A THEORETICAL ANALYSIS OF TWO-PHASE FLOW/FUEL PIN STRUCTURAL DYNAMICAL INTERACTIONS

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Fuel pin system vibrations have been intensively investigated to obtain knowledge useful in the design of fuel pin assemblies. Pressurized-tube reactors have also motivated research on two-phase-flow-induced vibrations in fuel pin systems. Several papers on experimental results of two-phase-flow-induced vibrations in fuel pin systems are available at present; however, due to the complexity of the two-phase flow, little theoretical work has been done on the problem.

This paper deals with a theoretical analysis of dynamical interactions between an air-water two-phase flow and a fuel pin structure for the case of slug flow, where air- and water slugs travel alternately along the pin or large bubble flow and where the two-phase-flow-induced vibrations become extraordinarily violent. The equation of motion of a single elastic rod, considered as the model of a fuel pin structure and inserted parallel to the two-phase flow, is formulated by accounting for 1) the rod's elastic restoring force, 2) the two-phase-fluid's centrifugal force, 3) gravitation, 4) Coriolis' force, 5) momentum change of the two-phase flow due to mass fluctuation, 6) the rod's inertial force, 7) inertial force due to the two-phase-flow's virtual mass, and 8) the pressure difference between both sides of the rod surface as an external force to the vibration system. The equation of the j-th mode vibration is obtained by applying Galerkin's method, including the effect of dynamical interactions between the two-phase flow and the rod. Dynamical interactions appear in the items 2), 4), 5), 7), and 8), above, however the effect of momentum change, 5), on dynamical interactions vanishes if the two-phase flow is assumed to be a non-mixing one.

The four above effects are theoretically evaluated by employing the following mathematical model for the two-phase-flow virtual mass distribution:

$$m(x,t) = m_0 + m_1 \sum_{n=1} \left\{ a_n \sin 2\pi n\left(\frac{t}{T} - \frac{x}{\lambda}\right) + b_n \cos 2\pi n\left(\frac{t}{T} - \frac{x}{\lambda}\right) \right\},$$

where T: arrival period of the water slug in the two-phase flow, and λ: distance between each two water slugs. Quantitative evaluation of the dynamical interactions shows the following major results: 1) effects of virtual mass and centrifugal force become strongest under the condition $2ln/\lambda = i + j$, or $\pm (j - i)$; l = rod length, i and j = intergers indicating i-th and j-th modes, 2) Coriolis's force magnitude shows almost the same feature as the above two quantities; however, its sign is opposite to those above, and 3) differential pressure effect becomes maximum at $2ln/\lambda = j$.

1. INTRODUCTION

The importance of parallel two-phase-flow-induced vibrations has been recognized in the design of vibration prevention devices in fuel pin systems. However, the very complex features of the two-phase flow have led, on the one hand, to a remarkable difference[1] between experimental results obtained up to the present and results from empirical formulas. A certain discrepancy also exists among those few experimental results[1~4], including the author's[4], depending on experimental conditions and fuel pin bundle geometry. On the other hand, even worse is the fact that few theoretical works on fuel pin vibration problems are available because of the great difficulty in mathematically formulating two-phase flow/structure interactions. Although the author[4] showed theoretical results about excitation mechanisms of vibrations induced by a parallel two-phase flow in a single fuel pin model, his theory can be used to evaluate nothing about vibration amplitudes. More complete information useful to designers is needed for evaluating vibration amplitudes under such conditions.

This paper confines itself to a theoretical study of dynamical interactions between a two-phase flow and a fuel pin in a slug flow, in which gas- and liquid-slugs travel alternately along a fuel pin or in a large bubble flow, and where two-phase-flow-induced vibrations become extraordinarily violent[4]. The equation of motion for a single elastic rod, considered as the model of a fuel pin structure and inserted parallel to the two-phase flow, is formulated by accounting for 1) the rod's elastic restoring force, 2) the two-phase fluid's centrifugal force, 3) gravity force, 4) Coriolis' force, 5) momentum change of the two-phase flow due to virtual mass fluctuation, 6) the rod's inertial force, 7) inertial force caused by the two-phase flow's virtual mass, and 8) pressure difference between the two sides of the rod surface as an external force. Dynamical interactions appear in items 2), 4), 7), and 8), above. The four effects are theoretically evaluated by expressing virtual mass distribution along the rod through a Fourier progressive wave series, showing that 1) effects of virtual mass' inertial force and centrifugal force become strongest under the conditon $2ln/\lambda = i + j$, or $\pm(i - j)$, 2) Coriolis' force magnitude shows almost the same feature as the above two quantities, but its sign is the reverse of those above, and 3) the differential pressure effect becomes maximum at $2nl/\lambda = j$, where i and j mean i-th and j-th vibration modes, respectively.

2. BACKGROUND KNOWLEDGE ON TWO-PHASE FLOW PATTERNS

2.1 Steam-Water Two-Phase Flow Patterns

Two reports on flow patterns in a high pressure steam-water two-phase flow within a vertical rod bundle are now available: Bergles et al.[5] showed the existence of bubble, slug, and annular flow patterns in a square channel consisting of 4 rods(12,7 mm OD) arranged in a pattern with a 17.8 mm square pitch and a 5,8 mm spacing between the shrouds and rods. The rods were adiabatically heated and pressure was set at $68{,}7 \times 10^5$ Pa. Steam and water were premixed and allowed an additional development length (1.2 m) before reaching the rods. Williams and Peterson[6] recently presented different experimental results: with 28×10^5 Pa pressure, flow configurations were bubble, slug, froth and annular flows in the bundle, which consisted of 4 vertical rods (6,35 mm OD) arranged in a single row at a 8.6 mm pitch, but under 67.8×10^5 Pa, no slug flow was observed. In hteir experiments, rods (600 mm long) were uniformly heated.

Depending on heating conditions and rod bundle geometry, a steam-water two-phase flow might well take different configurations.

2.2 Void-Signal/Pressure Correlation

A void-signal, expressing the gas or liquid in a two-phase flow, rather strongly correlates with slug flow pressure fluctuation[7]. Fig.1 shows, for example, a similarlity in power spectral density distribution shape for void-signals and for pressure fluctuation in a vertical slug flow. It might be said, however, that void-signal and pressure fluctuation are poorly correlated for other patterns such as bubble, froth, or annular flows.

3. FUNDAMENTAL EQUATIONS

To investigate the dynamical interactions between a two-phase flow and a fuel pin, that is, the influences of a two-phase flow on vibrational characteristics of a fuel pin system, the vibration equation of a straight rod inserted parallel to a two-phase flow is formulated, making the following assumptions:

1) the two-phase fluid flows uniformly along the rod with a velocity U,
2) the influences of viscosity and steady pressure of the two-phase flow are neglected,
3) the rod's motion is represented as an Euler type, having a rectangular cross section,
4) both ends of the rod are simply supported.

First, considered is the small lateral motion of the rod about its position of rest, which is taken to coincide with x-axis. The forces acting on a small element, dx, of the rod, which has undergone a small lateral displacement y(x,t), are shown in Fig.2. Force balances in the x- and y- directions are

$$\partial T/\partial x - Mg = 0 \quad \text{----}(1) \qquad \partial Q/\partial x + F_H + \partial(T\,\partial y/\partial x)/\partial x - B(P_1 - P_2) - M\,\partial^2 y/\partial t^2 = 0 \quad \text{-----}(2)$$

where T = tensile force; M = mass of the rod per unit length, Q = shear force, F_H = inviscid hydrodynamic force, P_1 and P_2 = pressure, and B = rod width.

The inviscid hydrodynamic force acting on the rod, F_H, is given by

$$F_H = (\partial/\partial t + U\,\partial/\partial x)\left[m(\partial y/\partial t + U\,\partial y/\partial x)\right] \quad \text{--------}(3)$$

as proposed by Lighthill . In eq.(3), m(x,t) is the virtual mass distribution of the two-phase flow. From assumption 3) the shear force is described as

$$Q = -\partial(EI\,\partial^2 y/\partial x^2)/\partial x \quad \text{--------}(4)$$

Integrating eq.(1) and substituting eqs.(3) and (4) into eq.(2), we obtain the equation of motion for a rod undergoing small lateral vibrations;

$$EI\frac{\partial^4 y}{\partial x^4} + mU^2\frac{\partial^2 y}{\partial x^2} - Mgx\frac{\partial^2 y}{\partial x^2} - Mg\frac{\partial y}{\partial x} + 2mU\frac{\partial^2 y}{\partial x \partial t} + \left(\frac{\partial m}{\partial t} + U\frac{\partial m}{\partial x}\right)\left(\frac{\partial y}{\partial t} + U\frac{\partial y}{\partial x}\right) + (M+m)\frac{\partial^2 y}{\partial t^2} = B(P_2 - P_1) \quad \text{----}(5)$$

In eq.(5) the 1st term on the left is the rod's restoring force, the 2nd the centrifugal force of the two-phase fluid, the 3rd and 4th gravitation force effects, the 5th Coriolis' force, the 6th the effect of two-phase flow momentum change, and the 7th the inertial force due to the rod mass and the two-phase flow virtual mass. The right hand side expresses an external force caused by pressure differences between the two sides of the rod surface.

From assumption 4) displacement y can be expanded to

$$y = \sum_{i=1} Y_i(t)\sin(i\pi x/l) \quad \text{--------}(6)$$

where $\sin(i\pi x/l)$ is the i-th vibration mode function of a straight rod, both ends of which are simply supported. Substituting eq.(6) into eq.(5) and applying the Galerkin's method, the equation of motion, eq.(5), can be described in the form of a system of ordinary differential equations:

$$EI\left(\frac{j\pi}{\ell}\right)^4 Y_j + \sum_{i=1}^{\infty}\left(\frac{i\pi}{\ell}\right)^2\left\{Mg X_{ji} - U^2 m_{ji}\right\}Y_i + 2U\sum_{i=1}^{\infty}\left(\frac{i\pi}{\ell}\right)m_{j\bar{i}}\dot{Y}_i + \sum_{i=1}^{\infty}\left\{m_{ji}^{(\cdot)} + U m_{ji}^{(\prime)}\right\}\dot{Y}_i +$$
$$+ U\sum_{i=1}^{\infty}\left(\frac{i\pi}{\ell}\right)\left\{m_{j\bar{i}}^{(\cdot)} + U m_{j\bar{i}}^{(\prime)}\right\}Y_i + M\ddot{Y}_j + \sum_{i=1}^{\infty} m_{ji}\ddot{Y}_i = P_j \quad (j = 1, 2, \cdots) \text{ -----(7)}$$

Here, the following quantities express the effects of dynamical interactions between the two-phase flow and rod structure;

$$m_{ji}(t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\, m(x,t)\sin\frac{i\pi x}{\ell}dx, \quad m_{j\bar{i}}(t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\, m(x,t)\cos\frac{i\pi x}{\ell}dx, \quad m_{ji}^{(\prime)}(t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\frac{\partial m(x,t)}{\partial x}\sin\frac{i\pi x}{\ell}dx,$$
$$m_{ji}^{(\cdot)}(t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\frac{\partial m(x,t)}{\partial t}\sin\frac{i\pi x}{\ell}dx, \quad m_{j\bar{i}}^{(\cdot)}(t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\frac{\partial m(x,t)}{\partial t}\cos\frac{i\pi x}{\ell}dx, \quad m_{j\bar{i}}^{(\prime)}(t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\frac{\partial m(x,t)}{\partial x}\cos\frac{i\pi x}{\ell}dx, \text{ -----(8)}$$
$$X_{ji} \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\, x\sin\frac{i\pi x}{\ell}dx, \quad P_j(t) \equiv \frac{2B}{\ell}\int_0^{\ell}(P_2 - P_1)\sin\frac{j\pi x}{\ell}dx.$$

Thus, what is needed is mathematical expression of the two-phase flow virtual mass distribution and pressure distribution for evaluating eq.(8).

4. MATHEMATICAL TWO-PHASE FLOW MODEL

Because two-phase flow configurations differ quite markedly from bubbleflow, slug flow, or annular flow and will be necessary to build different mathematical models for different flow patterns. For a fine bubble flow it might be possible to adopt a uniform flow (or single phase flow) model; for annular flow a separated or stratified flow model would be applicable. However, flow configurations differ greatly from the above for slug flow or large bubble flow. Photo 1 depicts a typical slug flow, showing that gas and liquid flow alternately. Almost the same can be said for large bubble flow.

For slug and large bubble flows, it is assumed that the gas and liquid phases travel alternately along the rod with a constant velocity of U. Thus, a mathematical model of two-phase flow virtual mass distribution can be proposed

$$m(x,t) = m_0 + m_1 f\left(\frac{t}{T} - \frac{x}{\lambda}\right) \;;\; \lambda = UT \text{ --(9)}$$

where m_0 is the average value of the virtual mass per unit rod length, m_1 the virtual mass deviation , andfunction $f(t/T - x/\lambda)$ expresses a traveling wave in the flow direction. The function f is here assumed to be periodic, although f might be, in reality, a random function with a dominant period T. Expanding the function f by progressive plane wave $\sin(2\pi n(t/T - x/\lambda))$ and $\cos(2\pi n(t/T - x/\lambda))$, eq.(9) is rewritten

$$f\left(\frac{t}{T} - \frac{x}{\lambda}\right) = \sum_{n=1}^{\infty}\left\{a_n \sin 2\pi n\left(\frac{t}{T} - \frac{x}{\lambda}\right) + b_n \cos 2\pi n\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right\} \text{ --(10)}$$

The two-phase flow pressure for our cases might strongly correlate with the alternate traveling of both phase fluids, refered to Fig.1. Consequently. it might be said that the pressure differences between the two sides of the rod are mathematically written in the same manner as in eq.(10):

$$(P_2 - P_1) = K m_1 f(t/T_p - x/\lambda_p) \text{ --(11)}$$

where T_p and λ_p are the period of pressure fluctuation and its wave length.

5. EVALUATION OF TWO-PHASE FLOW/STRUCTURE DYNAMICAL INTERACTIONS

Substituting eqs.(10) and (11) into eq.(8), there can be obtained the concrete form of each quantity, m_{ji}, $m_{j\bar{i}}$, ----- , x_{ji}, or P_j. Before going into the theoretical evaluation, it should be noted that the following two relations between $m_{ji}^{(\cdot)}$ and $m_{ji}^{(\prime)}$, or between $m_{j\bar{i}}^{(\cdot)}$ and

$m_{j\bar{i}}^{(\prime)}$ always hold for a case when both fluids would not mix while traveling along the rod:

$$m_{ji}^{(\cdot)} + U m_{ji}^{(\prime)} = 0, \qquad m_{j\bar{i}}^{(\cdot)} + U m_{j\bar{i}}^{(\prime)} = 0 \qquad \text{(12)}$$

Thus, the 4th and 5th terms in eq.(7) vanish for a large bubble or slug flow. The influence of the two-phase fluid virtual mass appear in terms m_{ji} and $m_{j\bar{i}}$. The concrete form of the quantities m_{ji} and $m_{j\bar{i}}$ are then written:

$$m_{ji}(t) = m_0\delta_{ji} + m_1\sum_{n=1}^{\infty}\{a_n I_s^{ji}(n,t) + b_n I_c^{ji}(n,t)\}, \quad m_{j\bar{i}}(t) = m_1\sum_{n=1}^{\infty}\{a_n II_s^{ji}(n,t) + b_n II_c^{ji}(n,t)\} \qquad \text{(13)}$$

where I_s^{ji}, I_c^{ji}, II_s^{ji}, and II_c^{ji} are defined by the following integrations:

$$I_s^{ji}(n,t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\sin\frac{i\pi x}{\ell}\sin 2\pi n\left(\frac{t}{T}-\frac{x}{\lambda}\right)dx, \quad I_c^{ji}(n,t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\sin\frac{i\pi x}{\ell}\cos 2\pi n\left(\frac{t}{T}-\frac{x}{\lambda}\right)dx,$$
$$II_s^{ji}(n,t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\cos\frac{i\pi x}{\ell}\sin 2\pi n\left(\frac{t}{T}-\frac{x}{\lambda}\right)dx, \quad II_c^{ji}(n,t) \equiv \frac{2}{\ell}\int_0^{\ell}\sin\frac{j\pi x}{\ell}\cos\frac{i\pi x}{\ell}\cos 2\pi n\left(\frac{t}{T}-\frac{x}{\lambda}\right)dx \qquad \text{(14)}$$

The integrations in eq.(14) can be easily expressed when amplitudes A_n^{ji}, B_n^{ji} and phase angle θ_n are introduced and evaluated by the following equations:

$$A_n^{ji} \equiv \frac{1}{\sqrt{2}\pi\ell}\left\{\frac{1}{\frac{(j+i)}{\ell}+\frac{2n}{\lambda}} - \frac{1}{\frac{(j+i)}{\ell}-\frac{2n}{\lambda}} - \frac{1}{\frac{(j-i)}{\ell}+\frac{2n}{\lambda}} + \frac{1}{\frac{(j-i)}{\ell}-\frac{2n}{\lambda}}\right\}\sqrt{1-(-1)^{j+i}\cos\frac{2n\pi\ell}{\lambda}}$$
$$B_n^{ji} \equiv \frac{1}{\sqrt{2}\pi\ell}\left\{\frac{1}{\frac{(j+i)}{\ell}+\frac{2n}{\lambda}} + \frac{1}{\frac{(j+i)}{\ell}-\frac{2n}{\lambda}} + \frac{1}{\frac{(j-i)}{\ell}+\frac{2n}{\lambda}} + \frac{1}{\frac{(j-i)}{\ell}-\frac{2n}{\lambda}}\right\}\sqrt{1-(-1)^{j+i}\cos\frac{2n\pi\ell}{\lambda}} \qquad \text{(15)}$$
$$\tan\theta_n \equiv (-1)^{j+i}\sin\frac{2\pi n\ell}{\lambda}\Big/\left\{1-(-1)^{j+i}\cos\frac{2\pi n\ell}{\lambda}\right\}$$

Thus, I_s^{ji}, I_c^{ji}, II_s^{ji}, and II_c^{ji} are written using the simple functions $\cos(2\pi nt/T + \theta_n)$, $\sin(2\pi nt/T + \theta_n)$, and $\cos(2\pi nt/T - \theta_n)$, as follows:

$$I_s^{ji}(n,t) = A_n^{ji}\cos\left(\frac{2\pi nt}{T}+\theta_n\right), \qquad I_c^{ji}(n,t) = -A_n^{ji}\sin\left(\frac{2\pi nt}{T}+\theta_n\right)$$
$$II_s^{ji}(n,t) = B_n^{ji}\sin\left(\frac{2\pi nt}{T}+\theta_n\right), \qquad II_c^{ji}(n,t) = B_n^{ji}\cos\left(\frac{2\pi nt}{T}-\theta_n\right) \qquad \text{(16)}$$

Next calculated is x_{ji}. From a simple computation of an integral in eq.(8),

$$X_{ji} = -\frac{\pi^2}{\ell}\sum_{j\neq i,\, j-i=\text{even}}\left\{\left(\frac{i}{j-i}\right)^2 - \left(\frac{i}{j+i}\right)^2\right\} \qquad \text{(17)}$$

Finally, the pressure effect on the j-th vibration mode P_j can be expressed as

$$P_j \equiv K m_1\sum_{n=1}^{\infty}\{a_n J_s^j(n,t) + b_n J_c^j(n,t)\} \qquad \text{(18)}$$

where J_s^j and J_c^j are easily calculated by using $\sin(2\pi nt/T + \theta_n)$ and $\cos(2\pi nt/T + \theta_n)$ as

$$J_s^j(n,t) = C_n^j\sin\left(\frac{2\pi nt}{T_p}+\theta_n\right), \qquad J_c^j(n,t) = C_n^j\cos\left(\frac{2\pi nt}{T_p}+\theta_n\right) \qquad \text{(19)}$$

Here, amplitude C_n^j is defined by the eq.(20):

$$C_n^j \equiv \frac{\sqrt{2}}{\pi\ell}\left\{\frac{1}{j/\ell + 2n/\lambda_p} + \frac{1}{j/\ell - 2n/\lambda_p}\right\}\sqrt{1-(-1)^j\cos\frac{2\pi n\ell}{\lambda_p}} \qquad \text{(20)}$$

In eq.(20) the phase angle is equal to that in eq.(15) with i = 0.

Quantities A_n^{ji}, B_n^{ji}, C_n^j, and θ_n are the functions of the integers i, j, and wave length $\tilde{\lambda}$, and the rod length l, in which $\tilde{\lambda} = \lambda/n$. The two-phase flow centrifugal and inertial forces can be evaluated through m_{ji}, the Coriolis'force through $m_{j\bar{i}}$, and pressure fluctuation through P_j. These four forces represent the two-phase flow/structure dynamical interactions for the case of fuel pin vibrations in parallel gas-and liquid two-phase flow.

6. NUMERICAL RESULTS AND DISCUSSION

Fig.3 shows the absolute values of A^{ji} against the integer i, i-th vibration mode, with parameters l = 100cm, $\tilde{\lambda}$ = 80 cm, and j = 1, 3, 5. The solid symbols stand for positive value and blank symbols for negative value. The amplitude A^{ji} has one or two maximum values, which correspond to the general conditions: $j + i = 2l/\tilde{\lambda}$, or $j - i = \pm 2l/\tilde{\lambda}$. Speaking more precisely in the case of j = 1, the maximum value appears near the values i = 1 and i = 3, for j = 3, near the values i = 1 and 5, and for j = 5, i = 3 and 7. Thus for A^{ji}, only two of the relations shown above hold.

Fig.4 shows the absolute value distribution of B^{ji} against the integer i with the same parameters as in Fig.3. Here it should be noted that the sign of B^{ji} is exactly opposite to that of A^{ji}, meaning that when A^{ji} is positive B^{ji} becomes negative and vice versa. The maximum values of B^{ji} appear also near where the same conditions as shown above hold.

The amplitudes C^{j} is shown in Fig.5 against the value i with the same parameters as above. The peak appears in this figure near where the condition: $j = 2l/\tilde{\lambda}$ holds. Where $\tilde{\lambda}$ = 80 cm, $\cos(2\pi nl/\lambda) = 0$. Then the square root in eq.(20) attains a value of one. For $\tilde{\lambda}$ = 25 cm, $\cos(2\pi nl/\lambda) = 1$, thus for j = even integers, the square root vanishes, for $\tilde{\lambda}$ = 15 cm, the value of the term varies according to j = even or odd integers.

Fig.6 shows the phase angle against $2l/\tilde{\lambda}$ with the conditions j + i = even or odd.

Finally, in Table 1, the assymptotic values of A^{ji}, B^{ji} and C^{j} are shown when the value $2l/\tilde{\lambda}$ approaches the values j + i, j - i, i - j, or j.

In applying these results to two-phase-flow-induced vibrations in fuel pin systems, if knowledge of the two-phase fluid virtual mass distribution -- namely the function f(x,t) -- can be obtained, it is very easy to evaluate a_n and b_n in eq.(10). Then employing numerical values from Figures 3 and 4 for A_n^{ji}, B_n^{ji}, and the phase angle from Figure 6, influences of the two-phase fluid virtual mass on centrifugal force, inertial, and Coriolis' forces can be concretely evaluated. Like the virtual mass, two-phase flow pressure fluctuation can be accounted for with consideration of the vibrations. However more experimental information is definitely needed about the two-phase flow with regard to spacial correlation of pressure fluctuation, virtual mass and damping effect distribution.

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Table 1 Assymptotic values of A^{ji}, B^{ji}, and C^{j} when 21/ approaches j + i, j - i, -j + i, or j (- means that the value approaches them from lower, and + for from higher)

	j + i		j - i		-j + i		j	
$2l/\tilde{\lambda}$	-	+	-	+	-	+	-	+
A^{ji}	1/2	-1/2	-1/2	1/2	1/2	-1/2		
B^{ji}	-1/2	1/2	1/2	-1/2	-1/2	1/2		
C^{j}							-1	1

Photo 1 A typical slug flow in a vertical circular channel

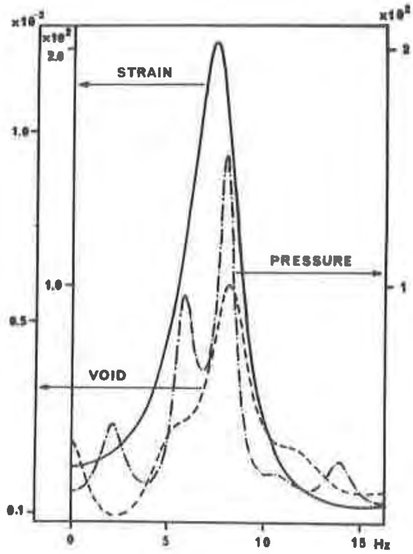

Fig.1 A similarity in power spectral density distribution shape for void-signal and for pressure fluctuation

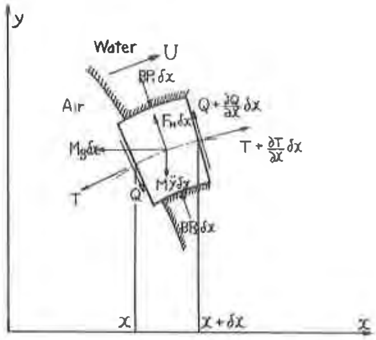

Fig.2 An element dx of rod showing forces acting on it

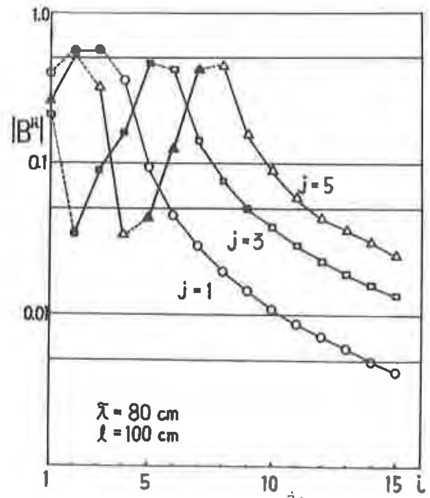

Fig.4 Absolute values of B^{ji} against the integers i, i-th vibration mode for l = 100 cm, $\widetilde{\lambda}$ = 80 cm

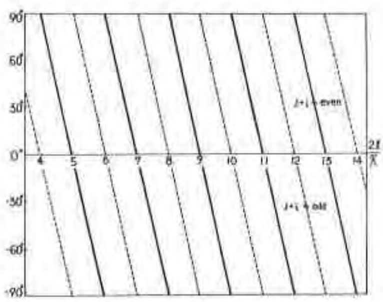

Fig.6 Phase angle θ_n and $2l/\widetilde{\lambda}$ with j + i = even and j + i = odd

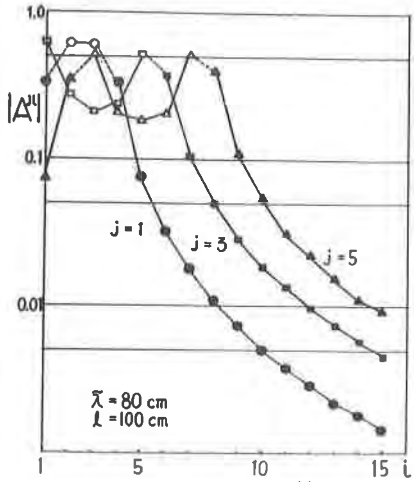

Fig.3 Absolute values of A^{ji} against the integers i, i-th vibration mode for l = 100 cm, $\widetilde{\lambda}$ = 80 cm

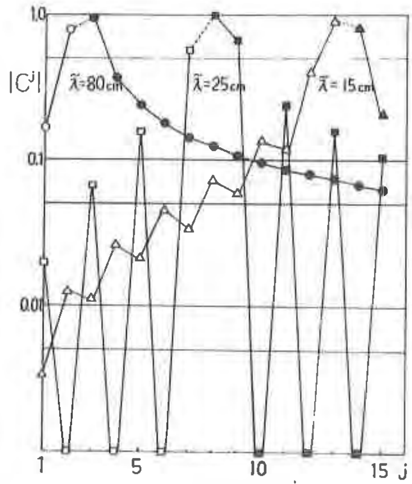

Fig.5 Absolute values of C^j against the integers j, j-th vibration mode