DAMAGE MODELLING FOR PREDICTION OF PLASTIC OR CREEP FATIGUE FAILURE IN STRUCTURES

J. LEMAITRE

Laboratoire de Mécanique et Technologie,
Université P. et M. Curie (Paris VI), 61, av. du Président-Wilson, F-94230 Cachan, France

SUMMARY

Mechanical theories of damage are developped from thermodynamics concepts by means of KACHANOV's effective stress. For isotropic three-dimensional damage, an equivalent scalar stress is introduced as a linear combination of mean stress and octahedral shear stress. Then, constitutive equations are written for plastic, creep and fatigue damage phenomena. Test and methods necessary to identify these three models, for each material, are described, damage measurements are taken from damage coupling with elastic behavior (or plastic, or viscoplastic). Some indications, on methods to solve damage equations to predict local failure, are given.

I The problem of local failure prediction on Structures.

Local failure of a metallic structure is here defined as initiation of a macro-crack, its main size being 1 mm of order of magnitude. The prediction of loading conditions at which such a crack appears (loading amplitude, time or number of cycles of loading) consists schematically in two steps :

- In a stress-strain analysis of the structure, the three-dimensional stress and strain fields histories are calculated for a given history of external loading. The material properties are represented by stress-strain constitutive equations : elasticity, plasticity or visco-plasticity. From the local failure point of view, this stress and strain state has to be known only for the few zones where the combination of stress or strain, temperature and time gives the most severe conditions in regard to rupture. For cyclic external loading the stress-strain state must be known at stabilization.
- In the failure analysis, calculation deals only with a small part of the structure : one volume element. The problem is to solve damage failure constitutive equations, differential in most cases. Stress, strain and temperature histories are the inputs ; time or number of cycles or stress to failure are the outputs.

This separation in two successive problems is built on uncoupling hypothesis between damage and stress-strain behavior : stress and strain states calculated in the structure analysis do not depend on damage evolution. This assumption is physically wrong, the coupling, small when damage is small, can be very important near the rupture conditions, but this is the only way, actually, to perform prediction with moderate computer time and price. It is difficult to estimate the order of magnitude of such errors which concern essentially the redistribution of stress due to damage. We may say that it is an open problem !

2. Mechanical theories of damage

From the physical point of view, damage is considered as initiation and development of micro-voids and micro-cracks in the crystal lattice to give rise to a macro-crack. From the mechanical point of view, ignoring the mode shape of micro-cracks and micro-voids growth, damage is considered as a state variable which affects the strength of the material : strength to strain and to failure in the sense of an effective stress introduced by KACHANOV and RABOTNOV [1] [2] and generalized by LECKIE-HAYURST [3] , HULT [4], CHABOCHE-LEMAITRE [5] and others.

2.1 Effective stress

This fundamental hypothesis of effective stress is the following :

It exists an "operator damage" $\mathbb{\Delta}$ applied to stress tensor σ which defines an effective stress $\tilde{\sigma}$:

$$\tilde{\sigma} = \mathbb{\Delta} . \sigma$$

to be introduced in constitutive equations instead of current stress σ, to describe elastic, plastic or visco-plastic strain behavior of damaged material. In the following, developments are restricted to isotropic damage. Any damaging loading gives the same amount of damaging effect in all directions. In other words, all the stress components are modified by the same value to define the effective stress components : $\mathbb{\Delta}$ is reduced to a scalar Δ classically written as $\Delta = \frac{1}{1-D}$

$$\boxed{\tilde{\sigma} = \frac{\sigma}{1-D}}$$

D is the scalar measure of isotropic damage ;
D = 0 corresponds to virgin material ;
D = 1 corresponds to the maximum value that can be taken at failure. $[1 - D]$ is thus interpreted as a relative area which resists effectively to the stress, taking into account the micro-voids, the micro-cracks and the corresponding field of stress concentration factors. This last point suggests the weakness of the effective stress concept : it is known that stress concentration factors fields of defected materials are different in elasticity plasticity or visco-plasticity. Unfortunately they are not well established for real defects in metals and, here again, simplicity suggests to take a unique variable to define damage.

2.2 Thermodynamics

Thermodynamics of irreversible processes is used to define variables and to precise the general form of constitutive equations [6]. In order to describe phenomena of elasticity, plasticity, or visco-plasticity and coupled damage, the following set of variables (observable, internal, associated) is introduced [7] [8]:

Total strain tensor $\mathcal{E}$

Plastic strain tensor $\mathcal{E}^p = \mathcal{E} - \mathcal{E}^e$

VARIABLES	Obs.	Int.	Ass.	VARIABLES
Elastic strain	$\mathcal{E}^e$	$\longrightarrow$	σ	Stress
Temperature	T	$\longrightarrow$	s	Entropy
Temperature gradient	$\overrightarrow{\mathrm{Grad}}\,T$	$\longrightarrow$	$\vec{q}$	Heat flux
Strain hardening		$\alpha_p \rightarrow$	A_p	Dual stresses
Damage		$D \rightarrow$	Y	Damage energy rate.

All the associated variables derive from the free energy potential. $\rho\Psi$ with respect to observable variables ; in particular :

$$Y = -\rho \frac{\partial \Psi}{\partial D}$$

Within hypothesis of uncoupling between plastic part of free energy and elastic-damage part, Y is also defined as one half the variation of elastic strain energy W_e corresponding to a variation of damage under constant stress and temperature :

$$Y = \frac{1}{2} \left.\frac{dW_e}{dD}\right)_{\sigma, T}$$

Y has a similar meaning as strain energy release rate **G** in fracture mechanics, which suggests the damage rupture criterion :

$Y = Y_c$ (characteristics of material) $\Longrightarrow$ rupture.

Existence of a pseudopotential of dissipation φ [9] extended to fracture enables us to write complementary damage evolution laws :

$$Y = \frac{\partial \varphi}{\partial \dot{D}}$$

φ is a convex function of all observable and internal variables, and their first time derivative but it cannot be identified directly from tests results. It justifies only the existence of a constitutive equation like :

$$\boxed{\dot{D} = f(\mathcal{E}^e, T, \alpha_p, D, \dot{\mathcal{E}}^e, \dot{\alpha}_p, Y)}$$

A large amount of empirical work has been done to write mathematical expressions of damage evolution under the one dimensional case, each of them having its own range of applicability [10]. The generalization of these models to the three-dimensional case is still an open problem mainly due to the lack of basic experimental results in order to choose between several hypothesis.

2.3. Equivalent reduced stress.

The simplest solution seems to define a scalar equivalent stress, in the sense of LECKIE and HAYHURST [3] [11] for creep rupture or SINES [12] and CROSSLAND [13] for fatigue rupture.

Within hypothesis of isotropy, damage is assumed to be the result of two main mechanisms : growth of voids due to hydrostatic stress, when positive, and growth of cracks due to octahedral shear stress [14]. The equivalent stress σ^* is a function of these two invariants :

$\sigma_m = \frac{1}{3}\sigma_{ii}$ with the usual convention summation.

$\bar{\sigma} = \sqrt{\frac{3}{2} s_{ij} s_{ij}}$ with $s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$

A linear combination is assumed and choosen to give $\sigma^* = \sigma$ in tension-compression.

$$\boxed{\sigma^* = \langle 3C\,\sigma_m + [1-C]\,\bar{\sigma} \rangle}$$

C is a void-crack sensibility coefficient depending of the material. This expression which depends of only one coefficient is a particular case of equivalent stresses defined in reference [7..10], Its value being positive or zero: $\langle x \rangle = 0$ if $x < 0$

Regarding influence of Temperature an interesting concept can be used, this is the temperature reduced stress ratio $\frac{\sigma}{\sigma_u(T)}$ (σ_u being the ultimate rupture stress in tension, function of temperature) introduced by J.L. CHABOCHE [6] and CHRZANOWSKI [15] to fit fatigue rupture test results at different temperatures.

Extending this concept to the three-dimensional case and for any kind of isothermal process of damage, temperature effect is represented by

$$\frac{\sigma^*}{\sigma_u(T)}$$

Then, in the general damage constitutive equation, if elastic strain $\mathcal{E}^e$ is replaced by the stress σ by means of linear elasticity relation :

$$\dot{D} = f(\sigma^*, T, \alpha_p, D, \dot{\sigma}^*, \dot{\alpha}_p),$$

if effective equivalent reduced stress is introduced, and influence of strain hardening variables α_p are neglected, damage constitutive equation is reduced to :

$$\boxed{\dot{D} = f\left(\frac{\sigma^*}{[1-D]\,\sigma_u(T)}, D, \dot{\sigma}^*\right)}$$

2.4 Constitutive equations.

It is well admitted now to distinguish between three phenomena : plastic damage due to large plastic strain, fatigue damage due to accumulation of small irreversibilities in each cycle and creep damage due to time exposure to stress.

Plastic damage

One of the simplest model is to consider that damage rate depends only upon effective equivalent reduced stress, $\dot{\sigma}^*$ acting as a multiplicator (as in plasticity) :

$$\dot{D} = f_p\left(\frac{\sigma^*}{[1-D]\,\sigma_u}\right).\dot{\sigma}^*$$

$\dot{D}$ is positive to satisfy the second law of thermodynamics [6] (and its physical meaning!) and occurs under loading only. If f_p is particularized by the one-dimensional model established in ref [16]

$$\boxed{\begin{array}{l} dD = \left[\dfrac{\langle \sigma^* - \sigma_D \rangle}{[\sigma_u - \sigma_D][1-D]}\right]^{\mathcal{A}} \dfrac{d\sigma^*}{\sigma_u - \sigma_D} \quad \text{if } \begin{cases} \sigma^* = \mathrm{Sup}(\sigma^*_{(t)}) \\ \text{and} \\ d\sigma^* > 0 \end{cases} \\ dD = 0 \qquad \text{if } \sigma^* < \mathrm{Sup}(\sigma^*_{(t)}) \text{ or } d\sigma^* < 0 \end{array}}$$

σ_D is a damage threshold, material dependent together with the coefficient $\mathcal{A}$ and the function $\sigma_u(T)$.

Creep damage .

Creep damage is a time dependent effect, its rate depends upon actual stress :

$$\dot{D} = f_c\left(\frac{\sigma^*}{[1-D]\,\sigma_u(T)}, D\right)$$

The generalization of the one dimensional KACHANOV model [1] is simply :

$$\boxed{\dot{D} = \left[\frac{\sigma^*}{a_o\,[1-D]\,\sigma_u}\right]^{r_o}}$$

or, in order to introduce possibility of non linear cumulation [17]:

$$\dot{D} = \left[\frac{\sigma^*}{a\,[1-D]\,\sigma_u}\right]^{r}[1-D]^{r-k(\sigma^*)}$$

a_o, r_o, $\sigma_u(T)$, (or a , r , $k(\sigma^*)$, $\sigma_u(T)$) are material dependent. Maximum principal stress criterion [18] is not contained in these models but $C = 0.5$ gives almost the same numerical values.

Under one-dimensional creep strain controlled cyclic conditions, it is possible to express stress as a function of plastic strain rate and to integrate KACHANOV's damage rate equation over one period to obtain cyclic damage amount $\frac{\delta D}{\delta N}$ as a function of total strain range $\Delta\varepsilon$ [19]:

$$\boxed{\frac{\delta D}{\delta N} = \frac{[1-D]^{-q}}{[q+1]\; N_c(\Delta\varepsilon)}}$$

$N_c(\Delta\varepsilon)$ is the function giving the number of cycles to failure of the material at very law frequency.

q is a material dependent coefficient.

Fatigue damage.

Pure fatigue is defined as a cyclic damaging process occuring when plastic strain is zero or small in each cycle (high frequency). It is possible to start with the same type of equation as for plastic damage.

$$\dot{D} = f_F\left(\frac{\sigma^*}{[1-D]\,\sigma_u}\right)\dot{\sigma}^*$$

and to integrate over one cycle, defined by maximum value σ^*_{MAX} and minimum value σ^*_{min} of the stress $\sigma^*(t)$ [20]. Note that σ^*_{min} is positive or zero due to the definition of σ^*.

$$\frac{\delta D}{\delta N} = 2\int_{\sigma^*_{min}}^{\sigma^*_{MAX}} f_F\left(\frac{\sigma^*}{[1-D]\,\sigma_u}\right)d\sigma^*$$

WOEHLER-MINER's model corresponds to a power function for f_F :

$$\frac{\delta D}{\delta N} = \begin{cases} \dfrac{B}{\sigma_u}\left[\sigma^{*\,b}_{MAX} - \sigma^{*\,b}_{min}\right] & \text{if } \sigma^*_{MAX} > \sigma_f \\ 0 & \text{if } \sigma^*_{MAX} \leqslant \sigma_f \end{cases}$$

σ_f is the fatigue limit in tension, B and b are constants, characteristic of each material

CHABOCHE's model [21] taking into account non linear cumulation corresponds to :

$$\boxed{\frac{\delta D}{\delta N} = \left[1-[1-D]^{\beta+1}\right]^{\alpha\left(\frac{\sigma^*_{MAX}}{\sigma_u}, \frac{\bar{\sigma}^*}{\sigma_u}\right)} \left[\frac{\sigma^*_{MAX} - \sigma^*_{min}}{M\left(\frac{\bar{\sigma}^*}{\sigma_u}\right)\sigma_u\,[1-D]}\right]^{\beta}}$$

where β , α , M , σ_u are material dependent, $\bar{\sigma}^*$ is the mean equivalent stress : $\bar{\sigma}^* = \frac{\sigma^*_{MAX} + \sigma^*_{min}}{2}$

For one-dimensional fatigue strain controlled cyclic conditions, it is also possible to express the stress as a function of strain in order to write damage relation as a function of strain range $\Delta\varepsilon$ in a similar manner as for creep damage [19]

$$\boxed{\frac{\delta D}{\delta N} = \frac{[1-D]^{-p}}{[p+1]\; N_F(\Delta\varepsilon)}}$$

p is a material dependent coefficient, $N_F(\Delta\varepsilon)$ is the function giving the number of cycles to failure under pure fatigue conditions (high frequency) at the considered temperature.

3. Tests and numerical identification .

The main hypothesis of isotropic damage enables us to determine the values of all material coefficients needed in the models from one dimensional tests, except coefficient C in the equivalent stress relation. Let us see what is the minimum number of tests to perform in order to know all these numerical informations.

3.1. Identification of equivalent stress relation

$$\sigma^* = 3C\sigma_m + [1-C]\,\bar{\sigma}$$

The determination of coefficient C for a particular material is one of the most difficult problems in damage models identification. Micrography observations of damaged material near failure condition can show if damage defects are :

- mainly voids $\longrightarrow C \simeq 1$
- mainly microcracks $\longrightarrow C \simeq 0$

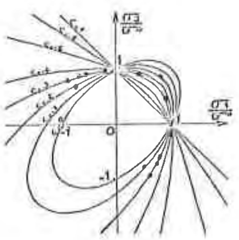
If both are present, there is no serious criterion to chose (take $\frac{1}{4} < C < \frac{1}{2}$!)

- Multi-dimensional fracture tests are difficult to perform. Nevertheless if some results can be obtained, under bi-axial stresses for example (σ_1, σ_2), they can be compared to the set of curves of figure 1 which represents $\frac{\sigma_2}{\sigma^*}$ as a function of $\frac{\sigma_1}{\sigma^*}$ for different values of C .

FIGURE 1
Biaxial equivalent stress.

$$\sigma^* = c\,[\sigma_1 + \sigma_2] + [1-c]\,[\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2]^{\frac{1}{2}}$$

Experimental points from Ni Cr Alloy, Moly and low Al. Steels.
Ref [11]

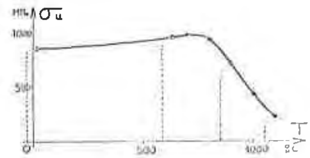


For plastic rupture of a given material or for one value of time to creep rupture (isochronous curves) or one number of cycles to rupture in fatigue, value of C to be taken is the one for which the corresponding curve fits experimental datas. On this figure, experimental isochronous points taken from reference [11] show that $C = 0.25$ is a good value for these three materials (at least for creep damage fracture).

3.2 Ultimate stress function of temperature.

The function $\sigma_u(T)$ defining reduced stress ratio may be obtained from the classical strain hardening tension test at strain rate $\dot{\varepsilon} \simeq 10^{-3}\, s^{-1}$. Figure 2 shows an example :

FIGURE 2
Tensile ultimate stress IN 100 refractory alloy.



3.3 Quantitative estimation of damage.

It is possible to identify the models either by indirect methods using complex history of loading [21] or direct methods by measure of damage or both. Physical methods of damage measure (counting of cracks or voids area, variation of density [22], resistivity) correspond to different definitions of damage. To be in accordance with thermo-mechanical definition developped here, damage must be estimated by its coupling with strain behavior, through the effective stress concept :

- Elasticity : the one-dimensional constitutive equation for linear elasticity of damaged material is :

$$\tilde{\sigma} = E\varepsilon^e \quad \text{or} \quad \frac{\sigma}{1-D} = E\varepsilon^e$$

which means that, knowing YOUNG modulus E, damage may be obtained by measurement of the elastic stiffness $\frac{\sigma}{\varepsilon^e}$ from any one-dimensional specimen :

$$D = 1 - \frac{1}{E}\,\frac{\sigma}{\varepsilon^e}$$

Figures 3-4-5 show examples of damage evolution during a strain hardening tensile test (plastic damage), a cyclic creep test (creep damage) and during a fatigue test (fatigue damage).

FIGURE 3
Plastic damage evolution during a tensile test copper T = 20°C

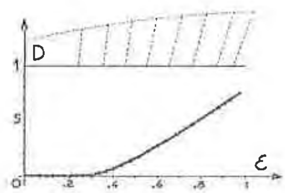


FIGURE 4
Creep damage evolution during a cyclic tension-compression creep test
ASI 316 T=550°C
$\varepsilon = \pm\ 10^{-2}$

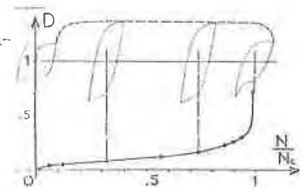
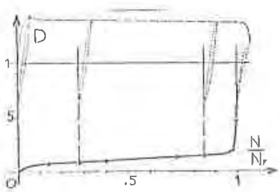


FIGURE 5
Fatigue damage evolution during a tension-compression fatigue test
ASI 316 T= 20°C
$\varepsilon = {+\ 0,7 \atop +\ 0}\ 10^{-2}$



- Plasticity : Fatigue damage evolution during uniaxial fatigue test is also measured by its coupling with cyclic plasticity behavior (when plastic damage is negligible)[5]. Assuming constitutive equation between plastic strain and stress amplitudes ($\Delta\varepsilon_p, \Delta\sigma$), after stabilized conditions, to be :

$$\Delta\varepsilon^P = \left[\frac{\Delta\tilde{\sigma}}{K_c}\right]^{M_c} \quad \text{or} \quad \Delta\varepsilon^P = \left[\frac{\Delta\sigma}{K_c[1-D]}\right]^{M_c}$$

Damage derives from stress controlled fatigue test by :

$$D = 1 - \left[\frac{\Delta\varepsilon^P_o}{\Delta\varepsilon^P}\right]^{\frac{1}{M_c}} \quad \left(\Delta\varepsilon^P_o = \Delta\varepsilon^P \text{ at stabilized conditions}\right)$$

or from strain controlled fatigue test by :

$$D = 1 - \frac{\Delta\sigma}{\Delta\sigma_o} \quad \left(\Delta\sigma_o = \Delta\sigma \text{ at stabilized conditions}\right)$$

- Visco-Plasticity : Based on similar methods, creep damage evolution, during monotonic or cyclic creep tests, is estimated by its coupling with visco-plastic behavior[5]. Assuming NORTON's relation, written through effective stress to be valid for secondary and tertiary creep :

$$\dot{\varepsilon}^P = \left[\frac{\tilde{\sigma}}{K_s}\right]^N = \left[\frac{\sigma}{K_s\,[1-D]}\right]^N$$

If $\dot{\varepsilon}^P_o$ is the secondary plastic strain rate ($D \simeq 0$) it is easy to show :

$$D = 1 - \left[\frac{\dot{\varepsilon}^P_o}{\dot{\varepsilon}^P}\right]^{\frac{1}{N}}$$

For strain controlled cyclic creep test, integration of NORTON's relation with some hypothesis gives the same relation as for fatigue damage coupled with plasticity :

$$D = 1 - \frac{\Delta\sigma}{\Delta\sigma_o}$$

3.4 Critical value of damage at rupture

Rupture criterion has been defined from thermodynamics by a critical value of damage energy rate $Y = Y_c$. It is more convenient to define a rupture criterion as a critical value of damage D_c : Using $Y = \frac{1}{2}\frac{dW_e}{dD}$ for a tensile test for which failure conditions are $Y = Y_c$, $\sigma = \sigma_u$, $D = D_c$

$$Y = \frac{1}{2}\,\frac{\sigma^2}{E\,[1-D]^2} \longrightarrow D_c = 1 - \frac{\sigma_u}{\sqrt{2EY_c}}$$

Usual values of σ_u and Y_c for metals gives

$$0.5 < D_c < 0.8$$

Non linearities in damage evolutions allow to take $D_c = 1$ in most practical cases

3.5 Calculation of material dependent coefficients

The three-dimensional equivalent stress expression is choosen to be equal to the one-dimensional stress σ of tension experiments

$$\sigma^* = \left\langle 3C\,\frac{\sigma}{3} + [1-C]\,\sigma \right\rangle = \langle\sigma\rangle$$

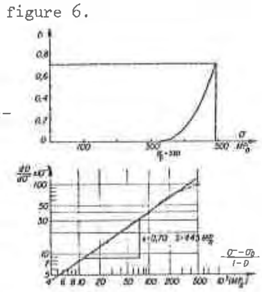
Identification of three-dimensional models is then completely identical to identification of isothermal tension-compression case. The mathematical forms choosen for the models, are mainly power-functions which means that a log-plot of damage measurements, as shown before, is the main work to be done to calculate the exponent as the slope of the best straight line fitting experimental results and the coefficient as the scale factor.

- Plastic damage :

$$dD = \left[\frac{\langle\sigma^* - \sigma_D\rangle}{[\sigma_u - \sigma_D]\,[1-D]}\right]^{s} \frac{d\sigma^*}{\sigma_u - \sigma_D}$$

- σ_D is estimated as a threshold
- s and $[\sigma_u - \sigma_D] = S$ are calculated from damage evolution as explained above. Identification of example of figure 4 is shown on figure 6.

FIGURE 6
Plastic damage characteristics
Pur Copper T = 20°C
Ref [16]



- Creep damage : Damage evolution during monotonic creep tests enables us to identify KACHANOV-RABOTNOV model or non-linear cumulation creep damage model:

Calculation of z_0 or $k(\sigma)$:

- Only one stress independent curve fits KACHANOV-RABOTNOV model :

$$\dot{D} = \left[\frac{\sigma^*}{a_0\,\sigma_u\,[1-D]}\right]^{z_0} \longrightarrow \frac{t}{t_c} = 1-\left[1-\frac{t}{t_c}\right]^{\frac{1}{z_0+1}}$$

where t_c is the time to rupture :

- Several stress dependent curves fit the second model :

$$\dot{D} = \left[\frac{\sigma^*}{a\,\sigma_u\,[1-D]}\right]^{z}[1-D]^{z-k(\sigma)} \longrightarrow \frac{t}{t_c} = 1-\left[1-\frac{t}{t_c}\right]^{\frac{1}{k(\sigma)+1}}$$

Then, $a_0.\sigma_u$ or $a.\sigma_u$ and z are determined for one temperature by times to rupture (isochronous curves)

$$t_c = \frac{1}{z_0+1}\left[\frac{\sigma}{a_0\,\sigma_u}\right]^{-z_0} \quad or \quad t_c = \frac{1}{k(\sigma)+1}\left[\frac{\sigma}{a\,\sigma_u}\right]^{-z}$$

Examples are given in reference [3].

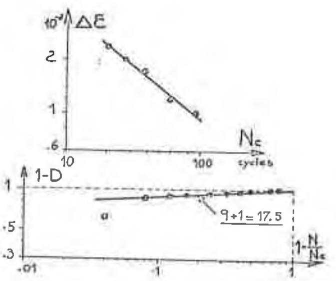
Identification of the one-dimensional cyclic strain controlled creep damage model requires a classical $N_c(\Delta\varepsilon)$ curve at very low frequency ($<.01$ Hz) and a curve $D(\frac{N}{N_c})$ such as the one of figure 4 to determine q by :

$$\frac{\delta D}{\delta N} = \frac{[1-D]^{-q}}{[q+1]\,N_c(\Delta\varepsilon)} \longrightarrow D = 1-\left[1-\frac{N}{N_c}\right]^{\frac{1}{q+1}}$$

An example is given on figure 7 :

FIGURE 7

Cyclic creep damage characteristics
OFHC Copper
T = 540°C
Ref [19]



- Fatigue damage :To identify CHABOCHE's model, the following informations are sufficient : - the simplest expression of function α taking into account fatigue limit as well as high stress levels can be choosen as [6]:

$$\alpha = 1-\gamma\,\frac{\sigma^*_{MAX}-\sigma_f}{\sigma_u-\sigma^*_{MAX}}$$

where σ_f is the fatigue limit (under alternating conditions) σ_u the tensile rupture stress. Under this hypothesis, the exponent β can be identified with the WOEHLER curves through integration of the model, giving rise to ($\bar{\sigma} = 0$) :

$$\left[\frac{\sigma_{MAX\,1}}{\sigma_{MAX\,2}}\right]^{\beta} = \frac{N_{F2}}{N_{F1}}\;\frac{\sigma_{MAX\,2}-\sigma_f}{\sigma_{MAX\,1}-\sigma_f}\;\frac{\sigma_u-\sigma_{MAX\,1}}{\sigma_u-\sigma_{MAX\,2}}.$$

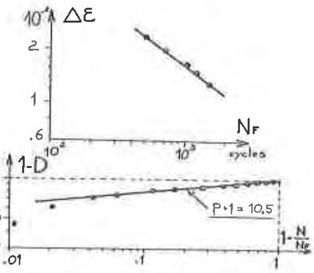
N_{F1} and N_{F2} being the measured numbers of cycles to failure under alternating stresses $\sigma_{MAX\,1}$, $\sigma_{MAX\,2}$.

Coefficient M and it dependence on mean stress is obtained from classical fatigue test results. The only coefficient, which needs damage measurement, is γ. Refs [6] and [21] gives the details of the identification procedures. Examples are described in Ref [5]. As for creep, the one-dimensional cyclic strain controlled fatigue damage model requires a classical $N_F(\Delta\varepsilon)$ curve at high frequency (>5 Hz) and a curve $D(\frac{N}{N_F})$ such as the one of figure 5 to determine p :

An example is given on figure 8 :

FIGURE 8

Fatigue damage characteristics
OFHC Copper
T = 540°C
Ref [19]



4. Methods to solve damage equations to predict Local failure.

Assume that a structure calculation corresponding to an history of loading gives stress tensor $\sigma_{ij}(t)$ and strain tensor $\varepsilon_{ij}(t)$ in a point where local failure conditions are to be checked. History of temperature is also given $T(t)$. The most general case to be solved is when the three kinds of damage occur simultaneously. Only one damage variable has been introduced by thermodynamics, this means that damage increment dD is function of $d\sigma^*$ for plastic damage, dt for creep damage and $d\sigma^*$ (or δN) for fatigue damage, that is the summation of the three increments :

$$dD = f_p\,d\sigma^* + f_c\,dt + f_F\,d\sigma^*$$

This equation, together with the models described, contains :

- plastic damage resulting of metal forming
- plastic-fatigue damage interaction
- non linear creep-fatigue interaction.

4.1 Initial damage conditions.

In general, plastic damage evolution in structures in service conditions is zero or negligible because strains are limited at small values for geometrical reasons. It is not the case during the forming process such as deep drawing or drop hammer forming. This damage has to be taken into account as an initial condition D_o for the structure computation. If maximum values of σ_m and $\bar{\sigma}$ (giving σ^*) during the forming process are known :

$$D_o = 1 - \left[1 - \left[\frac{\sigma^* - \sigma_D}{\sigma_u - \sigma_D}\right]^{s+1}\right]^{\frac{1}{s+1}}$$

If a thermal treatment occurs after forming it is necessary to estimate D_o by a test : a measure of damaged elasticity modulus for example.

4.2 Creep-fatigue failure prediction.

$\sigma^*(t)$ is easy to calculate from $\sigma_{ij}(t)$ with :

$$\sigma_m = \frac{\sigma_{ii}}{3}, \quad s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}, \quad \bar{\sigma} = \sqrt{\tfrac{3}{2} s_{ij} s_{ij}}$$

$$\sigma^* = \langle 3c\,\sigma_m + [1-c]\,\bar{\sigma} \rangle$$

$T(t)$ given introduced in $\sigma_u(T)$ gives $\sigma_u(t)$. Then, damage is governed by differential equation in time, for example ;

$$dD = \left[\frac{\sigma^*}{A[1-D]}\right]^{r} [1-D]^{r-k}\, dt + \left[1-[1-D]^{\beta+1}\right]^{\alpha} \left[\frac{\sigma^*_{max} - \sigma^*_{min}}{M\sigma_u [1-D]}\right]^{\beta} \delta N$$

Creep damage relation is solved step by step in time from a first term TAYLOR expansion by numerical procedure :

$$D_{i+1} = D_i + \dot{D}\,[t_{i+1} - t_i] \quad , \quad D_{(t=0)} = D_o$$

For cyclic loading the cheapest way is to integrate creep damage relation over one cycle of period Δt, assuming D constant during that cycle, and to use differential equation in number of cycles.

$$\frac{\delta D}{\delta N} = [1-D]^{-k} \int_0^{\Delta t} \left[\frac{\sigma}{a}\right]^{r} dt + \left[1-[1-D]^{\beta+1}\right]^{\alpha} \left[\frac{\sigma^*_{max} - \sigma^*_{min}}{M\sigma_u [1-D]}\right]^{\beta}$$

This equation is also solved step by step in number of cycles

$$D_{j+1} = D_j + \frac{\delta D}{\delta N} \quad , \quad D_{(N=0)} = D_o$$

Reference [23] gives indications to save computer time by an "optimal choice" of steps. Time or number of cycles to rupture is the one for which:

$$t_F = t\,(D = D_c)$$

or

$$N_F = N\,(D = D_c)$$

If the loading can be considered as cyclic and uniaxial, simpler relation is used [19]

$$\frac{\delta D}{\delta N} = \frac{[1-D]^{-q}}{[q+1]\,N_c(\Delta\varepsilon)} + \frac{[1-D]^{-p}}{[q+1]\,N_F(\Delta\varepsilon)}$$

with the same numerical procedure. In some cases over load effect leads plastic damage, if so, it is necessary to work with the three simultaneous relations : plastic, creep and fatigue damages

References [24] [25] contain results of application of these methods or similar from which we may deduce that accuracy of prediction deals within a factor 2, at most, on life time of structures.

REFERENCES

1 KACHANOV L.M. Izv Akad Nauk SSE. Otd. Tekh Nauk Nr 8, 1958, 26-31.

2 RABOTNOV Y.N Proc. XII Int.Cong.App.Mech. 1968.

3 LECKIE F.A-HAYHURST D.R Proc. Roy. Sty London Vol 340 N° 1622 1974, p. 323-347.

4 HULT. J.-BROBERG.H. Proc. II Bulg. Nat. Cong. Theo. App. Mech. VARNA 1973;

5 LEMAITRE J.-CHABOCHE J.L. Jour. Mec. Appl. Ed. GAUTHIER-VILLARS. Vol. 2 N°3 1978.

6 CHABOCHE JL. Thèse d'Etat. Juin 1978. Université P. et M. CURIE (Paris VI).

7 KRATOCHVIL J. Publ. Acad. Sc.Pologne P.PERZYNA 1974.

8 CORDEBOIS JP.-SIDOROFF F.Modèle d'endommagement tridimensionnel (to be published 1979).

9 GERMAIN P. Cours Mec. Mil. Cont. MASSON Paris 1973

10 O'NEIL MJ.Report. ARL/SM 326, ADSS MELBOURNE 1970.

11 HAYHURST DR. J.Mec.Phy. Sol.Vol 20 N°6

12 SINES G. NACA T.N.3495 1955.

13 CROSSLAND B.Int. Conf. Fat.Metl. LONDON 1956 P138.

14 FRANCOIS D. Ecol.Eté Fat.Mat. SHERBROOKE Québec 1978.

15 CHRZANOWSKI M.Int.J. Mec. Sc.Vol18 1976 p;69-73.

16 LEMAITRE J.-DUFAILLY J 3è Cong.F. Mec.Grenoble 1977

17 LEMAITRE J-CHABOCHE JL. IUTAM GOTEBORG.Suède 1974

18 MURAKAMI S. OHNO N. Eurom. Colloq. 111.Marienbad Tchécoslovaquie 1978.

19 LEMAITRE J.-PLUMTREE A. ASME/CSME Press. Vess. Pip. Conf. MONTREAL 1978.

20 LEMAITRE J. Sém. Mat. Struct.sous Charg.Cycl. Ecole Polytechnique 1978.

21 CHABOCHE JL. Rev. Fr. Meca.N°50-51, 1974 TP. ONERA. N° 1975-53.

22 BELLONI G.-BERNASCONI G. PIATTI G. (to be published in MECCANICA, 1977.)

23 SAVALLE S.-CULIE JP. Rech. Aéro. 1978-148.

24 CHABOCHE JL-STOLTZ C. Revue Franc. Meca. N° 52 1974, p. 37-47

25 OSTERGREN WJ-KREMPL E. ASME/CSME, Pressure Vessels and piping Conference, MONTREAL (Canada) 1978.