, investment (not shown) and output. Output increases more than absorption and as a result inflation falls below the Central Bank's target<sup>26</sup>. The monetary authority responds reducing the nominal interest rate. For a given level of external debt and due to the increase in output, the risk premium falls and so does the interest rate that the economy

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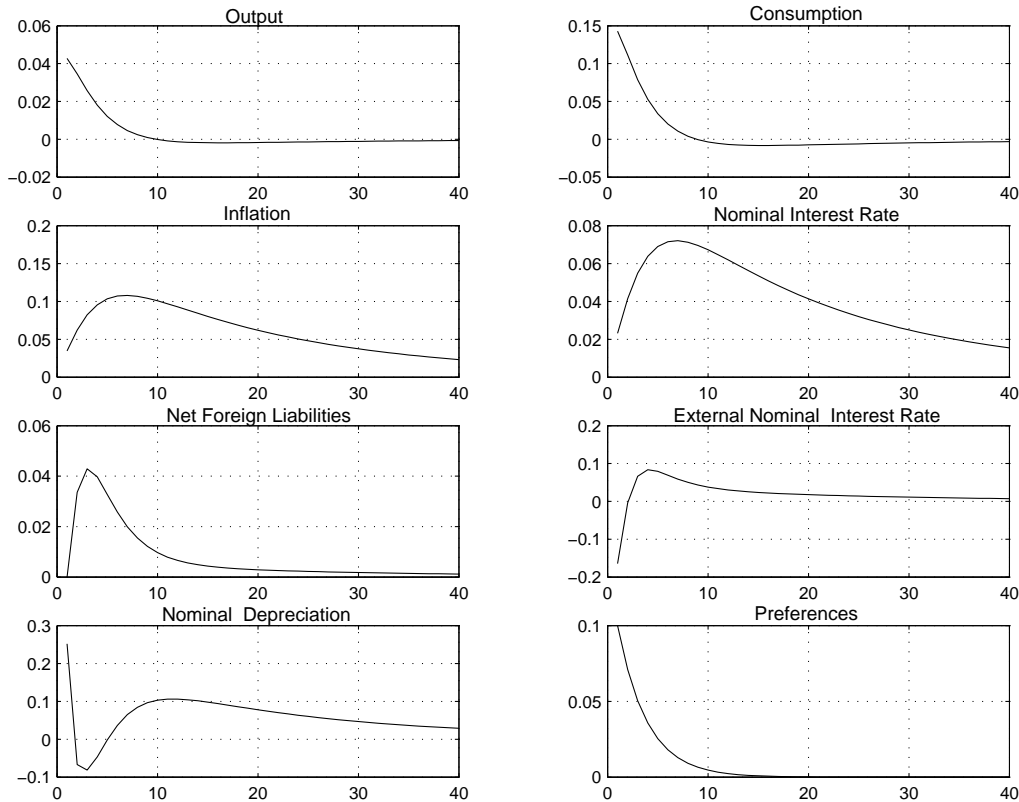
<sup>26</sup>The inflation that is falling is that of heterogeneous goods, and the output that increased was that of homogeneous goods, so the price of homogeneous goods decreased first and as the price of the heterogeneous good depends on the former, then it also falls.

Figure 4.1: Productivity Shock



faces externally. Capitals flow out of the country, that is the economy accumulates net foreign assets. In the balance of payments, the trade balance improves because output increases more than absorption and the net factor payments abroad fall. This is a standard result in small open economies: during productivity-driven booms the economy prepays external debt and this is reflected in a current account surplus. One interpretation is that the expectations of near-future debt repayments depreciate the nominal exchange rate on impact. Once the economy resumes the debt accumulation, the exchange rate appreciates. Note that between period one and two the exchange rate is appreciating although agents are demanding more foreign assets (in this case one would expect to see the exchange rate depreciating), between this two periods what happens is that there is another transmission mechanism that's acting: as output increased more than absorption, by market clearing conditions the price of homogeneous goods decreases and this is reflected as a whole in the exchange rate.

Figure 4.2: Preference Shock



What happens to the economy when hit by a preference shock is shown in Figure 4.2. All of a sudden agents decide to consume more and so output increases. This demand driven expansion generates inflationary pressures, to which the Central Bank responds by increasing the interest rate. The trade balance deteriorates because the increase in consumption is higher than the increase in output and so agents will finance consumption with higher indebtedness. Initially, the external interest rate falls, since the external debt to output ratio falls. As indebtedness increase so does the external interest rate faced by agents. Inflationary pressures increase price level of homogenous goods, by PPP the nominal exchange rate depreciates on impact, since debt increase next period (net foreign assets decrease), the nominal exchange rate appreciates, from there on the economy starts to pay back debt (increase net foreign assets) making the nominal exchange rate depreciate. The increase in foreign assets drives down the external interest rate.

Another interesting experiment is to analyze the impact of a transitory, but per-

sistent government expenditure shock. Figure 4.3 shows the results. Recall that in the model, the government finances government consumption by using net lump sum taxes. So, net transfers to agents fall (net taxes increase). In order to finance government expenditures agents take more external debt. The increase in government purchases “crowds out” consumption and investment, but still aggregate demand increases. Although equilibrium employment increases and so does output, this is not enough to compensate the absorption increase, so the trade balance deteriorates. This demand-driven shock increases inflation, calling for an interest rate hike by the Central Bank. Also the external interest rate falls, since the risk premium falls (recall that total external debt is given at the time of the shock and output has increased). Since debt is going to increase next period (net foreign assets decrease), the nominal exchange rate appreciates on impact. Between period one and period three approximately, the PPP mechanism is acting, so the increase in the price of homogeneous goods depreciates the exchange rate. From there on the economy starts to pay back debt (increase net foreign assets) making the nominal exchange rate depreciate and then slowly go back to its steady state level. The increase in foreign assets drives down the external interest rate.

Finally Figure 4.4 shows the case of a Risk Premium shock. When the risk premium increases it causes an increase in the external nominal interest rate. As debt becomes more expensive, agents are going to want to repay debt, this expectations make the nominal depreciation rate to depreciate on impact. In order to be able to pay debt, agents reduce their consumption and investment (not shown), and decide to work more (not shown). The increase in hours worked increases output. An imperfect pass-through is observed from the nominal exchange rate into the prices of heterogeneous goods, which causes inflation to rise. As a response, the monetary authority increases the nominal interest rate. As government expenditure is constant and consumption and investment decreased, the increase in output causes an excess of supply that generates a fall in the prices of the homogeneous good, this is what causes an appreciation and the following behavior of the exchange rate.

### 4.3.3 Sensitivity Analysis

We now study the properties of the dynamic response of the model to two key parameters: the degree of price stickiness and the sensitivity of the risk premium to the external debt to output ratio.

Figure 4.3: Government Expenditure Shock

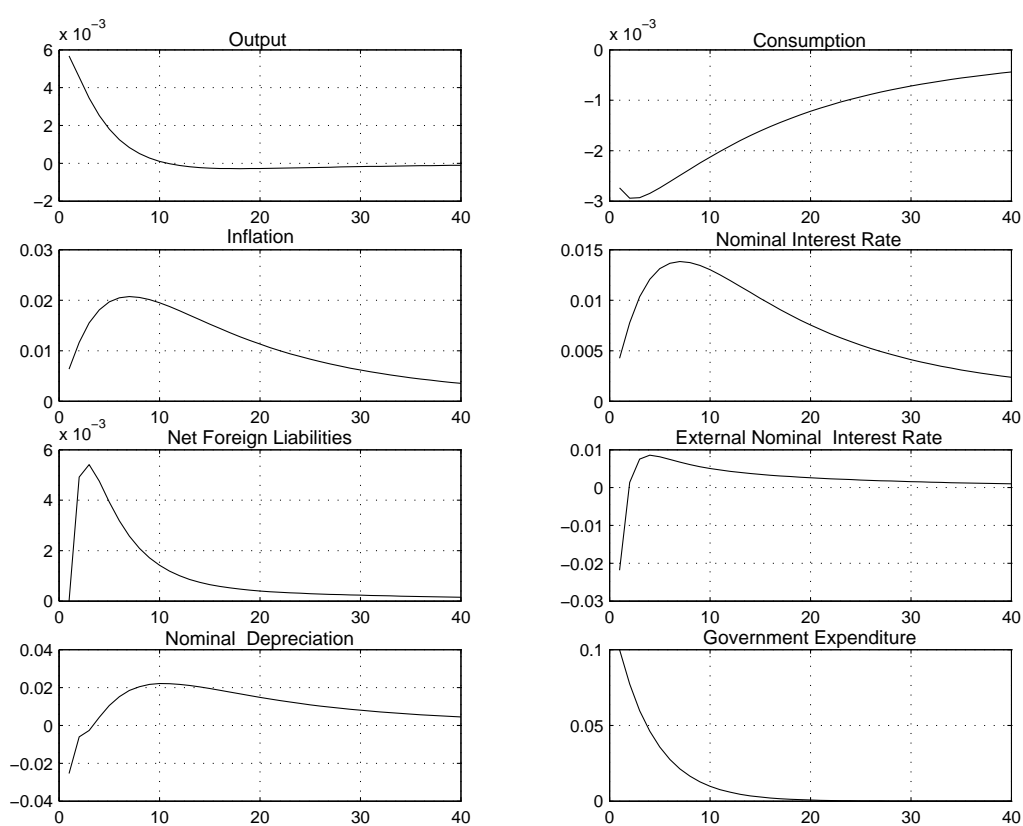
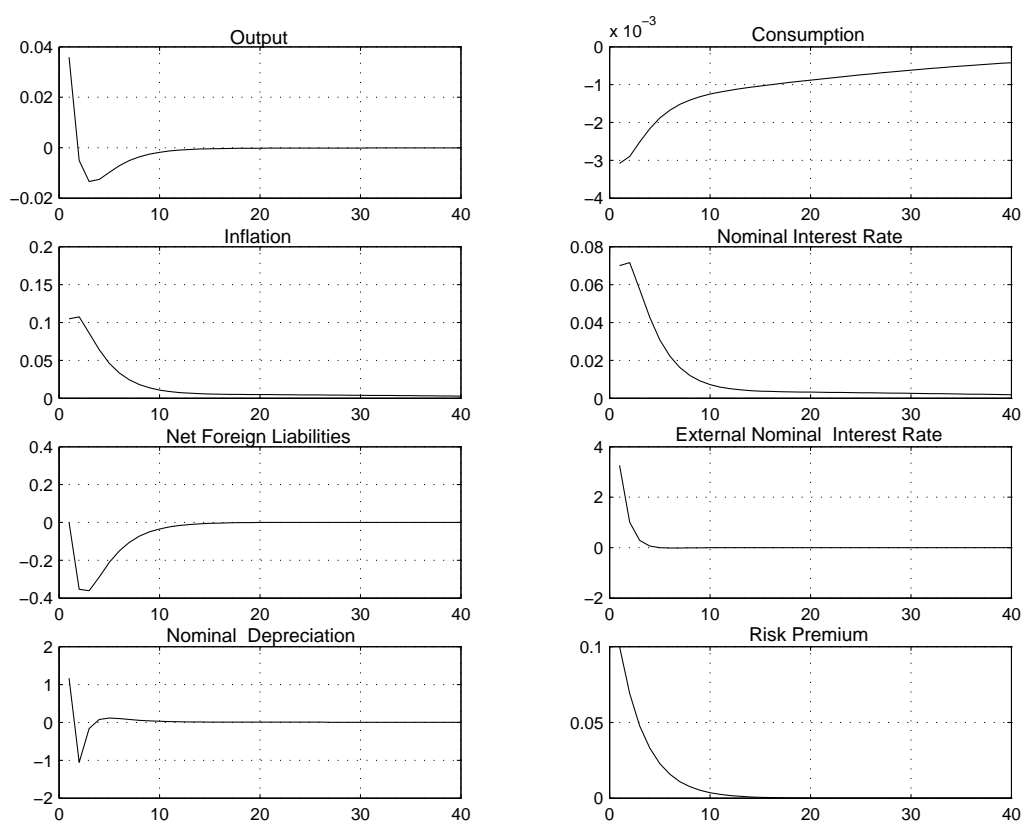


Figure 4.4: Risk Premium Shock



### More Flexible Prices

In our benchmark calibration we had set  $\epsilon = 0.75$ , so that retailers adjust prices every year. Now we show how the dynamics of the model change as retailers adjust prices every 6 months ( $\epsilon = 0.5$ ). Figures D.1 to D.3 in appendix 4 show the results. By increasing the degree of price flexibility we change the persistence and volatility of most of the nominal variables. Most of the real variables remain unchanged. So, as prices are more flexible:

1. Inflation becomes more responsive to all types of shocks. The response is considerably higher when the sources of the shock comes from the demand side (preferences and public expenditures). However, for more flexible prices the persistence is slightly lower.
2. As a consequence, the Central Bank adjusts the nominal interest rate more but for a shorter period of time.
3. On impact, the nominal depreciation is less responsive to productivity shocks and more responsive to public expenditure shocks. However, the subsequent adjustment process is more aggressive when prices are more sticky and the degree of persistence is higher.
4. The persistence of real consumption falls, for the productivity and public expenditure shocks.
5. There is little effect on the response of output, net foreign assets and the external nominal interest rate for all types of shocks.

### More Debt-Elastic Risk Premium

In the baseline calibration we had that ( $\omega_3 = 0.1$ ) and we study what happens as the degree of sensitivity to the risk premium is higher ( $\omega_3 = 0.5$ ). Figures D.4 to D.6 in appendix 4 show the results. By increasing the degree of response of the risk premium to debt, we change the persistence and volatility of the variables related to the external sector. There is little effect on domestic variables. So, as the risk premium becomes more sensitive to debt:

1. The external nominal interest rate becomes more responsive for all types of shocks.

2. The net foreign assets are less responsive for all shocks.
3. There is little or no effect on the rest of the variables for productivity, preferences and government expenditure shocks.
4. In the risk premium shock output, consumption, inflation and the nominal interest rate are less responsive for the higher elasticity.

#### 4.4 Validating the Model

As mentioned earlier, we deviate from the standard practice in the Real Business Cycle literature of comparing a subset of second moments of observed and artificial data. In order to assess the extent to which the calibrated model replicates salient and/or interesting features of the actual economy, we follow a frequency domain methodology proposed by Diebold et al. (1998,[50]).

We analyze the model's ability to reproduce the data by comparing the complete set of second moments at the frequency domain. There are several advantages of a frequency domain approach to compare the artificial and real data:

1. Working in the frequency domain enables the decomposition of variation across frequencies. This is useful to identify what portion of the variance is explained at what frequency or period.
2. The multivariate approach facilitates the comparison of cross-variable correlations and lead-lag relationships.
3. Allows to compare a full set of second moments, unlike in the standard approach in which only a subset of second moments are compared.
4. The spectrum is a graphical tool, so that we can assess visually and quickly the performance of the model.
5. We use a simple non-parametric bootstrap algorithm to perform statistical inference.

In sum, the methodology is to compare the model and data spectra. For a given set of calibrated parameters, the spectra can be used to assess the performance of our model.



As we will show in the next subsections, this tool is very useful to determine the advantages and limitations of our model.

#### 4.4.1 The Theoretical Model Vs. the Observed Data

In this subsection we summarize the methodology and present our results concerning the agreement of the data spectrum with the model spectrum. The methodology consists of five steps. First, we took a series for the Gross Domestic Product (GDP) and another one for inflation<sup>27</sup>, logarithms were applied to the GDP series and then both series (inflation and output) were seasonally adjusted using the X12 filter and then filtered using Hodrick and Prescott, so that frequencies beyond eight years were eliminated. Second, we estimate the sample data spectrum and compute its uncertainty using bootstrap techniques. Third, from the estimated spectrum and its uncertainty we determine the salient and/or interesting features that we expect the theoretical model to reply. Fourth, we compute the model's theoretical spectrum. Finally, we compare the theoretical and observed estimated spectrums at the required frequencies. The methodology proposed by Diebold et al. goes a little further by proposing an spectral maximum likelihood estimation technique to calibrate the model parameters by minimizing the disagreement between sample and theoretical spectrums at pre defined frequencies. This step is left for future work.

#### 4.4.2 Estimating the Spectra

For an N-variate linearly regular covariance stationary process with population autocovariance matrices  $\mathbf{\Gamma}(\tau) = E[(\mathbf{Y}_{t+\tau} - \mu)(\mathbf{Y}_t - \mu)^T]$ , the population spectra at frequency  $\omega$  is defined as

$$\mathbf{F}_{\mathbf{Y}}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \mathbf{\Gamma}(\tau) \exp(-i\omega\tau)$$

for  $\omega \in [-\pi, \pi]$ . An important property of the spectra is that

$$\int_{-\pi}^{\pi} \mathbf{F}_{\mathbf{Y}}(\omega) \exp(i\omega\tau) d\omega = \mathbf{\Gamma}_{\tau}$$

which is particularly useful when  $\tau = 0$ . See Hamilton (1994,[27]) Chapters 6 and 10.

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<sup>27</sup>The series for the GDP was constructed as follows: For the period 1994-2003, the quarterly data was taken from the national accounts statistics reported by the colombian national department of statistics (DANE). For the period 1977-2003, this series was backward-chained using the quarterly growth rate reported for this period by the national department of planning (DNP). The inflation series is from the Central Bank.

The diagonal elements of  $\mathbf{F}_Y(\omega)$ ,  $f_{kk}(\omega)$ , are the univariate spectra. According to the spectral representation theorem, areas under this curve are the relative contribution of the frequencies to the total unconditional variance of the  $k^{th}$  variable.

Off diagonal elements,  $f_{kl}(\omega)$ , are the cross spectral densities, and can be expressed in polar form as

$$f_{kl}(\omega) = ga_{kl}(\omega) \times \exp\{i \times ph_{kl}(\omega)\}$$

where

$$ga_{kl}(\omega) = \sqrt{re^2(f_{kl}(\omega)) + im^2(f_{kl}(\omega))}$$

is the gain and

$$ph_{kl}(\omega) = \arctan\{im(f_{kl}(\omega))/re(f_{kl}(\omega))\}$$

is the phase at a frequency  $\omega$ . The gain tells us by how much the amplitude of  $y_l$  has to be multiplied in order to reach the amplitude of  $y_k$  at a same frequency  $\omega$ . The phase measures the lead of  $y_k$  over  $y_l$  at frequency  $\omega$  (The phase shift in time units is  $ph(\omega)/\omega$ ). Instead of using the gain it is customarily to report the coherence defined as  $coh_{kl}(\omega) = ga^2(\omega)/(f_{kk}(\omega) \times f_{ll}(\omega))$ , which measures the squared correlation between  $y_k$  and  $y_l$  at a frequency  $\omega$  (See Hamilton (1994,[27]) Chapter 10).

An obvious non parametric way to estimate the population spectra based on a sample  $\{\mathbf{Y}_t\}_{t=1}^T$ , is to replace the population autocovariances and mean vector  $\mu$  with sample quantities so that the sample autocovariance at lag  $\tau$  becomes

$$\hat{\Gamma}_\tau = \frac{1}{T} \sum_{t=\tau}^{T-1} (\mathbf{Y}_{t-\tau} - \bar{\mathbf{Y}}) (\mathbf{Y}_t - \bar{\mathbf{Y}}) \quad \text{for } -(T-1) \leq \tau \leq (T-1)$$

and the estimated spectra

$$\hat{\mathbf{F}}_Y(\omega_j) = \frac{1}{2\pi} \left\{ \sum_{\tau=-(T-1)}^{T-1} \hat{\Gamma}_\tau \exp(-i\omega_j\tau) \right\}$$

evaluated at frequencies  $\omega_j = 2\pi j/T$  for  $j = 1, 2, 3, \dots, T/2 - 1$ . However, this sample estimate is not consistent. A consistent estimate may be found by windowing the autocovariances sequence using the Blackman-Tuckey approach which gives an estimated spectra of the form,

$$\hat{\mathbf{F}}_Y^*(\omega_j) = \frac{1}{2\pi} \left\{ \sum_{\tau=-(T-1)}^{T-1} \Lambda(\tau) \hat{\Gamma}_\tau \exp(-i\omega_j\tau) \right\}$$

where the window function  $\mathbf{\Lambda}(\tau)$  is a matrix of lag windows<sup>28</sup>. By adjusting the lag window according to the sample size we can simultaneously reduce the bias and variance of the spectra estimate and hence obtain a consistent estimator of the population spectra. This approach is the same as smoothing the estimated sample periodogram using an equivalent spectral kernel. From this spectrum estimate we obtain estimates of the population coherence and phase.

#### 4.4.3 Assessing Sample Variability

In order to assess the sampling variability of this estimator, Diebold et al. propose to use a resampling algorithm called the Cholesky factor bootstrap. If the vector sequence  $\varepsilon^{(i)}$  is a random sample of an  $NT$  dimensional standard distribution,  $(\mathbf{0}_{NT}, \mathbb{I}_{NT})$ , then

$$\mathbf{z}^{(i)} = \bar{\mathbf{z}} + \mathbf{P}^* \varepsilon^{(i)} \sim (\mathbf{1}_T \otimes \mu, \mathbf{\Sigma}^* = \mathbf{P}^* \mathbf{P}^{*T})$$

where  $\bar{\mathbf{z}} = \mathbf{1}_T \otimes \bar{\mathbf{Y}}$  and  $\mathbf{\Sigma}^*$  is the corresponding variance covariance matrix obtained from the estimated autocovariance matrices multiplied by the corresponding window functions.

For each iteration ( $i = 1, 2, 3, \dots, R$ ) we randomly draw  $\mathbf{z}^{(i)}$  and from this we compute  $\hat{\mathbf{F}}^{*(i)}(\omega_j)$  for  $\omega_j = 2\pi j/T$  ( $j = 1, 2, 3, \dots, T/2 - 1$ ) and then construct the confidence intervals for the spectra, cross-spectra, coherence and phase of the vector.

#### 4.4.4 The Theoretical Model Spectra

Given that the model can be written in a State Space Form

$$\mathbf{Y}_t = \mathbf{H}\mathbf{x}_t\mathbf{x}_{t+1} = \mathbf{M}\mathbf{x}_t + \mathbf{R}\eta_{t+1} \quad (4.28)$$

where the innovation vector  $\eta_{t+1}$  is iid $(\mathbf{0}, \mathbf{\Omega})$ , it is straightforward to compute the theoretical model spectra by simple spectral density arithmetic (See Hamilton (1994,[27]) Chapter 10). Notice that equation (4.28) is closely related to (4.27).

When this is not possible (that is when  $\mathbf{\Omega}$  is singular or when observed data and model are assumed to arise from different sets of transformations), it is advisable to generate

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<sup>28</sup>A window lag matrix is a generally truncated symmetric and positive, weighting function for the lags. The truncation lag defines the window size, and outside this window the weights are zero. By giving small or zero weights to long lagged autocovariance matrices (the poorly estimated ones), the estimated spectra becomes smoother and consistent at the cost of some small sample bias.

a very long simulated path of the variables subject to continuous innovations, and estimate the spectra from this simulation. If the simulation is long enough, the sampling errors are negligible.

In our case we followed the second methodology. We generated artificial data and filtered it with Hodrick and Prescott, then we took the observed data and filtered it with Hodrick and Prescott as well in order to have two groups of series in the same frequencies to be able to compare them.

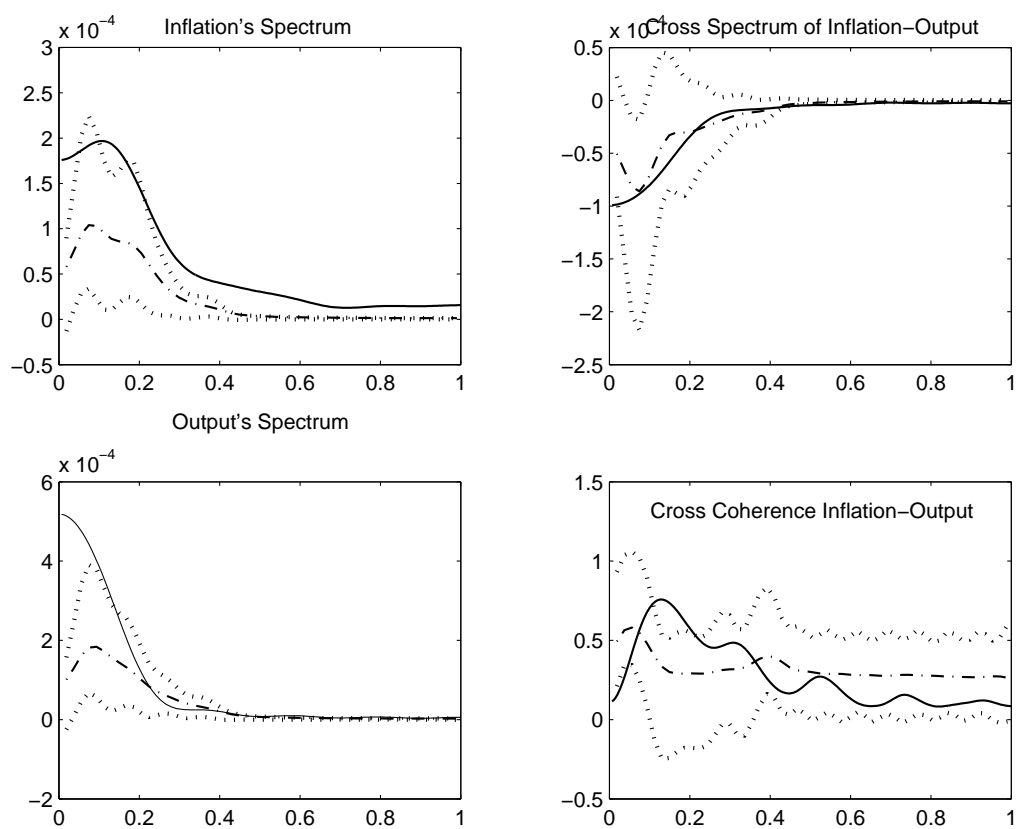
#### 4.4.5 Results

Figure 4.5 contains, on the upper and lower left panels, the estimated inflation and output gap spectrums along with the corresponding 95% uncertainty bands and the model theoretical spectra. The upper right panel contains the estimated inflation and output gap cross spectral density together with its uncertainty bands and the theoretical cross spectrum. On the lower right panel we find the estimated coherence of inflation and output gap, its uncertainty and the theoretical one.

From the two left panels, that is the univariate spectra, we find that the spectral density is statistically significant for frequencies between  $0,04\pi$  and  $0,4\pi$  which correspond to periods between 2 and 25 quarters, and variations along these frequencies explain at least 80% of the observed sample variability. The estimated spectra shows a peak for frequencies between  $0,08\pi$  and  $0,1\pi$  which correspond to periodic movements between 10 and 12 quarters. The estimated cross periodogram is negative for all frequencies and significant for frequencies between  $0,04\pi$  and  $0,1\pi$ , that is for periods between 10 and 25 quarters. The population coherence is statistically significant at frequencies of up to  $0,092\pi$ , that is for periodic movements beyond 11 quarters, and is not dominated by any particular frequency although it presents a peak at  $0,05\pi$ , with a correlation of 0.74 for movements around 20 quarters. It is also scattered significant for some high frequencies .

From these figures we derive the salient features of the data that the model has to mimic. First, inflation and output gap are dominated by periodic movements between 2 and 25 quarters with a peak between 10 and 12 quarters, which could show some degree of stickiness or persistence. The cross spectrum and coherence show results in the same direction. The population coherence does not seem to be dominated by a particular set of frequencies. However, there is a peak correlation of 0.74 for movements around 20 quarters.

Figure 4.5: Spectrums and Coherence



The solid line corresponds to the model theoretical spectrum. The dotted line corresponds to the estimated data spectrum. The dashed lines correspond to the 95% upper and lower bands of the data spectrum respectively.

The theoretical model frequency analysis shows some persistence both in the univariate spectra as well as in the cross spectrum, with monotone spectrum for output gap and cross spectrum. The inflation spectrum peaks at a frequency of  $0,10\pi$ , that is, for periodic movements between 9 and 10 quarters. The model's theoretical coherence presents clear dominance in frequencies between  $0,05\pi$  and  $0,45\pi$ , that is periodic movements between 2 and 20 quarters, with a maximum coherence at  $0,12\pi$ , that is periodic movements between 8 and 9 quarters.

The comparison between sample and theoretical spectra and cross spectra reveals important similarities. The theoretical spectra and cross spectra fall into the sample uncertainty bands for frequencies beyond  $0,05\pi$ , that is for periodic movements of inflation and comovements of inflation and output gap of up to 20 quarters, that is 5 years, and for periodic movements of output gap of up to 10 quarters (2 and a half years). For shorter frequencies the spectra and cross spectra of the model are significantly different from the sample ones. The model's coherence falls into the uncertainty bands for most of the frequencies but the ones surrounding the peak of the model's coherence, and very long run periodic movements.

We conclude that the model theoretical spectra and cross spectra does not differ statistically from the respective population quantities for, at least, frequencies beyond  $0,05\pi$ , which correspond to periodic movements of up to at least 10 quarters. Population's coherence is not statistically different from the model's coherence at most of the frequencies, it is only statistically different at the peak of the model's theoretical coherence and for very short frequencies (very long run period movements).

## 4.5 Final Remarks

This paper evaluates *quantitatively* how much of the observed dynamics of the Colombian macroeconomic data can be explained by a model in which the presence of nominal rigidities is important in the context of a small open economy. We explore the macroeconomic effects of different types of shocks (productivity, preference, government expenditure and risk premium shocks) and the implications for monetary policy when operating under an Inflation Targeting framework. The main monetary policy instrument is the nominal interest rate. We also study the macroeconomic effects of higher price flexibility

and a lower sensitivity of the risk premium to the debt to output ratio.

We find that as prices become more flexible, inflation becomes more responsive to all types of shocks and so the Central Bank has to respond with higher interest rates. However, the effects of the shocks are less persistent although this difference is not substantial. In addition, the dynamic response of output, net foreign assets and the external interest rate doesn't change significantly. When the risk premium is more sensitive to external debt to output ratio, net foreign assets become less sensitive and nominal depreciation becomes more sensitive to all shocks, but the productivity one.

We go further and take the model to data. In particular, we evaluate the ability of our calibrated model to reproduce the behavior of observed cyclical inflation and output gap, when productivity shocks are the main source of fluctuations. We follow a frequency domain comparison methodology proposed by Diebold et al. (1998,[50]). The Colombian data is characterized by: first, inflation and output gap are dominated by periodic movements between 2 and 25 quarters with a peak between 10 and 12 quarters. The cross spectrum and coherence show results in the same direction. Second, the coherence does not show any significant dominance of frequencies for the cross movements but the correlation jumps to 0,6 for periodic movements around 5 quarters. These facts are compared to the data simulated from the model. We conclude that the simulated data spectra and cross spectra does not differ statistically from the respective population quantities for at least 10 quarters. The model spectra presents more persistence than the observed data and the population coherence is captured for most frequencies but the ones around the peak of the model's theoretical coherence and very long run periodic movements. It is also possible that the data displays a high degree of persistence due to the fact that the Colombian economy has suffered a long gradual disinflation period. A long disinflation period may induce a high degree of persistence in the data, that may not be present in the future. Subsequent research will address these issues.

At the theoretical level, a number of extensions are left for future work. First, one can evaluate the efficiency and welfare effects of alternative monetary policy rules. We have focused here on Taylor rules, but we can explore alternative specifications such as Inflation Forecast rules. In fact some recent work of Laxton and Pesenti (2003, [38]) and Levin, Wieland and Williams (2001, [39]) has evaluated the efficiency of alternative monetary policy rules. Second, we have used this model for an Emerging Market economy like Colombia. However, there is nothing particular in our model that pertains to an Emerging Economy. In

fact our model can also be used for a small open developed economy. Emerging Markets are characterized by a number of imperfections, like borrowing constraints, domestic financial markets imperfections, a high share of non-tradable sector and balance sheet effects of nominal depreciations to name a few. It would be interesting to explore many of these issues.



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## Appendix A

# Algorithm for Solving Endogenous Country Risk Models

The characterization of the equilibrium of the model presented in chapters 2 and 3 does not yield a closed form solution. However, the recursive nature of the model allows the computation of the equilibrium and the study of its long-run characteristics using numerical methods. The computational method used for the solution of the model is similar to that of Huggett (1993, [29]) and Aiyagari (1994, [1]). The main differences are that here there is an additional discrete state and action (default-repay) and the market clearing condition. To simplify notation, let  $x = g(s)$  denote the policy functions  $(a', d') = (a(s), d(s))$ . The method is based on the following steps:

1. Start with an arbitrary probability of default function  $\lambda^{(i)}$  and borrowing limit  $\underline{a}^{(i)}$
2. Given  $\lambda^{(i)}$  compute  $r^{(i)}$  using equation (4.17) and compute the optimal policy function  $g(s)$  and the optimal transition probability matrix  $P(s, B)$  using the equations (2.2)-(2.4).
3. Update  $\lambda^{(i)}$  to  $\lambda^{(i+1)}$  using the optimal transition probability matrix  $P(s, B)$  and update the borrowing limit to  $\underline{a}^{(i+1)}$ .
4. Check whether the zero profit condition is approximately satisfied. If not, use  $\lambda^{(i+1)}$

and  $\underline{a}^{(i+1)}$  and repeat steps 2 to 4.

5. After convergence of the algorithm, given the optimal policy  $g(s)$  and the optimal transition probability matrix  $P(s, B)$  iterate on  $\psi^{(i+1)}(B) = \int_S P(s, B) d\psi^{(i)}$  from an arbitrary initial distribution  $\psi^{(0)}$  to obtain the stationary probability distribution.

The first step of the algorithm is solved by policy function iteration on equations (2.2)-(2.4), for a given  $r^{(i)}$  function. The second step requires to obtain  $P(s, B)$ . Notice that the optimal policy  $g(s)$  and the Markov chain  $\pi$  on  $y$  induce a Markov chain on  $s$ ,  $P(s, B)$  via the formula:

$$\Pr[s_{t+1} = s' | s_t = s] = \Pr[d_{t+1} = d' | s_t = s] \Pr[a_{t+1} = a' | s_t = s] \Pr[y_{t+1} = y' | y_t = y]$$

or:

$$\Pr[s_{t+1} = s' | s_t = s] = \iota(d', a', s) \pi(y, y') \quad (\text{A.1})$$

where  $\iota(d', a', s) = 1$  if  $a' = a(s)$  and  $d' = d(s)$  and 0 otherwise. This indicator function identifies the time  $t$  states  $s$  that are sent into  $(a', d')$  at time  $t + 1$ . Equation (A.1) defines an  $n \times n$  matrix  $P$  where  $n$  is the number of total possible states. The matrix  $P$  is used to compute the ergodic distribution,  $\psi$  and the probability of default.

The computation of the probability of default in the third step deserves some comment. Typically, when there is no default option, all the states of the Markov Chain associated with  $P$  will be recurrent and its stationary distribution,  $\psi$ , can be interpreted as the fraction of time that the country spends in each state. When the possibility of default exists there are two types of states: repayment states, which are transient and defaulting states which are absorbent (once one of the states of this set of states has been reached, the state of the system moves only among them). The numerical results show that the matrix  $P$  has the following structure:

$$P = \begin{bmatrix} Q & R \\ 0 & U \end{bmatrix}$$

where  $Q$  is a matrix that yield the probability of moving within a set of transient states (repayment) in the next period,  $R$  is a matrix whose elements express the probability of moving from a repayment state to a defaulting state in the next period, and  $U$  is a matrix whose elements yield the probability of moving within the defaulting states. Note that the matrix 0 indicates that once the decision of default has been taken there is no possibility



to reach a repayment state in the next period. The optimal probability of default in period  $t + 1$  at a given time  $t$  can be computed as<sup>1</sup>:

$$\lambda_{t+1} = \sum_{\tau=0}^t Q^{t-\tau} R U^{\tau} \quad (\text{A.2})$$

where  $R$  is the probability of default at  $t = 1$ .

Step 4 updates  $r$ , using the zero profit condition. If  $\|\lambda^{(i+1)} - \lambda^{(i)}\| < \sqrt{\varepsilon}$  then stop. Otherwise continue iterating on the previous steps. The last step is the standard computation of the ergodic probability distribution.

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<sup>1</sup>For a proof of this result, see Medhi (1994, [42]) pages 116-117.

## Appendix B

# Demand for the differentiated consumption good

The following is the problem that has to be solved in order to find the demand function:

$$\max_{c(z)_t} P_t^c * c_t$$

s.t.

$$\int_0^1 p^c(z)_t c(z)_t dz$$

or what is the same

$$\max_{c(z)_t} P_t^c \left[ \int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

s.t.

$$\int_0^1 p^c(z)_t c(z)_t dz$$

deriving with respect to  $c(z)$

$$P_t^c \left[ \int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{1}{\theta-1}} c(z)_t^{-\frac{1}{\theta}} = p^c(z)_t$$

$$\frac{P_t^c}{p^c(z)_t} \left[ \int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{1}{\theta-1}} = c(z)_t^{\frac{1}{\theta}}$$

$$\left(\frac{P_t^c}{p^c(z)_t}\right)^\theta \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} = c(z)$$

as

$$c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

then

$$c(z)_t = \left(\frac{P_t^c}{p^c(z)_t}\right)^\theta c_t$$

## Appendix C

### Optimal price chosen by retailers

$$\max_{p^c(z)_{t+j}} E_t \sum_{j=0}^{\infty} (1-\varepsilon) \varepsilon^j \Delta_{t+j} c_{t+j} \left[ \left( \frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{1-\theta} - \varphi_{t+j} \left( \frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} \right]$$

In period  $t$  the firm is going to choose a price for the whole horizon of time so  $p^c(z)_{t+j} = p^c(z)_t$  (they choose prices from now on):

$$\max_{p^c(z)_{t+j}} E_t \sum_{j=0}^{\infty} (1-\varepsilon) \varepsilon^j \Delta_{t+j} c_{t+j} \left[ \left( \frac{p^c(z)_t}{P_{t+j}^c} \right)^{1-\theta} - \varphi_{t+j} \left( \frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} \right]$$

deriving with respect to  $p^c(z)_t$

$$E_t \sum_{j=0}^{\infty} (1-\varepsilon) \varepsilon^j \Delta_{t+j} c_{t+j} \left[ \frac{(1-\theta)}{P_{t+j}^c} \left( \frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} - \theta \frac{\varphi_{t+j}}{P_{t+j}^c} \left( \frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta-1} \right] = 0$$

$$E_t \sum_{j=0}^{\infty} (1-\varepsilon) \varepsilon^j \Delta_{t+j} c_{t+j} \theta \frac{\varphi_{t+j}}{P_{t+j}^c} \left( \frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta-1} + E_t \sum_{j=0}^{\infty} (1-\varepsilon) \varepsilon^j \Delta_{t+j} c_{t+j} \frac{(1-\theta)}{P_{t+j}^c} \left( \frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} = 0$$

$$(1-\varepsilon) (p^c(z)_t)^{-\theta-1} \theta E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \frac{\varphi_{t+j}}{(P_{t+j}^c)^{-\theta}} + (1-\varepsilon)(1-\theta) (p^c(z)_t)^{-\theta} E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} (P_{t+j}^c)^{\theta-1}$$

$$(p^c(z)_t)^{-\theta-1} \theta E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \frac{\varphi_{t+j}}{(P_{t+j}^c)^{-\theta}} = (\theta-1) (p^c(z)_t)^{-\theta} E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} (P_{t+j}^c)^{\theta-1}$$

rewriting for  $p^c(z)_t$  to obtain the optimal price

$$p_t^{opt} = \frac{\theta}{\theta - 1} E_t \left[ \frac{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \varphi_{t+j} \left( P_{t+j}^c \right)^\theta}{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left( P_{t+j}^c \right)^{\theta-1}} \right]$$

dividing both sides by  $P_t^c$  and multiplying and dividing by  $\frac{1}{(P_t^c)^\theta}$

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta - 1} E_t \left[ \frac{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \varphi_{t+j} \left( \frac{P_{t+j}^c}{P_t^c} \right)^\theta}{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left( \frac{P_{t+j}^c}{P_t^c} \right)^{\theta-1}} \right] \quad (C.1)$$

From the numerator:  $E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \varphi_{t+j} \left( \frac{P_{t+j}^c}{P_t^c} \right)^\theta$ , so we define

$$\begin{aligned} E_t \Theta_{t+1} &= E_t \left( \Delta_{t+1} c_{t+1} \varphi_{t+1} \left( \frac{P_{t+1}^c}{P_t^c} \right)^\theta \right) + \varepsilon E_t \left( \Delta_{t+1+1} c_{t+1+1} \varphi_{t+1+1} \left( \frac{P_{t+1+1}^c}{P_t^c} \right)^\theta \right) \\ &+ \varepsilon^2 E_t \left( \Delta_{t+1+2} c_{t+1+2} \varphi_{t+1+2} \left( \frac{P_{t+1+2}^c}{P_t^c} \right)^\theta \right) + \dots \text{ and} \\ \Theta_t &= \Delta_t c_t \varphi_t \left( \frac{P_t^c}{P_t^c} \right)^\theta + \varepsilon E_t \left( \Delta_{t+1} c_{t+1} \varphi_{t+1} \left( \frac{P_{t+1}^c}{P_t^c} \right)^\theta \right) + \varepsilon^2 E_t \left( \Delta_{t+2} c_{t+2} \varphi_{t+2} \left( \frac{P_{t+2}^c}{P_t^c} \right)^\theta \right) + \dots \end{aligned}$$

so

$$\begin{aligned} E_t \left( (P_{t+1}^c)^\theta \Theta_{t+1} \right) &= E_t \left( \Delta_{t+1} c_{t+1} \varphi_{t+1} (P_{t+1}^c)^\theta \right) + \varepsilon E_t \left( \Delta_{t+1+1} c_{t+1+1} \varphi_{t+1+1} (P_{t+1+1}^c)^\theta \right) \\ &+ \varepsilon^2 E_t \left( \Delta_{t+1+2} c_{t+1+2} \varphi_{t+1+2} (P_{t+1+2}^c)^\theta \right) + \dots \\ (P_t^c)^\theta \Theta_t &= \Delta_t c_t \varphi_t (P_t^c)^\theta + \varepsilon E_t \left( \Delta_{t+1} c_{t+1} \varphi_{t+1} (P_{t+1}^c)^\theta \right) + \varepsilon^2 E_t \left( \Delta_{t+2} c_{t+2} \varphi_{t+2} (P_{t+2}^c)^\theta \right) + \dots \\ (P_t^c)^\theta \Theta_t &= \Delta_t c_t \varphi_t (P_t^c)^\theta + \varepsilon E_t \left( (P_{t+1}^c)^\theta \Theta_{t+1} \right) \end{aligned}$$

dividing both sides of the equation by  $(P_t^c)^\theta$ :

$$\begin{aligned} \Theta_t &= \Delta_t c_t \varphi_t + \varepsilon E_t \left( \left( \frac{P_{t+1}^c}{P_t^c} \right)^\theta \Theta_{t+1} \right) \\ \Theta_t &= \Delta_t c_t \varphi_t + \varepsilon E_t \left( (1 + \pi_{t+1}^c)^\theta \Theta_{t+1} \right) \end{aligned}$$

In a similar way, from the denominator of C.1  $E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left( \frac{P_{t+j}^c}{P_t^c} \right)^{\theta-1}$  one can obtain:

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left( (1 + \pi_{t+1}^c)^{\theta-1} \Psi_{t+1} \right)$$

## Appendix D

# Figures for Sensitivity Analysis of Impulse Responses

Figure D.1: Productivity shock when prices are more flexible

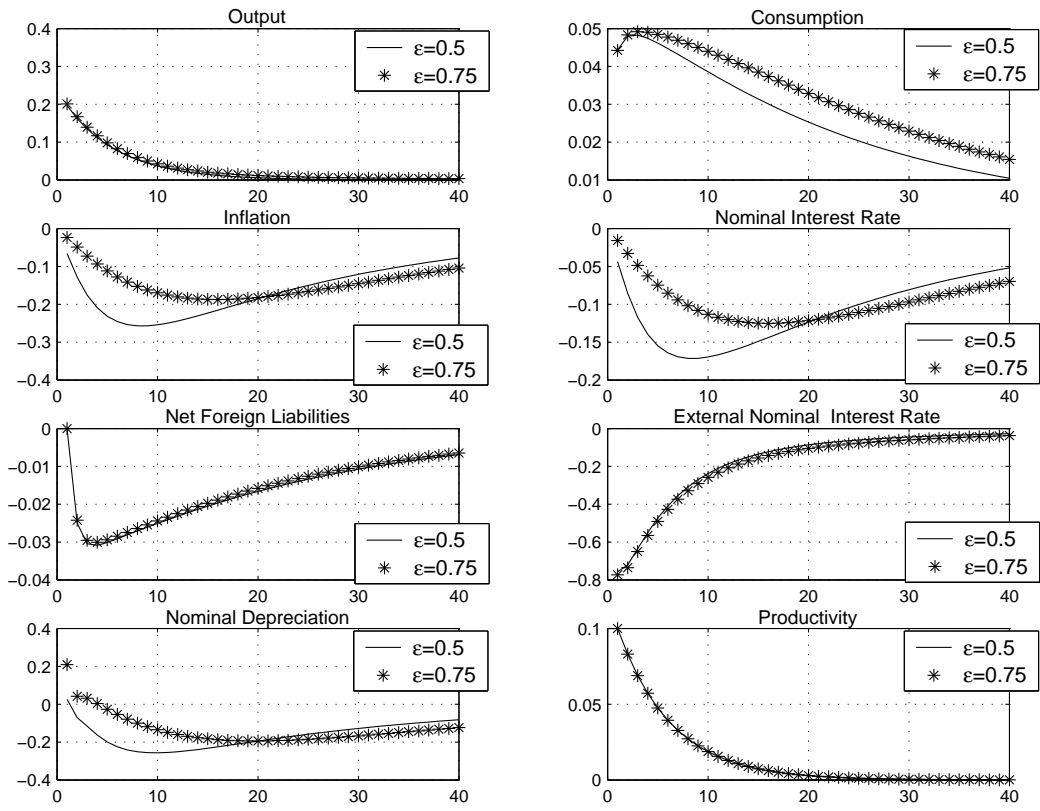


Figure D.2: Preferences shock when prices are more flexible

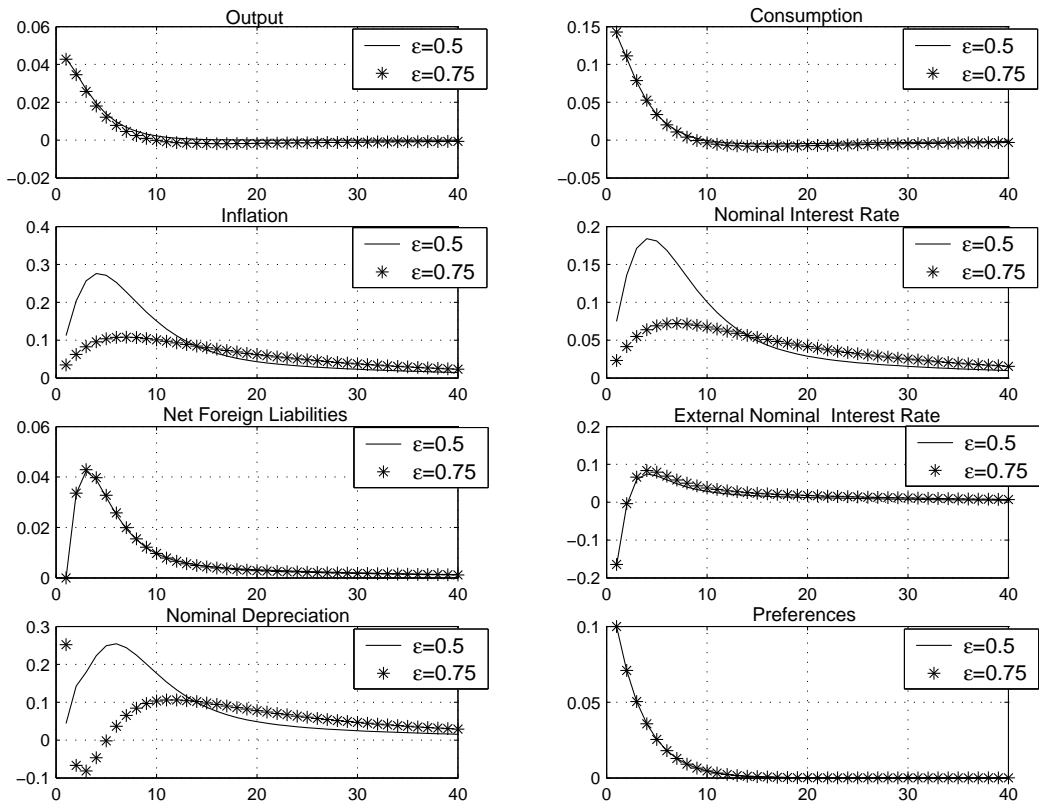




Figure D.3: Government Expenditures shock when prices are more flexible

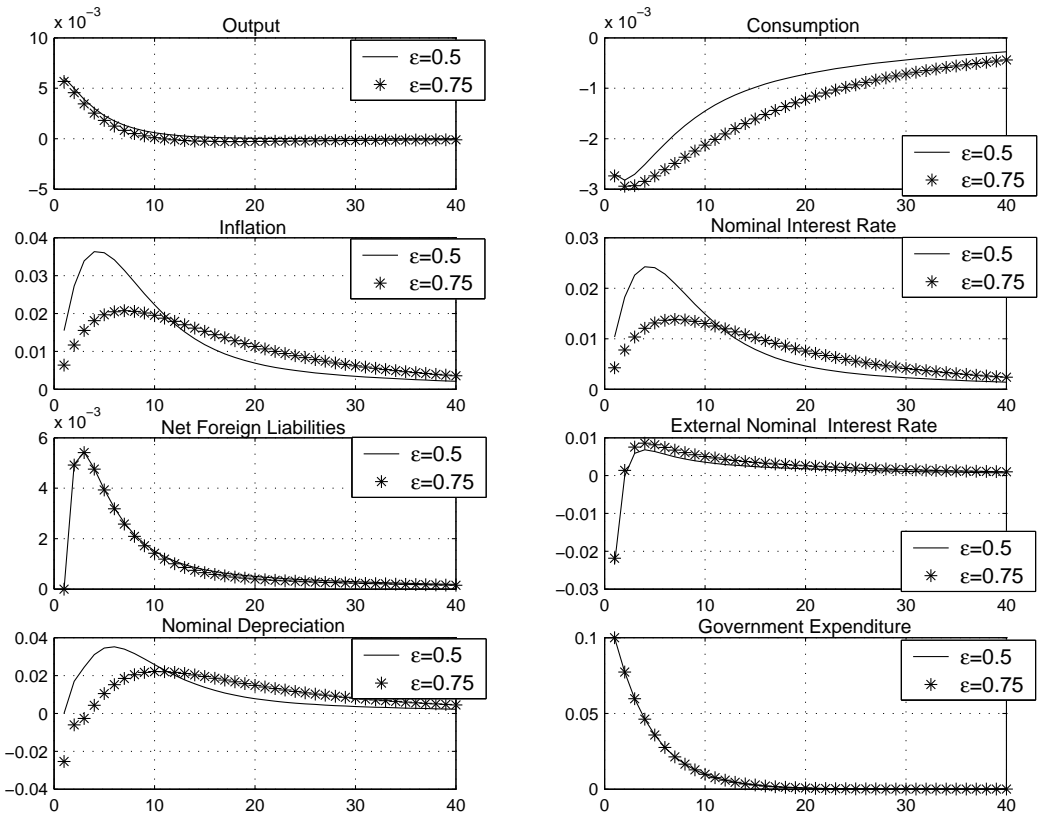


Figure D.4: Productivity shock when the risk premium is more elastic

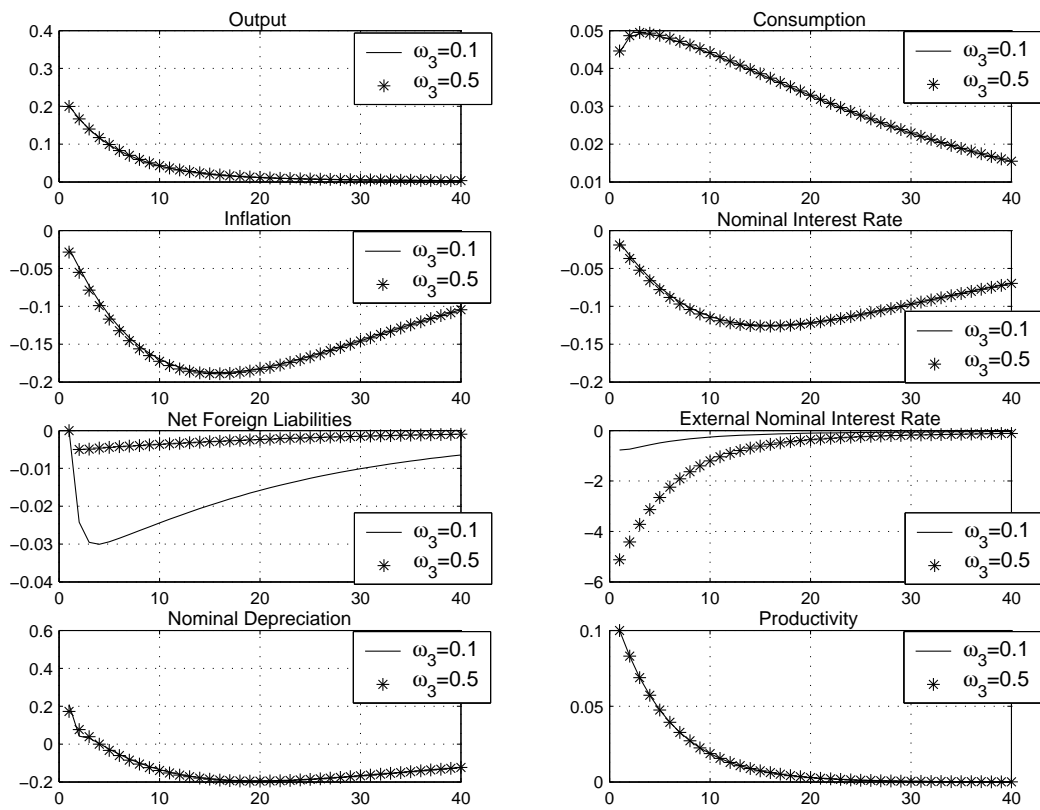


Figure D.5: Preferences shock when the risk premium is more elastic

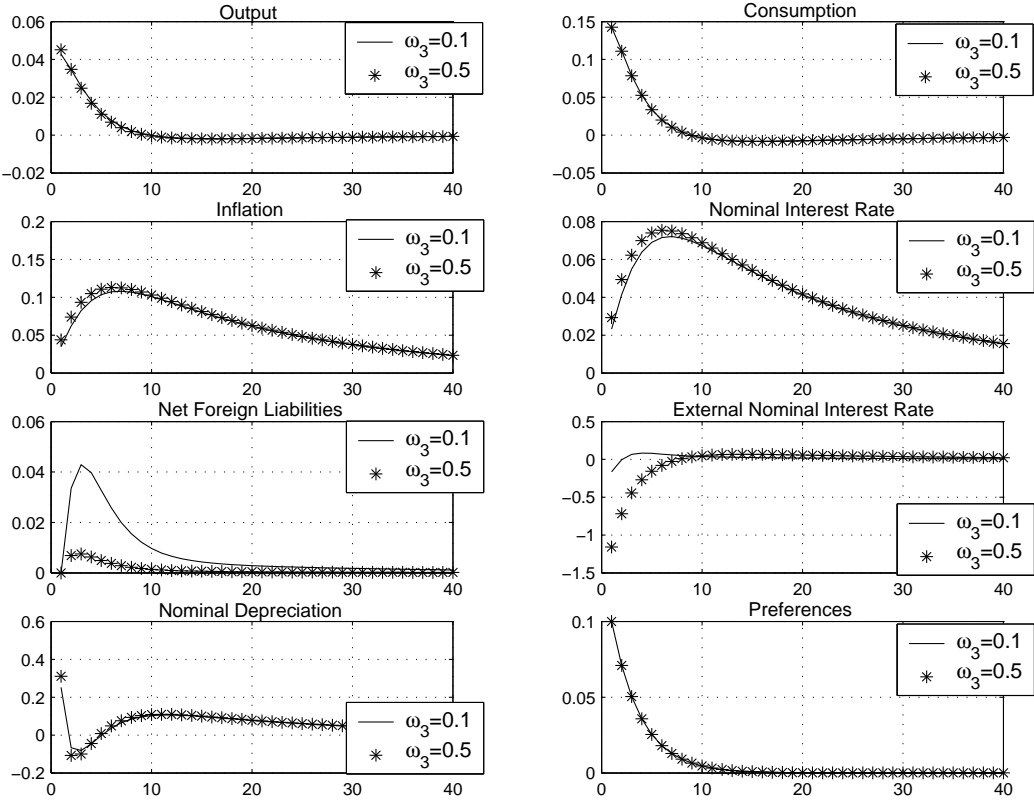


Figure D.6: Government Expenditures shock when the risk premium is more elastic

