Analysis and modelling of engineering structures in frequency domain

K.Ishtev
Department of Automatics and Systemotechniques, High Electro-Mechanical Institute 'Lenin', Sofia, Bulgaria
Z.Bonev
Department of Structural Mechanics, High Institute of Civil Engineering, Sofia, Bulgaria
P.Petrov
Central Laboratory of Seismic Mechanics and Earthquake Engineering, Bulgarian Academy of Sciences, Sofia, Bulgaria
P.Philipov
Sofia, Bulgaria

1 INTRODUCTION.Analysis and modelling of engineering stru ctures often requires a large number of unknown parameters and thus a lot of computational efforts well known from the literature are needed.We shall point out only a few of them such that a large amount of high speed storage required, quite a long computational time, numerical stbility problems associated with the numerical responses.Sometimes, when dynamical behaviour of a discretized structure is studied some of displacements for instance are not of interest but others are at the some time impor tant to make the final conclusions.A commanly used practi ce from this point of view is specifying a number of para meters as master which are tracing the motion of the stru cture.All of the rest parameters ar expressed in impli cit form and are specified as slave.It is proposed [1] an algorithm which was used in order to eleminate slave para meters.The most important advantage of this technique is that the number of equations is significantly reduced as a result of elemination.Also, elemination phase does not require further simplifications associated with neglec ting of inertial forces thus it is performed without loss of accuracy.However that leads to rising the order of ti me derivatives in master equations.

This paper deals whith some possible aplications for modelling and analysis of engineering structures, basing on technique, mentioned above.The governing system of equations is written by using frequency domain approach since elemination technique has computational signifi cance in this field.Modelling is made basing on the well known relationship Y(jw)=W(jw)*X(jw).Here X(jw) is a com plex Fourer spectra associated with the imput signals being defined as earthquake, wind, hydrodynamic, control or other type of action.W(jw) is a matrix complex transfer function which reveal the correlation between imput X and output Y spectra.Y(jw) represents a complex Fourier spectra of output signals.Input and output signals are both associated with master degrees of freedom, thus matrix transfer function is composed of elements in such a manner that slove unknown parameters are implemented implicitly.It is available an integration algorithm of "condensed" system of equations.It is also shown that computational time needed for integration phase does not depend significantly on the order of equations, but is depening on the number of equations.The efficiency of that approach can be expressed in decreasing of high speed storage becouse of the reduced number of equations.Another merit is decreasing of the total amount of computational time needed for the processing.In particular, of course, we can evaluate some of the properties of the structure having matrix transfer functions as an intermediate product of computations.

2.MODELLING AND ANALYSIS OF ALGORITHMS.The dynamical behaviour of engineering structurs can be studied by using a system of differential equations of the type:

$$A_\ell \frac{d^\ell y^*}{dt^\ell} + \cdots\cdots + A_1 \frac{dy^*}{dt} + A_o y^* = B_m \frac{d^m X}{dt^m} + .. + B_o X \quad (1)$$

where Y* is the displacement vector associated with master degrees of freedom.X represnts the input signal vector.Equation (1) can be obtained starting from the initial system of equations in which elemination is not yet performed:

$$M.\frac{d^2y}{dt^2} + C\frac{dy}{dt} + K.y = - M.R.\ddot{X} \quad (2)$$

It is worth to note, that both descriptions (1) and (2) have the same order of accuracy and do not require neglecting of certain inertial effects.However the general distinction of input and output signals in (1) is higher than the order of derivatives in (2).Another difference between (1) and (2) is that the number of equations in (1) is reduced, respectivly the dimensions of the matrices Ai and Bi.

If we apply Laplace transform over the system of equations (1), then we shall obtain the following expression

$$A(p).y^*(p) - A_{i.c.}(p) = B(p).X(p) \quad (3)$$

where A(p) and B(p) are matrices of polinomial type:

$$\begin{aligned} A(p) &= A_o + p.A_1 + \cdots\cdots + p^\ell.A_\ell \\ B(p) &= B_o + p.B_1 + \cdots\cdots + p^m.B_m \end{aligned} \quad (4)$$

and vector Aic(p) should be determined from the initial conditions.It is easy to obtain Fourier transform of (1) from equation (3) on substituting p by jw:

$$A(j\omega).y^*(j\omega) = B(j\omega).X(j\omega) + A_{i.c.}(j\omega) \quad (5)$$

After left matrix multiplication of both sides of equation (5) by the inverse matrix A(jw)-1 we obtain:

$$Y^*(j\omega) = A^{-1}(j\omega).B(j\omega).X(j\omega) + A^{-1}(j\omega).A_{i.c.}(j\omega) \quad (6)$$

The matrix:

$$W(j\omega) = A^{-1}(j\omega).B(j\omega) \quad (7)$$

represents a frequency transfer function on the structure.

The process of modelling can be done also without searching for inverce matrix of A(jw).We note, that in equation (5) unknown is only the vector Y*(jw).The right side of this equality can previously be determined and as a result of that it will become a complex vector, such that:

$$A(j\omega).Y^*(j\omega) = C(j\omega) \quad (8)$$

If we rearrange real and imaginery parts of complex system of equations (8) into adjacent matrix blocks, we obtain a systm of equations but coefficients are real, so that:

$$\left[\begin{array}{c|c} A_r & -A_i \\ \hline A_i & A_r \end{array}\right] . \left[\begin{array}{c} Y_r^* \\ \hline Y_i^* \end{array}\right] = \left[\begin{array}{c} C_r \\ \hline C_i \end{array}\right] \quad (9)$$

where Ar, Y*r, Cr, Ai, Y*i, Ci are real and imaginery parts of the matrix A(jw) and vectors Y*(jw) and C(jw) respectively.Thus the problem of determination of output signals spectra leads to a standard problem of solving system of linear equations.

Similarly, we can determine matrix transfer functions of a structure; According to (7) that could be done by solving a system of equations:

$$A(j\omega).W(j\omega) = B(j\omega) \quad (10)$$

or

$$\left[\begin{array}{c|c} A_r & -A_i \\ \hline A_i & A_r \end{array}\right] . \left[\begin{array}{c} U \\ \hline V \end{array}\right] = \left[\begin{array}{c} B_r \\ \hline B_i \end{array}\right] \quad (11)$$

where U and V are real and imaginery transfer functions and Br and Bi are similarly real and imaginery part of the matrix B(jw).

3.THE STRATEGY OF THE ALGORITHMS.There exists a technique called "dynamic condensation" which is used in order to provide "condensed" description (1) instead of expanded description defined by (2).Some condensation algorithms were discussed in [2].

From the previously obtained metrices of condensed description Ai, i=0,...,l and Bi, i=0,...,m is possible to determine complex matrices A(jw) and B(jw):

$$A(j\omega) = A_r(\omega) + jA_i(\omega) = A_0 + j\omega . A_1 + \ldots\ldots + (j\omega)^{\ell} . A_{\ell}$$
$$B(j\omega) = B_r(\omega) + jB_i(\omega) = B_0 + j\omega . B_1 + \ldots\ldots + (j\omega)^{m} . B_m \quad (12)$$

To do that it is expedient to apply Horner`s scheme according to which further compupapions for instance for Ar and Ai can be transformed as follows:

$$A_r(\omega) = A_0 - \omega^2(A_2 - \omega^2(A_4 - \ldots - \omega^2(A_{n-2} - \omega^2 A_n)\ldots))$$
$$A_i(\omega) = \omega(A_1 - \omega^2(A_3 - \ldots - \omega^2(A_{n-3} - \omega^2 A_{n-1})\ldots)) \quad (13)$$

Horner`s rule, as could be seen from current computations tends to decreasing of computational time.Moreover, that rule is appropriate in order to avoid overflowing or loss of accurancy thus dirrect computations using (12) are recommended.

Matrix transfer functions are obtained as a result of solution of system (10) or (11).In case, when an equations solver with complex coefficients is available, then the use of (10) is resonable since matrix dimensions in (10) are two times smaller than the matrix dimensions in (11).

The response of structure can be determined basing on the equations (8) or (9).In connection with that the spectra of input signals are needed X(jw) by introducing input histories X(t) and then, after mapping output spectra Y*(jw) - next step is obtainin of desired time history of the output signal Y*(t).To achieve that some efficient cmputer programs can be employed - Fast Fourier Transform Development (Winograd`s algorithm [3] or others).

It can be concluded from the above consideration, assuming a linear system of equations, that digital operations associated with Fourier transform and condensation tecchnique will be fulfilled not more than one time.The process of computing complex matrices in corresponding equations and solving these equations should be repeated for each values of the frequency.That means computational time will generally depend on the operations mentioned above.Rising the order of derivatives yields to adding new terms when matrices A(jw) and B(jw) are determined - see formulae (12) and (13).In other words updating of existing matrices requires multiplication of matrices by a scalar and then consequent adding this result to the initial one.The general point here is that rising the order of derivatives does not strongly affect the global computatinal time.The effect of condensation thus can be found in reduced number of equtions, so that the efficiency of dynamic condensation (reducing of number of equations) is due to the relatively small final number of equations.

4.SOME APPLICATIONS.Dynamical properties of engineering and geological structures are reflecting in frequency transfer functions.Analysis of these functions shows frquencies which will be amplified and which will be suprssed [4].Here will be considered an shaking table (see figure 1) as an example to illustrate dynamic condensation attached to frequency domain approach.The existing situation is that the model is installed on the table and it must be subjected to a test signal at the common joints.Due to the flexibility of the table, the external signal U (as input) and desired test signal are not identical.

Theoretically both signals will be identical when shaking table is ideally rigid and does not allow any deformability.

Other situations, alternatively, impose using a "soft" table when some additional effects can be included - for example soil structure interaction.Rises the problem how to choose external control action, supplied by activator in such a way to immitate desired action at the junction between the model and the table.This problem can successfully be solved if we determine the matrix transfer function of the coupled system model - table and if we specify master unknown parameters Wxy(jw) to be located at the boundary between two parts.Then from the lelation:

$$X(j\omega) = W(j\omega).u(j\omega) \tag{14}$$

can be determined spectrum of the external signal u comming from the activator, so that spectrum needed in the master nodes is already obtained:

$$u(j\omega) = \left[W(j\omega)\right]^{-1}. X(j\omega) \tag{15}$$

The next step is to apply Fourier`s transform in order to determine the actual time domain history of input signal.

This manner can be expanded when we need to compute spectral density of control action Su(w) when desired spectral density Sx(w) of signal must be achived.We can do that by using the relation:

$$S_x(\omega) = \Psi(\omega).\, S_u(\omega) \tag{16}$$

where the elemnts of the matrix Ψ(jw)

$$\Psi_{ij}(\omega) = |W_{ij}(j\omega)|^2 \tag{17}$$

and Wij is i,j-th element of matrix transfer function.

Needed spectral density of the control signal can be obtained as follows:

$$S_u(\omega) = \Psi(\omega)^{-1}.\, S_x(\omega) \tag{18}$$

Note, that having spectral density (18) allows to generate an arbitrary number of random actions.That can easyly be done by applying [5,6].

5.REFERENCES:

[1].ISHTEV K.,Z.BONEV,PH.PHILIPOV,Linear Moddeling Using Dynamicl Condensation,SMiRT8,Brussels, Belgium,1985.
[2].ISHTEV K.,P.PETROV,Z.BONEV,PH.PHILIPOV,Structural Modelling of Systems throught Dynamic Condensation Technique,ICCM86,Tokyo,Japan,1986.
[3].Mc CLELLAN J.,CH.RADER,Number Theory in Digital Signal Processing,New Jersy,Prentice-Hall,1979.
[4].ISHTEV K.,L.HADJIKOV,P.DINEVA,PH.PHILIPOV,Solving Some Problems of Engineering Seismology by Structural Method,SMiRT7,Chicago,Illinois,1983.
[5].БЫКОВ В.В.,Цифровое Моделирование в Статистической радиотехнике,Сов.Радио,Москва,1971.
[6]САЧАНСКИ С.,К.ИЩЕВ,В.БОЖИЛОВ,Ф.ФИЛИПОВ,Генериране на сеизмични сигнали,Четвърти Национален Конгрес по Теоретична и Приложна Механика,Варна,1981.

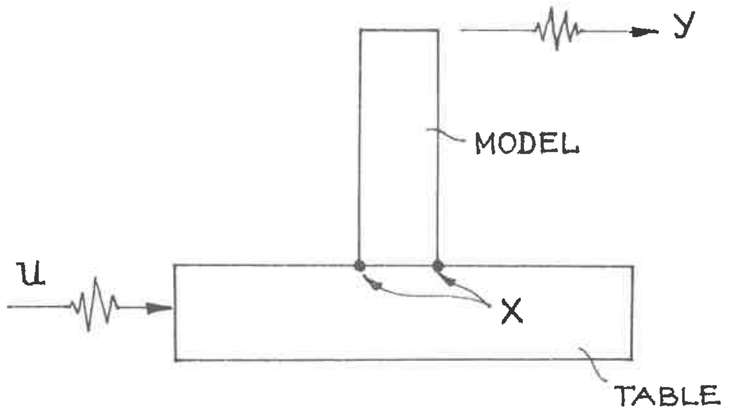

Fig. 1